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The Mathematical Theory of Categories in Biology and the Concept of Natural Equivalence in Robert Rosen

Franck VARENNE*

Abstract: The aim of this paper is to describe and analyze the epistemological justification of a proposal initially made by the biomathematician Robert Rosen in 1958. In this theoretical proposal, Rosen suggests using the mathematical concept of "category" and the correlative concept of "natural equivalence" in mathematical modeling applied to living beings. Our questions are the following: According to Rosen, to what extent does the mathematical notion of category give access to more "natural" formalisms in the modeling of living beings? Is the so-called "naturalness" of some kinds of equivalences (which the mathematical notion of category makes it possible to generalize and to put at the forefront) analogous to the naturalness of living systems? Rosen appears to answer "yes" and to ground this transfer of the concept of "natural equivalence" in biology on such an analogy. But this hypothesis, although fertile, remains debatable.
Finally, this paper makes a brief account of the later evolution of Rosen's arguments about this topic. In particular, it sheds light on the new role played by the notion of "category" in his more recent objections to the computational models that have pervaded almost every domain of biology since the 1990s.

Keywords: theory of categories; theoretical biology; Robert Rosen; Nicolas Rashevsky; computational models.

Résumé : *L'objectif de cet article est de rendre compte de la justification épistémologique de la proposition faite, dès 1958, par le biomathématicien Robert Rosen d'introduire le concept mathématique de « catégorie » et celui – corrélatif – d'« équivalence naturelle » dans la modélisation mathématique appliquée au vivant. Nos questions sont les suivantes : en quoi la notion mathématique de catégorie permet-elle, selon Rosen, de donner accès à des formalismes plus « naturels » pour la modélisation du vivant ? La naturalité de certaines équivalences (que la notion mathématique de catégorie sert justement à généraliser et à mettre en évidence) est-elle analogue à la naturalité des systèmes vivants ? Rosen semble faire fond sur cette dernière hypothèse, féconde, mais pourtant discutable.*

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Cet article propose ensuite de mesurer l'évolution des arguments de Rosen à ce sujet, en particulier dans ses conséquences apparemment décisives pour la critique des modèles computationnels du vivant, modèles aujourd'hui en pleine expansion.

Mots-clés : *théorie des catégories ; biologie théorique ; Robert Rosen ; Nicolas Rashevsky ; modèles computationnels.*

Introduction

The mathematical concepts of "category" and "natural equivalence" have recently been closely linked. A part of theoretical biology has been rapidly overtaken by the mathematical concept of category, with the precise aim of defending the naturalness of a particular form of mathematical modeling of theoretical type in biology,[1] and of fighting against what was seen as the artificial nature of modeling by calculation automata, and, more broadly, computer modeling, which had already begun its development in biochemistry, in physiology (metabolism), and in developmental biology (morphogenesis). In this, for some theoretical biologists, it was already a question of confronting the heart of the ontological and epistemological assumptions that implicitly underpin computational approaches to formalized and quantitative biology.

1 - In our specific context, and leaving aside the many characterizations and often competing epistemic functions of models, such as description, prediction, explanation, data reduction, and so forth, I will materially characterize a formal "model" as any kind of formal construct presenting a form of unity, simplicity, and homogeneity. For a review, see Jean-Marie Legay, *L'Expérience et le modèle* (Paris: INRA, 1997) and Franck Varenne, "Fragmenter les modèles: Simulation numérique et simulation informatique," in *Biologie du xxie siècle: Évolution des concepts fondateurs*, ed. Paul-Antoine Miquel (Brussels: De Boeck, 2008), 265–295. A glance at the literature on formal modeling—see Franck Varenne, "What Does a Computer Simulation Prove? The Case of Plant Modeling at CIRAD" in *Simulation in industry'2001: 13th European Simulation Symposium*, ed. Norbert Giambiasi and Claudia Frydman (Ghent, Belgium: SCS Europe, 2001), 549–554—quickly shows that even if, materially, two models may be similar (developing an identical mathematical formalism, for example), their epistemic roles may vary depending on the degree and variety of ontological commitment they enjoy. We qualify a model as theoretical when it refuses to be only predictive or phenomenological (descriptive at the level of observables) or even mechanistic (having only local interpretation and applicability) and makes no claim to directly formulate laws, fundamental axioms, or rules of formal deduction valid for a whole field of reality, as theoretical biologists believe that a theory of life will eventually do, but when it commits itself nonetheless ontologically at this level and claims to provide a first approximation of such principles or such laws for the entire domain. The morphogenetic models proposed by René Thom are of this type, for example. See René Thom, *Structural Stability and Morphogenesis* (Reading, MA: W. A. Benjamin Inc., 1975).

For the philosophy of biology in general and the epistemology of models in particular, as well as for the epistemology of what is known today as "computational biology," this attempt to implement the concepts of the mathematical theory of categories therefore merits wider understanding and discussion. Long before contemporary revivals (notably due to the newly favorable context of postgenomic and neosystemic biology) and the ever-developing trajectory of *mathematized theoretical biology* (which it might be more appropriate to term, more modestly, *mathematized conceptual biology*) occurring since the 1950s, a number of theorists in biology were already attempting to make use of very abstract mathematics indeed. They did so in such a way as to at least make explicit, in contrast, the reductionist constraints imposed by the formalization of the living world in computational systems.[2]

The overall objective of this article, which is primarily historically oriented, is to ask whether this particular critique, on the conceptual level, remains pertinent at a time when computational biology is expanding exponentially and appears to dominate the entire spectrum of modeling in biology. Despite the difficulties it has encountered, could an adaptation of mathematical categories to biology still be a source of inspiration for a mathematized conceptual biology in our time?

We can't hope to achieve this overall goal if we do not first try to reach a more accessible and particular objective, which we shall seek out in this article. It concerns the question of the true nature of this *naturalness* that theorists in biology of the 1950s believed they had recognized in the mathematical concepts of the theory of categories, to the point of seeing therein an ideal for any type of mathematical modeling of theoretical type in biology. What *naturalness* are we actually discussing, when talking about the mathematical

2 - These constraints (and others) are highlighted today in the work of Giuseppe Longo and Francis Bailly, for example. See Francis Bailly and Giuseppe Longo, *Mathematics and the Natural Sciences. The Physical Singularity of Life* (London : Imperial College Press, 2011)); Giuseppe Longo, "Des sciences exactes aux phénomènes du vivant, à partir de Schrödinger: Mathématiques, programme et modèles;" this article was originally published in 2008 in a book that is now out of print, but it has since been reprinted in Franck Varenne and Marc Silberstein, eds., *Modéliser & simuler: Épistémologies et pratiques de la modélisation et de la simulation* (Paris: Éditions Matériologiques, 2013), 1:83–111.

concept of categories, such as it seems to be applied to formal modeling in biology? Does *natural equivalence*, in the mathematical sense, have same nature as a modeling of life deemed "natural" by these theoretical biologists? If not, what is the nature of the difference? We will see, in this case, that the theory of categories could have been more than a repository of formalisms—as is commonly the case in most areas of mathematics when implemented as applied mathematics—and that it might have played a role as a veritable *epistemological infrastructure* for all future modeling in biology.

