The 'logic' of self-organizing systems

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Abstract

A totally new computational grammatical structure has been developed which encompasses the general class of self-organizing systems. It is based on a universal rewrite system and the principle of nilpotency, where a system and its environment have a space-time variation defined by the phase, which preserves the dual mirror-image relationship between the two. As briefly summarized in the paper, there is already substantial and hard evidence in favour of the application of this universal rewrite approach to quantum physics. Further applications in diverse fields suggest that, while the relationship between a self-organized system and its environment must be fully understood in quantum physical computational terms rather than digital ones, a new discrete approach to this quantum mechanical understanding can be described, which extends beyond the purely quantum range. It offers a new calculational means, within existing digital computational technology, to approach and validate the workings of self-organized systems, and may well encompass related but different computational methods used by other workers.

Introduction

The computer revolution which began in the mid-twentieth century was based on digital logic and the Turing machine. It has led to ever more powerful hardware and software, using syntactical processing, operating within fixed rules and fixed environments. However, it seems that we are still no nearer to answering the question: can machines think? And we are certainly not close at present to providing a machine which has the power to think in the way that humans do. The human brain does not appear to operate like a Turing machine, for a thinking machine requires semantic processing as well as syntactic. Is there, then, a 'logic' in the form of a computational grammar, that can be applied to something like the human or AI brain?

We will suggest that there is, and that it is not digital logic, which is just a particular outcome of this more fundamental one. The 'natural logic' or grammar that we will propose is based more on 0 and 0, than 0 and 1, and it uses the universal constraint of totality zero in a systematic way to create a process which appears to be similar to the way that many natural processes operate. This 'natural' grammar generates digital logic but is not confined to it. Rather it describes the internal organization of an open system / process of increasing complexity unguided or managed by any outside source, but which displays newly emergent properties at every stage consistent with those that have gone before and takes the self-similar form of a universal attractor of dimension 2. It therefore describes in every accepted sense a process of self-organization.

Rewriting and rewrite systems

All computer programming based on the Turing machine can be described in terms of a rewrite (or production) system. Rewriting is a general process involving strings and alphabets, classified according to what is rewritten. A rewrite system provides a set of equations that characterises a system of computation based on the use of rewrite rules. In setting up such a system we are concerned with much the same components as in languages and grammars, namely an alphabet (finite or otherwise) with symbols, strings or words, sentences and expressions; and also rules governing what is rewritten and how it is rewritten. A conventional rewrite system requires 4 fixed components:

- alphabet
- rewrite rules (productions)
- a start 'axiom' or symbol
- stopping criteria

As an example (following the suggestion of our colleague, Bernard Diaz), we may take a process for generating Fibonacci numbers. We have two rules: p1: A $\rightarrow$ B; p2: B $\rightarrow$ AB. We start with generation 0, and the single symbol A. Apply rule p1, and replace A with B. In generation 2, B becomes AB. Now, A becomes B, and B becomes AB … etc.

N=0	A	length	1
N=1	$\rightarrow$B		1
N=2	$\rightarrow$ AB		2
N=3	$\rightarrow$ BAB		3
N=4	$\rightarrow$ ABBAB		5
N=5	$\rightarrow$ BABABBAB		8
N=6	$\rightarrow$ ABBABBABABBAB		13
N=7	$\rightarrow$ BABABBAB...		

We may notice how the rules A $\rightarrow$ B and B $\rightarrow$ AB reproduce the structure of 3-dimensional algebra

$$\boldsymbol{i} \rightarrow \boldsymbol{j} \qquad \boldsymbol{j} \rightarrow \boldsymbol{i}\,\boldsymbol{j} = \boldsymbol{k}$$

A string like ABBABBABABBAB seems to be creating a fractal-like structure in 3-dimensional space – but in the plane (as for example in holography).

Universal system / alphabet

It is possible to create a universal alphabet – an alphabet used in a universal rewrite system (Rowlands and Diaz, 2002, Diaz and Rowlands, 2006). A universal rewrite system is one which allows all four elements – alphabet, start object, rules, stopping criteria – to be varied. At the beginning of such a system the alphabet is often the same as the start object. The alphabet might also be the rules.

One universal alphabet allows us to do this if we assume that the universe, or any alphabet within it, is always entirely nothing. The principal and only assumption is a zero totality state, with no unique description, i.e. one that is infinitely degenerate. In other words, we have to keep describing the alphabet in a way that ensures that it is always new, but still zero.

The start position is zero – but is any such state and not unique, and there is no limit or stopping criteria, because the final state is also always zero. We can conceive it as defining a *zero attractor*. Any non-zero deviation from 0, say R, necessarily incorporates an automatic mechanism for recovering the zero, say 'conjugate' R^*, but the zero totality (R, R^*), does not define a *unique* zero, and must always define a new structure.

How, then, do we know the new structure is new? We know this because it defines the position of (R, R^*) within it, and the process continues indefinitely. In effect, we define a series of *cardinalities*, but ones based on zero, rather than on infinity.

The process is most conveniently displayed (though not defined) by a 'concatenation' or placing together, with no algebraic significance, of the alphabet with respect to either its components ('subalphabets') or itself. If the alphabet describes a cardinality or totality, then anything other than itself will necessarily be a 'subalphabet' and the concatenation will yield nothing new. The only other option will be concatenation with itself, which, to ensure that the cardinality is not unique, must yield a new cardinality or zero totality alphabet.