The opportunity to reflect on the applicability of the mathematical notion of categories to biology will therefore involve illuminating the link (or lack thereof) between mathematical *naturalness* as it is strictly defined in the context of this mathematical theory, and the judgment of greater or lesser *naturalness* which can be made by the modeling biologist on the subject of any given type of mathematical formalization he or she may implement to describe a biological phenomenon.

In this paper, in order to narrow the question to the study of a specific corpus, we shall explore the pioneering work of Robert Rosen (1934–1998), who, in 1958, echoed an article by Samuel Eilenberg and Saunders MacLane.[3] To contextualize events, the first section looks briefly at the evolution of the epistemology of Nicolas Rashevsky (1899–1972), Rosen's colleague and predecessor in theoretical biology. The second section presents Rosen's first approach, based on notions of systems and graphs. The third section provides a review of some of the concepts of the theory of categories. The fourth section explains how Rosen came via these to his second approach, precisely that which brings into play the concepts of the mathematical theory of categories and the notion of natural equivalence. The final section presents more recent developments of Rosen's thought on these issues.

3 - Samuel Eilenberg and Saunders MacLane, "General Theory of Natural Equivalences," *Transactions of the American Mathematical Society* 58, no. 2 (September 1945): 231–294.

The "Biotopology" of Nicolas Rashevsky

In earlier work,[4] I explained how and why the theoretical biologist Nicolas Rashevsky gradually developed his epistemology of formalizations of the living world from molecular biophysics and population studies, inspired in the 1930s by Lotka,[5] towards a topology of life—his biotopology—in the 1950s, passing through an intermediate stage in which he worked on an epistemology of formalization advocating direct mathematical modeling of biological functions in the 1940s.

From 1933 onwards, Rashevsky's epistemology was clearly inspired by Lotka's physicalism,[6] along with that of D'Arcy Thompson. In his famous book *On Growth and Form* (1917),[7] Thompson had tried to fight against the hegemony of biometric and statistical approaches to the morphogenesis of living beings. Through his notion of *structural transformation*, he brought to light the structural affinities between the morphologies of different species. In this, he emphasized an idea that Rashevsky wished thereafter to relay and expand further, namely the idea that there is a unity to life beyond the full range of its structural manifestations. Rashevsky's goal was to achieve the conception of a general theory of biology and the "organism as a whole." In his first period, Rashevsky therefore attempted to go beyond the mechanicism of D'Arcy Thompson, taking physicalism seriously in all its generality—generalizations of physical theories other than the purely mechanical could be imagined, for example the electrical hypothesis of cell division, a hypothesis formulated and then explicitly rejected by Rashevsky when invalidated by experimental measurements. The epistemological model for a mathematized biological theory is, then, that of

4 - Franck Varenne, "Le Destin des formalismes: Pratiques et épistémologies des modèles face à l'ordinateur" (PhD. diss, Université Lyon-II, 2004); Franck Varenne, "Nicolas Rashevsky (1899–1972): De la biophysique à la biotopologie," *Cahiers d'Histoire et de Philosophie des Sciences* (2006): 162–163; Franck Varenne, *Formaliser le vivant: Lois, théories, modèles?* (Paris: Hermann, 2010), chapters 7, 14, and 15.

5 - Alfred J. Lotka, *Elements of Physical Biology* (Baltimore: Williams & Wilkins Co., 1925); Lotka, *Elements of Mathematical Biology,* 2nd ed. (New York: Dover Publications, 1956).

6 - We can talk of physicalism in the sense that it is assumed here that the laws by which living things grow are—or will be—all reducible to the laws of physics.

7 - D'Arcy Wentworth Thompson, *On Growth and Form* (Cambridge: Cambridge University Press, 1917), completely revised edition published in 1942 (Cambridge: Cambridge University Press), and reprinted in 1992 (New York: Dover).

Einstein's general relativity, with its revolutionary character in epistemological terms. In biology, for Rashevsky, an extended mathematical physics was required to renew the concepts relevant to any formalization of living systems.

It was in the 1940s that Rashevsky eventually accepted that strict physicalism must be amended or, at least, coupled with an approach proceeding to a direct formalization of biological functions, for example ingestion, digestion, sensitivity, locomotion, and so forth. In the 1950s, coming to consider that the essence of a living being is not so much its particular physical structure, or even all of the biological functions it implements, but rather *the system of dependency relationships between these functions*, Rashevsky was led to call for the emergence of a *relational biology*.[8] Relationships between functions are ultimately the essence of living forms because, according to him, they seem to present an invariance with respect to the diversity of the manifestations of these forms. An organization must be seen as a set of relationships between functions. And the unity of life is revealed by the fact that one can pass from the representation of one living system to another not by structural transformations, but rather, more generally, by "functional transformations."[9] Functions can be replaced by groups of functions—represented as subgraphs in the oriented graph of functions—which globally have the same local function in the set of functions manifested by an organism. The graph of relations between functions of an organism is oriented, because some functions are necessary in order to ensure others—for example, ingestion always precedes digestion, from paramecium to mammal.

What matters, therefore, to characterize a living being, is the topology of the graph of its biological functions. This topological characterization, while being accurate and loaded with biological meaning, does not make it irreducible for other organisms and species of living things. According Rashevsky, this new characterization, along with the sum of empirical knowledge already accumulated in biology, logically leads us to formulate what he calls

8 - Nicolas Rashevsky, "Topology and Life: In Search of General Mathematical Principles in Biology and Sociology," *Bulletin of Mathematical Biophysics* 16 (1954): 317–348.

9 - Rashevsky, "Topology and Life," 322.

the principle of *biotopological mapping*, or the *principle of epimorphism*—there exist different mechanisms which furnish the same organic property, and which a single mathematical concept (type of node in a graph, type of subgraph, and so forth) must be able to describe. This principle partly anticipates the more recent idea of multirealizability, of the same biological functions being operated on a diversity of physical substrates. It nevertheless mixes without distinction the fact of multirealizability, wherein there is a relationship between function and variable structures, with the topological transformation of a graph into a series of other smaller or larger graphs, also variable, where this time a relationship between the functions and the subgraphs of functions is found. A more stringent characterization of the principle of epimorphism by Rashevsky removes the ambiguity. Here there is no suggestion of an anticipation of multirealizability in the strict sense, since there is no mention of structures—only relationships between organisms, notably from a genealogical, or phylogenetic, perspective.

> We may now state somewhat more precisely the principle of biotopological mapping: there exists one, or very few, primordial organisms, characterized by their graphs; the graphs of *all* other organisms are obtained from this primordial graph or graphs by a transformation, which contains one or more parameters. Different organisms correspond to the different values of those parameters..[10]

According to Rashevsky, the most important information here is that we can assume that this topological transformation is uniform. The relational graph of functions of all living beings can be derived from a *single* primordial graph using a *single* topological transformation. And we can assume this from the moment we assume that life is sufficiently uniform. Henceforth it is only the parameters of this uniform transformation that change. According to Rashevsky, the transition to biotopology thus yields the language necessary to express the point in a sufficiently abstract manner for *the unity of life* to be accurately portrayed.

10 - Rashevsky, "Topology and Life," 329.

Robert Rosen's First Approach: Systems and Graphs

In 1958, Robert Rosen took up the challenge of the Rashevskian "relational biology" project, although interpreting it in systemic terms—taken from the theory of systems—and adapting it to formalization using "flow charts" and "block diagrams."[11] This representation itself developed out of circuit theory and the theory of automata. Each node in a diagram represents a block or "black box," in the sense that we can only know its input and output properties, but not the processes that take place inside. Any organism can be represented in its operation as a diagram connecting the inputs and outputs of blocks by oriented edges. According to Rosen, each block represents a systemic component. Each oriented edge is an oriented link—assuming that a flow of information or material occurs there—between the output of one component and the input of another. Rosen saw this as sufficient to represent any type of metabolism schematically and conveniently.[12]

To this, Rosen adds a consideration he deems critical. An organism is characterized not only by its metabolism[13] but also by an activity of *repair* to certain (not all) parts of its system. This repair activity explains the sure evolvability of a biological system at the same time as its relative durability in the face of phenomena which may oppose it.

Rosen takes the example of the individual biological cell. Within it must necessarily subsist components that may reconstruct other components that have been destroyed or inhibited. For this to constitute a veritable repair, and thus guarantee the durability of the cell, it is necessary for these repaired components themselves to form a part of the ensemble of components involved in the metabolism of the cellular organism in question. And for this to be possible in principle, Rosen continues, the only solution is that each of these metabolic components Mi is conjugated to a repair component Ri whose inputs are uniquely outputs of Mi and whose sole

11 - Robert Rosen, "A Relational Theory of Biological Systems," *Bulletin of Mathematical Biophysics* 20 (1958): 245–260.

12 - Rosen, "A Relational Theory of Biological Systems," 246.

13 - The metabolism summarized as anabolism and catabolism—molecular synthesis, molecular degradation.

output will be to provide a replica of Mi. No other relationships between Ri and other components are to be permitted. In addition, this ad hoc repair component should itself not be involved in metabolism itself, because were it to be so, it would by definition be a component Mj requiring its own Rj, and so forth.

After the construction of a plausible minimal graph (**M**, **R**) on the basis of these theoretical considerations, Rosen raises the question of the topological constraints that must weigh on such a graph for the overall repair to be possible. This amounts to asking what the topological conditions are for a fully connected graph to give way, after more or less partial repair, to another graph, also fully connected and able to provide outputs similar to those in the original graph. One of Rosen's results is that the system cannot always maintain its original topology. Sooner or later it will collapse to one of its subsystems (or subgraphs) which is topologically less complex, or even be completely destroyed. Rosen also demonstrates the existence of a component which is central to any system—of a type able to be partially repaired before being completely destroyed—the failure of which directly causes the failure of the entire system. He also shows the interest of having a topological segregation of a group of components dedicated to the reproduction of the entire system. This is, for Rosen, an a priori deduction of the advantage for a cell to be equipped with a nucleus, that is to say, to have a specific place—corresponding to a strongly connected subgraph—which brings together all the cell's reproductive material.

Having now broadly sketched Rosen's first theoretical approach, as inspired by Rashevsky, it may be instructive to read the critique of it that Rashevsky himself published that same year:

> In a recent paper, Robert Rosen applied topological considerations to the study of an organism as a whole. Those considerations have no direct relation to the principle of biotopological mapping. They rather represent a topological model of an organism, especially a model of the repair mechanisms which organisms possess for lost or impaired parts.[14]

14 - Nicolas Rashevsky, "A Note on Biotopology of Reproduction," *Bulletin of Mathematical Biophysics* 20 (1958): 275.

For Rashevsky, Rosen uses, just as he had himself, an abstract representation inspired by graph theory. However, the vertices of his graphs are not representations of biological functions—even if he had himself advocated such a use—but representations of mere parts of the organism. In chapter 33 of the third edition (1960) of his synthetic book *Mathematical Biophysics*,[15] Rashevsky offered concrete examples—these parts may be the eyes, stomach, kidney, and so on. According to Rashevsky, therefore, Rosen is going backwards. He only uses the topological tool of graphs to represent, in a different way, that which is but a local mechanistic model of the interactions between the components of the organism. We are far, here, from a veritable relational biology, as it is ultimately the relationship between structures—and not between functions—that is the subject of Rosen's block diagrams. Believing he was moving towards a renewed theory of life and its unity, that is to say, believing he was proposing a new theoretical model, Rosen was in fact doing no more than modeling the living world in mechanistic terms, thereby only managing to prove the local validity of his model.

According Rashevsky, Rosen was a victim of the mechanistic physicalism which, at the end of the 1950s, seemed to be reawakening, helped on by the parallel development of automated approaches—cybernetics—in other contexts,[16] issuing mainly from the work of John von Neumann and Stanisław Ulam.[17] Rashevsky remained convinced, however, of the interest of the mathematist and specifically topological thread of thought that Kurt Lewin had already called for in psychology in 1936. Lewin had in fact made a clear distinction between direct mathematization and physicalist modeling. For Lewin, the fact that mathematics had hitherto mainly been used in physics did not mean that calling upon mathematics in the life sciences or the humanities would mean being guilty of physicalism:

> Like an illustration the working out of a model can have a certain value. On the other hand it can, especially in psychology, involve serious dangers: a model usually contains much that is

15 - Nicolas Rashevsky, *Mathematical Biophysics—Physico-mathematical Foundations of Biology*, 2 vols. (Chicago: University of Chicago Press, 1938)

16 - Although the overlap of the two is only very partial.

17 - Varenne, *Le Destin des formalismes*; Varenne, *Formaliser le vivant*.

> purely arbitrary. One usually uses it like an illustration only in so far as the analogy holds, *i.e.*, really only as long as it is convenient. As soon as consequences ensue which do not agree with the real facts, one evades the difficulty by asserting that it is after all only a model or an illustration. One says "A comparison is not an equation". How far one uses the model for explanation and at what point one discards it as no longer binding is purely arbitrary. In this respect model and illustration are sharply distinguished from the mathematical representation which we are trying to attain. If one decides to represent a real fact by a mathematical concept then one is forced to acknowledge all the consequences which are involved in this concept. This certainly makes the task a difficult one. On the other hand science will obtain the real benefit of the application of mathematical concepts only if it uses them in an absolutely binding way.[18]

For Rashevsky, as for Lewin, by directly using the concepts of mathematical topology—directly meaning without the mediation of a prior physical or mechanical model—we seek a general and impeccable structure of inference in a relational context, not a partial and approximate system of analogies. One is not representing a substrate but conceptualizing relations. In particular, recourse to concepts associated with topology (such as the graph) should not assume that physical space and its constraints will be taken into account, constraints to which the structures of the organism are certainly subject. The fact that topology was being used to represent spatial forms—organizational structure—such as the segregation of the components of repair into the cell nucleus, was therefore deeply shocking to Rashevsky.

As we will see, Rosen's response would be to move to a second, more radical theoretical and epistemological approach, which accentuated the mathematist thread in theoretical biology. It is in this context that concepts related to the mathematical theory of categories were introduced for the first time. Before presenting his second proposal, and taking the measure of the role it gave to the notion of category, it will be necessary to review some of the concepts of this theory.

18 - Kurt Lewin, *Principles of Topological Psychology* (New York: McGraw Hill, 1936), 79.

Concepts of Category Theory

As noted at the beginning of the foundational article by Eilenberg and MacLane,[19] one of the interesting results of algebra is the theorem that a vector space L, with real values and finite dimensions, is isomorphic to its conjugate T(L).[20] However, to build an isomorphism between these two spaces, it is necessary each time to establish it on a particular vector basis of L. Thus no particular isomorphism may impose itself once it is known that its two structures are isomorphic.

From this, however, comes a more interesting result—that the same cannot be said for the isomorphism that also exists between this vector space L and the conjugate of its conjugate, written T(T(L)):

> For the iterated conjugate space T(T(L)), on the other hand, it is well known that one can exhibit an isomorphism between L and T(T(L)) without using any special basis in L. This exhibition of isomorphism $L \cong T(T(L))$ is "natural" in that it is given *simultaneously* for *all* finite-dimensional vector spaces L.[21]

Simultaneity or naturalness here means that the *same* definition works for *any* vector space, so much so that often L is identified with T(T(L)). The isomorphism is exhibited without need for recourse to anything but L alone and therefore without recourse to other parameters dependent on L.[22] In the following part of their introduction, Eilenberg and MacLane specify: "A discussion of the 'simultaneous' or 'natural' character of the isomorphism $L \cong T(T(L))$ clearly involves a simultaneous consideration of all spaces L and all [linear] transformations λ connecting them."[23] Immediately after which they add, significantly: "This entails the simultaneous consideration of the conjugate spaces T(L) and the induced transformations $T(\lambda)$ connecting them."[24]

19 - Eilenberg and MacLane, "General Theory of Natural Equivalences," 231–232.
20 - The conjugate of a vector space L is the vector space of all linear functions on L.
21 - Eilenberg and MacLane, "General Theory of Natural Equivalences," 232.
22 - This explanation is provided by Wilfrid Hodges and Saharon Shelah, "Naturality and Definability," *Journal of the London Mathematical Society* 33, no. 1 (1986): 2.
23 - Eilenberg and MacLane, "General Theory of Natural Equivalences," 233.
24 - Eilenberg and MacLane, "General Theory of Natural Equivalences," 233.

This is why they first introduce (generically) the linear transformations λ which transform any finite dimensional vector space into another space of finite dimension, and then the mappings induced between the conjugates. For two spaces (L_1, L_2), this is the linear transformation λ_1:

$$\lambda_1 : L_1 \rightarrow L_2$$

From this is drawn the induced mapping T (λ_1):

$$T(\lambda_1): T(L_1) \rightarrow T(L_2)$$

In presenting it this way, Eilenberg and MacLane decided to denote two different functions with the same letter T: one function operating on linear transformations and another function that operates on vector spaces. These two functions are the components of what they call a functor, the functor T.

To recap: between L_1 and $T^2(L_1)$, there is a natural isomorphism, as there is between L_2 and $T^2(L_2)$. Let them be called $\tau(L_1)$ and $\tau(L_2)$.

We therefore have:

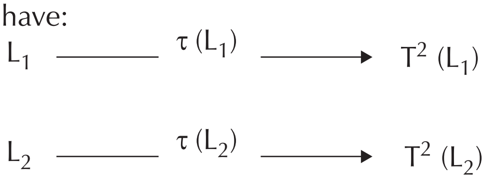

Figure 1

Moreover, between L_1 and L_2, we have the identity functor I(λ).[25] Finally, between $T^2(L_1)$ and $T^2(L_2)$, we have the induced transformation $T^2(\lambda)$. Hence the following full diagram:

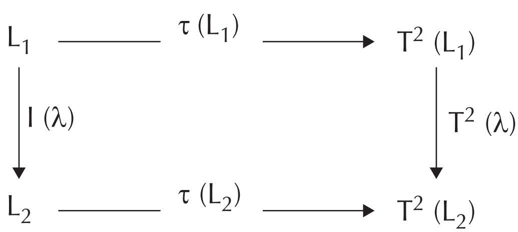

Figure 2

25 - Functor composed of two functions such as I(L) = L and I(λ) = λ.

This diagram is commutative. The two possible paths from L_1 to T^2 (L_2) are identical. This is what we may call the condition of "naturalness" or "simultaneity" for the family of natural isomorphisms L → T (T (L)). We also have what one might call a "natural equivalence" between the functor I and the functor T^2. This is a functor morphism which respects the composition of the morphisms.

Eilenberg and MacLane then noted that this condition of naturalness can be generalized. It does not occur only between a vector space and its iterated conjugate but also between groups and their homomorphisms, or between topological spaces and their continuous mappings, and so on.[26]

Considering this generality, it is proposed to introduce the concept of category. A category consists of this first set of data:

1 / A collection of objects to be designated by A, A′ . . . ; these objects can be sets.

2 / A function assigning to each pair (A, A′) of objects in the category, a set denoted H (A, A′), whose elements are termed *mappings* or *transformations* or *morphisms* or *arrows*. If *f* is an element of H (A, A′), we say that A is the *domain* or the source of *f*, and A′ is the *range*, or *goal* of *f*.

These first two properties constitute the minimum basis necessary to construct the theory of a set of mappings. Required are: 1 / objects upon which the mappings act; 2 / the mappings themselves. However the definition of the concept of category does not stop there, otherwise it would have zero operational interest. It is in fact necessary to introduce ways to "combine" or "compose" these mappings. We then construct an algebra on these mappings *f* to make comparisons between them, as well as the structures that emerge from them:

3 / Within a category, there must also be a function called a *composition* which assigns, to any pair of mappings (*f*, *g*) such as $f \in$ H (A, A′) and $g \in$ H (A′, A″) a mapping *gf* in the set H (A, A″). This gives a *commutative* diagram[27] between A, A′, and A″ (see figure 3).

26 - Eilenberg and MacLane, "General Theory of Natural Equivalences," 234.
27 - It should be noted that Rosen was to identify precisely this diagram with his previous block diagram (see below).

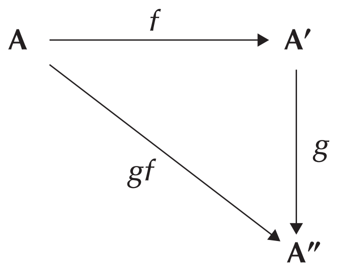

Figure 3

Finally, in order for the concept of category to be applicable to concepts we already recognize, three axioms must be added:[28]

C1 – Any category mapping has one and only one domain and one and only one range.

C2 – The composition of mappings is associative: $h \circ (g \circ f) = (h \circ g) \circ f$.

C3 – There is an identity mapping i_A of any object A onto itself.[29]

The concept of subcategory can be defined. A subcategory preserves at once the identity mapping and the composition between mappings, as well as the domains and the ranges of its mappings.

Starting from the concept of category, we can also define the concept of the functor. If **A** and **B** are two categories, then a functor T of **A** onto **B** is a pair of mappings which, to any object of **A** associates an object **B**, and to any mapping of **A** associates a mapping of **B**, and where $T(g \circ f) = T(g) \circ T(f)$, and $T(iA) = i_{T(A)}$. Recall that i_A is the identity mapping of any object A onto itself. A functor appears, then, as a generalization of the notion of a mapping.

28 - This is Rosen's annotation in Robert Rosen, "The Representation of Biological Systems from the Standpoint of the Theory of Categories," *Bulletin of Mathematical Biophysics* 20 (1958): 322.

29 - More precisely, the identity mapping is a mapping $i_A \in H(A, A)$ such that for any object A′ of this category and for each pair of mappings (f, g) such that $f \in H(A, A')$ and $g \in H(A', A)$, then $f \circ i_A = f$ and $i_A \circ g = g$. See Rosen, "The Representation of Biological Systems," 322, and Eilenberg and MacLane, "General Theory of Natural Equivalences," 237.

From this point of view, a functor is a type of correspondence between categories that preserves the category structure. This notion also generalizes homomorphisms (between groups) or continuous mapping (between topological spaces), for example.

Rosen's Second Approach: Categories and Living Organisms

Now let us ask the following question: How does Rosen justify his use of category theory to build a theory of the representation of living beings? He was undoubtedly hurt by Rashevsky's criticisms, but his second article in 1958[30] answered them indirectly, by giving replies to a self-interrogation, wherein Rosen emphasized other limits of his previous approach, using graphs.

First of all, in a real organism, one output of a component can be the input to several other components. Thus, for example, an endocrine gland can secrete a hormone—a *single* output, which will affect *many* organs. Therefore a number of arrows can be shown exiting the formal representation of a single output of this gland (see figure 4).

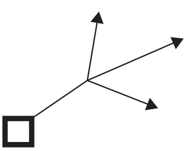

Figure 4

Symmetrically, a same component can send several different outputs to another component. This would be represented by a single arrow connecting the two, and information about this diversity will be lost. Thus, the particular endocrine gland, known as the pituitary or hypophysis, sends *different* hormones to the *same* organ. (See figure 5)

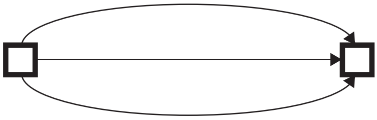

Figure 5

30 - Rosen, "The Representation of Biological Systems," 322.

In this regard, Rosen shows that a living being is certainly a relational entity, but adds that the internal relations manifested are in reality very rarely "binary," that is to say bilateral, with a transmitter → receptor. They are most often multilateral.[31] The approach by graphs may take into account this fact, but at the cost of some complication.

The representation of the organism by a block diagram has another drawback, according to Rosen. By introducing the "environment of the organism," denoted E, a component is introduced that is not a real component of the organism but nevertheless sends inputs to a large number of components of the graph and receives all outputs not connected to the organism's internal components. As its status is not well defined, and as it is also at the borders of other components, by its formal behavior at its limits during the demonstration of general theorems, it demands separate treatment, which complicates reasoning by particularizing and weakening any intuition that may be gained concerning them.

According to Rosen, it would be valuable to have a representation of the environment that would allow it to stand alongside other components.[32] This choice indicates that Rosen wishes to chase away symbolic dispersion and is searching, like his master, for a unification and translation of intuition. He is on the path to the categories: "Although we may to a certain extent overcome the difficulties we have mentioned by the introduction of a number of technical devices, the theory which results will have lost the intuitional clarity which constituted a large part of its appeal."[33]

Having formulated these self-criticisms, Rosen then presents a positive reason which justifies, in his view, the choice of using the mathematical theory of categories for the elaboration of a general theory of formal representations of life.

> It will be seen that, although the theory which results seems at the outset to be considerably more complicated than our previous treatment, we can formulate our results, and even our definitions,

31 - Rashevsky emphasized this point later, especially in his article "Physics, Biology and Sociology: A reappraisal," *Bulletin of Mathematical Biophysics* 28 (1966): 292.
32 - Rosen, "The Representation of Biological Systems," 318.
33 - Rosen, "The Representation of Biological Systems," 318.

> in a simpler, more intelligible and more precise fashion than is possible through any refinement of our other approach.[34]

Thus, paradoxically, at first sight, Rosen hopes that by raising the level of mathematical abstraction of the representation of life by another degree, we might regain the ability to intuit its behavior in a manner that is at once formal, accurate, and biologically meaningful. It is in this sense that he thinks this theoretical gesture, using the mathematical concept of "natural transformation" will give a certain naturalness to the formalisms of life. But what is the implicit hypothesis upon which Rosen builds?

Rosen notes first that the theory of categories shows the "naturalness" of transformations of one functor to another, when they have the property of being commutative. They are called "natural" (in the mathematical sense) because they leave the "good mathematical constructs" invariant.[35]

When Rosen announces that these mathematical concepts would be useful for a general theory of representations of biological systems, he seems to suppose that the approach of mathematics through the concept of categories is particularly well suited to the identification of *mathematical naturalness* of certain mathematical transformations—unlike other approaches, such as the one using set theory, for example—and at the same time provides evidence that the use of this concept of category, this time for formalizing life, ensures the *naturalness* of this formalization. It is not artificial, not built, not stitched on to real phenomena occurring in the living world.

34 - Rosen, "The Representation of Biological Systems," 319.

35 - René Lavendhomme, *Lieux du sujet: Psychanalyse et mathématique* (Paris: Seuil, 2001), 274. Lavendhomme states: "In general, we can say that a good mathematical construction must be resistant to natural transformations [that is to say, a commutative transformation from one functor to another, see Lavendhomme, *Lieux du sujet*, 273–274]. It must be 'natural.' It is with the idea of naturalness that Eilenberg and MacLane begin their reflections on categories. There are, in mathematics, structures that depend on certain choices in order to be executed, while others are, as they say, canonical, not being dependent on contingent decisions. It is these that give rise to natural transformations, and it is to clarify this idea that the notion of natural transformation was created." Lavendhomme, *Lieux du sujet*, 273–274. Confirmation of this remark can be found in Eilenberg and MacLane, "General Theory of Natural Equivalences," 247.

What is it exactly? How does he attempt to support this bold inference? He continues his preamble by introducing the concept of the faithful functor, inspired once again by Eilenberg and MacLane. A functor T is faithful if:

1/ T (f) = T (g) implies that $f = g$;

2/ when $g \circ f = i_{A'}$, and T (A) = T (A′) and T (f) = $i_{T(A)'}$, then A = A′.

In this latter case, *f* and *g* are called the *equivalences* of objects A and A′. In particular, if **A** is a category of groups, *g* and *f* are isomorphisms. If it is a category of topological spaces, they are homeomorphisms. He then clarifies the concept of the embedded category. A category **A** is embedded in another category **B** by a functor T, if T is a faithful functor from **A** to **B**. The image of **A**, denoted by T (**A**) is then a *subcategory* of the category **B**.

Rosen then recalls the formulation of a theorem he considers of great importance for his theory of representation of systems: "Theorem 1: Any abstract category **A** can be embedded as a subcategory of the category **S**, the objects of which consist of all sets[36] and the mappings of which are the totality of all set-theoretical many-one mappings of sets."[37] By choosing a faithful functor of any category **A** onto **S**, we can embed it in **S** and we can therefore consider the objects of this category **A** as ordinary sets, and its mappings as ordinary mappings as in set theory. Thus, we can also speak of the concepts of union, intersection, and Cartesian products between objects of any category.[38] It is this point which directs formalism towards the intrinsic consideration of the multilateral relations subsisting in the organism.

It is also this last point which allows Rosen to pass on to the phase which he called the "construction of the representation itself." He makes us see that with the theorem, it is possible, from an arbitrary

36 - Eilenberg and MacLane noted that while the notion of the "category of all sets" raises the paradoxes encountered in this respect in set theory, it does not add any extra paradoxes and can be treated, they say, either by deciding to adopt an intuitive approach or by using a logical foundation for avoiding just such paradoxes. See Eilenberg and MacLane, "General Theory of Natural Equivalences," 246.

37 - Rosen, "The Representation of Biological Systems," 323.

38 - Rosen, "The Representation of Biological Systems," 324.

category **A**, to form an *oriented graph* (or diagram) by selecting a collection of objects A_i from the category and a collection of mappings from the sets H (A_i, A_j). Two objects are said to be *connected* by an oriented edge *if and only if* there is a mapping that has one of them in his domain and the other in its range.

To generalize again and to adapt this representation to living organisms, Rosen notes that it is possible to weaken the constraint weighing on the construction of an oriented edge. We may demand only that the objects in the diagram *contain* the domains and ranges of the mappings.[39] We can see that in the perspective of a representation of life, the fact of considering the objects themselves simply as *sets* having the property of *containing* the domains and ranges of the mappings has a decisive weight in this abstraction that aspires to "naturalization," while not fundamentally changing the possibility of applying the general considerations in force for the categories to this representation.

Furthermore, Rosen considers that the "abstract block diagram"[40] (ABD) that we can build from there actually reverses previously adopted conventions: a vertex represented a physical component in the organism in his first representation by block diagram, when in fact it is now more convenient to consider the components as represented by the mappings,[41] that is to say by the oriented edges of the abstract block diagram. Respectively, the ABD objects do not represent components, but the inputs and outputs thereof. This type of representation has the advantage not only of resolving the issue of the multilateralism of relations but also that of the environmental component. It is no longer incongruously hypostatized. Only the relationships in which it operates are included in the diagram, and with exactly the same status as the other components.

After this first phase of construction, therefore, Rosen showed that the mathematical concept of category was rich and flexible enough to formalize any type of oriented graph representing a living being.

39 - Rosen, "The Representation of Biological Systems," 324.
40 - "Abstract," because not all these diagrams necessarily correspond to a physical system. See Rosen, "The Representation of Biological Systems," 325.
41 - Each component is now represented by a set of mappings. We thus escape the supposed physicalist reduction assumed to be necessary by Rashevsky as soon as one uses diagrams.

After a long section in which Rosen shows that it is possible to find a canonical form for an ABD representing a given living system, the search for naturalness is explicitly extended as he recalls that the functors are conceived precisely as "natural means"[42] to make comparisons between categories. He concludes:

> Similarly, it follows that any structures formed from the objects and mappings in various categories may likewise be compared by functors. Thus, given an abstract block diagram (which we may denote by **M**) of objects and mappings in a category **A**, we may apply a functor T: **A** → **B** and obtain in the category **B** the collection of images under T of the objects and mappings of **M**. We may write the image of the abstract block diagram **M** as T (**M**). One of the natural questions to ask concerning a given functor T is whether the image of an abstract block diagram by T is again an abstract block diagram.[43]

The problem[44] that Rosen explicitly raises is *whether it can be demonstrated that equivalent biological systems have the same representation within this mathematical formalism and, conversely, if two mathematically equivalent representations actually correspond to the same biological system*.[45] We will have then proved that the mathematical equivalence exhibited thanks to the formal technique of functors between categories corresponds to biological equivalence. And for Rosen, this would be one more sign in favor of the idea that the mathematical formalism of categories, or "natural equivalences," most naturally apply to the essence of that which is alive, its essence being demonstrated and mobilized here—at least by showing what it is not—in what is invariant between two biological systems deemed to be equivalent in view of their respective abstract block diagrams.

However, all that Rosen achieves in declaring in this article from 1958 is this last theorem: "Let **M** be an abstract block diagram

42 - Rosen, "The Representation of Biological Systems," 334.
43 - Rosen, "The Representation of Biological Systems," 334.
44 - We note that it is considered itself "natural" by Rosen, but here in the sense that it is a traditional, almost reflex mathematical interrogation. This research tactic is often considered "natural" by mathematicians since it is usually so fertile and powerful. According to George Pólya, for example, whereas a mathematician is someone who sees analogies where others do not, a great mathematician is one who manages to see analogies between analogies.
45 - Rosen, "The Representation of Biological Systems," 336.

which represents a definite biological system. Let T be a faithful functor. Then T (**M**) is an abstract block diagram which represents the system if and only if T is regular and multiplicative."[46] Ultimately it appears that it is not enough for the functor to be faithful in order to conserve structures imposed by operations of inclusion and intersection between sets. It must also allow conservation of the semiordered space, like that of the commutative semigroup (the Cartesian product) which the former possesses, in order to give rise to an ABD. It is necessary for the image by the functor to conserve the structure of set theory, the same which, through theorem 1, allowed the formalism of the diagrams to be transcribed into that of categories. Thus Rosen admits to not possessing the means to show if the biological equivalence between organisms is directly translatable in terms of mathematical categorical equivalence via a functor of this type. In other words, and contrary to his own expectation, Rosen does not know how to show if the hybrid diagram of biological and mathematical equivalences commutes.

From our point of view, we see that for *mathematical equivalence* between categories to effectively encompass within itself that which makes its own the equivalence in the biological sense (equivalence between biological systems from the relational point of view), and therefore that its property—at first sight impressive—of being categorical seems to be able to transit by the effect of an *equivalence between equivalences* from the domain of mathematics to that of living beings, *it is not sufficient* in reality for mathematical equivalence to be *simply natural* in the sense of the naturalness proper to the mathematical theory of categories. It is not sufficient for the functor to be only faithful. Highly constraining properties must be adjunct to it, which cause it to lose the character of generality of the approach based on categories, largely collapsing back into an approach based on sets. Thus, once more in the history of science, and undoubtedly to the chagrin of theoretical biologists, it appears that mathematics will not divulge, from within itself, how it applies to anything outside of itself—meaning, in effect, empirical reality.

46 - Rosen, "The Representation of Biological Systems," 334–335. A functor is regular if it satisfies the following conditions: 1/ if $A \in \mathbf{S}$ and $A \neq \emptyset$, then $T(A) \neq \emptyset$; 2/ if $A \subset B$, then $T(A) \subset T(B)$. A functor is multiplicative if for any pair of sets A_1 and A_2, we have $T(A_1 \times A_2) = T(A_1) \times T(A_2)$.

At the end of the article, however, there are practical results which are nothing short of remarkable. Rosen offers a formal translation using the block diagram approach of the general and logical theory of automata of McCulloch, Pitts, and von Neumann. A general automaton in the von Neumannian sense, consisting of a network of automata—each with only one input—can be represented as a category mapping whose range is the space A = {0, 1} of the game of heads or tails and whose domain is the Cartesian product of A multiplied by itself as many times as there are inputs to the general automaton.[47]

Thus, a general automaton can always be represented by an abstract block diagram in a suitably defined category. For Rosen, von Neumann's general automaton is a confirmation of his general theory of the representation of living systems. It is an illustration of it, in his words, because the graphical aspect—in the sense of graph theory—of von Neumann's general and logical theory of automata is no more, according to Rosen, than a consequence of the more general formalism, therefore assumed to be more natural, that he adopted himself with the theory of categories. For him, just like for Rashevsky, antibiotics could have been predicted by biotopology, and the general and logical theory of automata could have been derived entirely in the abstract from the categorical perspective he proposed. The automaton theory therefore appears to him neither necessary nor decisive, as it is not fundamental.

One lesson of this article is therefore epistemological, going as far as making a normative-type value judgment on the choices of formalization to be made, rather than testing those choices in the light of empirical data.[48] Category theory provides uniform criteria[49] to biologists which will allow them to assess a priori the relevance of their formalism. This relevance is itself characterized only in terms

47 - Rosen, "The Representation of Biological Systems," 337.

48 - In this way, we see that theoretical biology remains fairly directly dependent on extreme philosophical positions. This dependence certainly remains, although it is probably becoming less naively direct—becoming more interlaced with the empirical—in more advanced quantitative sciences.

49 - That is to say, independent of the context and the stakes of formalization. This assumption is severely challenged today by practitioners of applied formalisms. This is seen by the success of the opposed pragmatists, contextualist and perspectival epistemologies of models, and formalizations for complex systems.

of generality, this generality then itself being closely related to what Rosen wishes to consider as a "naturalness," this naturalness being assumed to characterize, in absolute terms, certain formalisms, as distinct from others.[50]

Categories for Life in Rosen's *Life Itself* (1991)

Faced with this rather underwhelming result, Rosen long hesitated before returning to category theory, although he never renounced his general epistemological discourse on the formalisms relevant to biology. In *Essays on Life Itself* (2000), for example, a book collated by his daughter after his death in 1998, the two foundational articles of 1958 are not even mentioned—for his family, or perhaps for himself, his bibliography only begins in 1959.[51] It is, in fact, from this date onwards that he began to question the very idea that it might be possible to formally represent a biological system by an automaton, or indeed any mathematical mechanism. This radical thesis was to become his favorite claim until his death. Rosen would warmly echo the positions taken by René Thom on the refusal of the hegemony of discrete models and other related matters.

In this final section, we will briefly summarize the evolution of Rosen's thoughts as reflected in *Life Itself* (1991) and the articles that followed. For the record, we note that the main argument of *Life Itself* is to be found in the conjunction of three consistent theses:

50 - In other contexts, without reference to mathematical categories, but sometimes simply to differential formalisms, other theoretical biologists will also claim this troubled idea of "natural formalisms" that is supposed to allow other formalisms to be condemned. Alongside the work of René Thom we might note the lesser-known case of Brian C. Goodwin, Conrad H. Waddington's student. See, on this point, Varenne, *Le Destin des formalismes*, 341: "In 1970 [Goodwin] published an article on biological stability which implicitly targeted Lindenmayer [inventor of L-systems, today extensively present in computational approaches], openly criticizing the choice of formalism of automata in developmental biology. His argument amounts seeing, in what is presented as a formalization by automata, not *a real formalization*, that is to say an effective formal representation ('natural' in the sense given by his qualification of differential 'natural models') or even an approximate translation, but a *simple analogy* of the gene to the computer which, in certain critical cases, fails to take into account some essential biological phenomena. It would not, then, actually be a formalism, but a mere 'formal analogy.'"

51 - See Robert Rosen, *Essays on Life Itself* (New York: Columbia University Press, 2000), 343.

1/ Newton's mechanistic approach—the hypothesis of states, the hypothesis of recursion—is to physics what logical formalization is to mathematics, that is to say, a reductionism.

2/ Mathematics had a Gödel to denounce this reductionism, while physics had no such iconoclast. For physics, we are awaiting new theoretical tools in the hope of achieving lucidity.

3/ These tools can come from mathematical biology.

According to Rosen, physics, along with all the natural sciences—their computational developments being merely the most obvious example—had implicitly learned the habit of giving substantive content to the thesis of Alonzo Church, which asserts, with good reasons but no certainty, that any mechanism (in the mathematical sense) can be perfectly emulated by a universal Turing machine—a classic computer with infinite memory. It is often concluded, by sophistry and therefore wrongfully, that every phenomenon with a physical manifestation is, or will be, capable of being simulated by a computer.[52]

Rosen asserts that there is an implicit and false epistemological thesis in the computational development of contemporary science. This thesis is based on a fallacy which assumes that by being physical, a phenomenon is reducible to a mechanism in the mathematical sense. Just as analytic philosophers since Gilbert Ryle have it, we can say that there is here a *category error*. Used to searching for equivalences between equivalences, Rosen rejects the foundational fallacy he perceives here, a fallacy that reduces us to a series of reductive formalisms.

Armed with this analysis, in his 1991 book, Rosen sought to reactivate his own proposition for the use of category theory. This brings us to certain theses concerning this subject that appear in *Life Itself*. Firstly, Rosen sees it as worthwhile to resist mechanistic reductionism because that standpoint requires a rejection of final cause in the Aristotelian sense. Rosen felt that final cause should be included

52 - Robert Rosen, *Life Itself* (New York: Columbia University Press, 1991), 204. For a partial explanation of this issue, see Franck Varenne, *Qu'est-ce que l'informatique?* (Paris: Vrin, 2009).

in the formalization of life. Following this initial update, he presents the mathematical theory of categories primarily as a rigorous manner for mathematics to model itself. It is a theory of modeling designed as a general analog relationship. A modeling relationship imposes representations with mutual implications. Rosen, as we have seen, assumes that in a living organism, both causal and inferential implications occur. Inferential implications are able to encompass final causes. The final cause of P requires both that its effect implies something, and that it *implies the implication of P itself*. However, he points out, this violation of temporal order shocks us only because it is something that seems to be prohibited in formalisms. But it is precisely that we only ever consider formalisms under the sole Newtonian model that prohibits the future affecting the present, and imposes that they must always take a classic computational form. But who imposes this kind of formalism?[53] Category theory gives us precisely the freedom necessary to break free and take charge of the implications of implications (arrows between arrows), and thus Aristotle's final cause.

As a mathematical language that speaks of mathematical language, category theory also shows that there is no general formalism that would tell us how causal implications and inferential implications could always be analogous. As a consequence, we cannot reduce any and all models of living systems to an approach that can only proceed by automata. There is not only one single mode of implication for understanding all things,[54] either purely causal or purely inferential.

Moreover, category theory is used to compare formalisms without being able itself to be formalized. This is a property it shares, disturbingly, with "natural language."[55] Rosen concludes from this that the theory of categories tends to have the same naturalness as natural language. And hence, the mathematical notion of category is joined to Aristotle's view in natural language ("The most general species of what is meant by a single word," *Categories* 2).

53 - Rosen, *Life Itself*, 48–49.
54 - Rosen, *Life Itself*, 132.
55 - Rosen, *Life Itself*, 45.

Rosen provides initial evidence of this set of (negative) theses by construction. He conceives of an "augmented abstract block diagram," that is to say taking into account "external implication"—a change of axiom in the formalism—and, hence, "finality."[56] However, this last of Rosen's projects was to remain a sketch only.

Conclusion

Let us form a rapid assessment of the evolution of the role given to mathematical categories in Rosen's theoretical biology. In 1958, category theory was conceived by him as a general method of formalization, including mechanistic formalization, that was legitimized but ultimately marginalized by that fact. It was seen as a suitable—"natural" in this sense—method of formalization for living systems, because it allowed a consideration of complex and multilateral relations. By practically illuminating, that is to say comparing, the functional equivalences between living systems with mathematical equivalences between formalisms, it also seems to have invoked a radical epistemological and normative judgment on the most relevant types of formalizations for life.

By 1991, however, far from helping to infer the approach of formalization of living systems by automated calculation, category theory instead primarily had shown the reductive nature of any computational formalization. It becomes a means of reintegrating finality into a formal language, judged more natural for describing life, being closer to natural human language. Note here the likely influence of René Thom, often cited by Rosen. Concerning the rehabilitation of finality, Thom would ultimately advocate the generality of his own mathematical theory of catastrophes.

In the end, we see that Rosen's work was built on durable philosophical convictions which were nonetheless refined over time. He himself admitted early on that he was unable to demonstrate that the hybrid diagram (living systems / / mathematical formalisms) was commutative, when considering the real system and its equivalent in the loop of the diagram. *Mathematical equivalence does not encompass biological equivalence* except to constrain it,

56 - Rosen, *Life Itself*, 138–139.

artificially and therefore *unnaturally*, ultimately collapsing back into an entirely artificial modeling.

To my eyes, what Rosen's work is successful in showing is that in this mathematized theoretical biology, despite the research being carried out tirelessly in this direction, mathematics remains unable to play the role to which it was called by Rashevsky, to have the same status it enjoys in theoretical—relativistic and quantum—physics, to be constitutive, rather than merely regulative (bolted on, and therefore modeling, in this sense), of the constituent concepts. Through Rosen's input, it appears that the goal—or perhaps the dream—of the theorist is not only, of course, to infer the real from the theory, but also, more radically, to seek to show that *mathematics is able to legitimize, by itself*, and without the intervention of another cognitive instance, *its own relevance and applicability to reality*. Mathematics would then no longer need an external epistemology. By advancing, independently, it would build unaided 1 / the scope of its applicability, along with 2 / the norms of its applicability.

It is therefore understandable that the particularly loaded notion of "category" came to be involved in this somewhat directionless strategy in contemporary theoretical biology. Not only would a certain mathematics be conceptually foundational for our knowledge of some parts of reality, but, moreover (and based on the assumption that at the limit a continuity would necessarily come about where the gap nevertheless lies[57]), it would be from the interior of mathematics which this conformity between reality and mathematics would be seen, a conformity thus supposed to be "natural," in a sense which could ultimately be both mathematical and physical, arising from a hypothetical commutation in the hybrid diagram.

Yet, the very evolution of the function of the enrollment of mathematical categories in Rosen's theoretical biology is enough to show that this enrollment has more of a reactive than a constructive role. Mobilizing a deliberately generalizing and rather intimidating

57 - Hypothesis according to which the greatest mathematical generality necessarily leads to the real, touching it, and, finally, merging with it. This fallacy — an *ad ignorantiam* fallacy— is common, as it is so attractive. Its formulation is roughly this: we do not know (or conceive) that which might escape that which allows the conception of all that is conceivable, and therefore it does not exist.

mathematical apparatus mainly served to respond to the growing hegemony of computational approaches with an unprecedented leap in mathematical abstraction, regardless of the precise function—sometimes alternative, and sometimes radically critical, as we have shown—that has been given, at one time or another, to this most abstract of mathematics.

This is undoubtedly true—categories are again with us, having been revived in recent years. Being aware of the pioneering works in this field and their fascinating character, as well as their limitations, should provide an opportunity to ensure that this return to fashion can be more than simply an opportunity to pursue academic battles and that it can nourish the search for real, alternative formalization solutions that are capable of confronting real-world data and, indeed, of going head to head with computational models themselves.