The condition *create* symbolised by $\Rightarrow$ means that every alphabet produces a new one which subsumes itself as a component. The condition *conserve* symbolised by $\rightarrow$ means that nothing new is created except by extending the alphabet. Creation is always of a new zero. The process can be recursive, creating everything E (all symbols) at once, or iterative, creating one symbol only. In fact it is both iterative and recursive. The process is fractal and can begin or end at any stage. Self-similarity exists at all stages. And as has been shown (Rowlands, 2007), it describes / creates both time and 3D space, and therefore, the process may be thought of as prior to both. That is, the condition of non-unique cardinality requires that

(subalphabet) (alphabet) $\rightarrow$ (alphabet)

there is nothing new

(alphabet) (alphabet) $\Rightarrow$ (new alphabet)

the zero totality not unique

The nature of the new alphabet produced by $\Rightarrow$ will always be determined by the need to satisfy $\rightarrow$ in all possible cases. We can only find out what a new alphabet will look like when we have worked out all the ways in which concatenation with its subalphabets will yield only itself.

Suppose, then, that our first zero totality alphabet has the form (R, R^*). Applying the conserve mechanism ($\rightarrow$) by concatenating it with its subalphabets should produce nothing new. So

$$(R)\,(R, R^*) \rightarrow (R, R^*)$$
$$(R^*)\,(R, R^*) \rightarrow (R^*, R) \equiv (R, R^*)$$

No concept of 'ordering' is required by concatenation, but each term must be distinct, so we can easily show that these concatenations lead to rules of the form:

$$(R)\,(R) \rightarrow (R)\,;\ (R^*)\,(R) \rightarrow (R^*)\,;$$
$$(R)\,(R^*) \rightarrow (R^*)\,;\ (R^*)\,(R^*) \rightarrow (R)$$

Now we need to show that the zero-totality alphabet (R, R^*) cannot be unique, and that concatenation with itself (or 'create') must produce a new conjugated alphabet. Something like (A, A^*), with the terms undefined, would be indistinguishable from (R, R^*), so the only way of creating a new alphabet which is distinguishable is by incorporating the old one. However, we must do this in such a way that the conserve mechanism still applies, that is, that the subalphabets yield nothing new. So we try

$$(R, R^*)\,(R, R^*) \Rightarrow (R, R^*, A, A^*)$$

Applying the conserve mechanism ($\rightarrow$) to this new alphabet, by concatenation with the subalphabets, produces

$$(R)\,(R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*) \equiv (R, R^*, A, A^*)$$
$$(R^*)\,(R, R^*, A, A^*) \rightarrow (R^*, R, A^*, A) \equiv (R, R^*, A, A^*)$$
$$(A)\,(R, R^*, A, A^*) \rightarrow (A, A^*, R^*, R) \equiv (R, R^*, A, A^*)$$
$$(A^*)\,(R, R^*, A, A^*) \rightarrow (A^*, A, R, R^*) \equiv (R, R^*, A, A^*)$$

The order of the terms is different (as it has to be) but the total is the same.

We soon realise that (R, R^*) and (A, A^*) can only be different if

$$A\,A = R^*, \text{ etc., while } R\,R = R$$

At the next stage, we have a problem, for

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*)$$

would lead to new concatenated *terms* like AB, AB^* when we apply the conserve mechanism ($\rightarrow$). So we must include these in advance, as in

$$(R, R^*, A, A^*)\,(R, R^*, A, A^*)$$
$$\Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$

However, a new complication arises when we successively perform the conserve operation with (R), (R^*), (A), (A^*), (B), (B^*), (AB), (AB^*), to leave the totality unchanged. The process is straightforward for the first six operations:

$$(R)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$

$$(R^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (R^*, R, A^*, A, B^*, B, AB^*, AB)$$

$$(A)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (A, A^*, R^*, R, AB, AB^*, B, B^*)$$

$$(A^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (A^*, A, R, R^*, AB^*, AB, B^*, B)$$

$$(B)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (B, B^*, AB, AB^*, R^*, R, A, A^*)$$

$$(B^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (B^*, B, AB^*, AB, R, R^*, A^*, A)$$

But there is still a problem, when we come to the operations of the concatenated terms, such as (AB) and (AB^*) on themselves and on each other. There are two clear possibilities for the concatenation (AB) (AB), and we can regard these as the 'commutative' and 'anticommutative' options:

$$(AB)\,(AB) \;\rightarrow\; (R) \qquad (\textit{commutative})$$
$$(AB)\,(AB) \;\rightarrow\; (R^*) \qquad (\textit{anticommutative})$$

However, there is really no choice, for *only the anticommutative option* leads to something new. The commutative option leaves A and B indistinguishable and so does not extend the alphabet. So we are obliged to default on the anticommutative option, and the last two concatenations become:

$$(AB)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (AB, AB^*, B, B^*, A, A^*, R^*, R)$$

$$(AB^*)\,(R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$\rightarrow (AB^*, AB, B^*, B, A^*, A, R, R^*)$$

This solves the problem for A and B, but it cannot be repeated to include new terms, such as (C), (D), etc., when the alphabet is extended at higher stages because some inconsistency will always reveal itself at some point in the analysis. Anticommutativity effectively produces a closed 'cycle' with components (A, B, AB) and their conjugates, and excludes any further C, D …-type term of anticommuting with them. However, the separate successive cycles, say, (A, B, AB), (C, D, CD), etc., can be introduced commutatively into the structure, and this can be continued indefinitely. All the terms have a unique identity *because they each have a unique partner*.

The alphabets generated by the create process ($\Rightarrow$) thus lead to a regular series of identically structured closed anticommutative cycles, each of which commutes with all others. It is the structure which is familiar to us as the infinite series of finite (binary) integers of conventional mathematics. The closed cycles produce an infinite ordinal sequence, establishing for the first time the meaning of both the number 1 and the binary symbol 1 as it appears in classical Boolean logic as a conjugation state of 0, and the alphabets structure themselves as an infinite series of binary digits.

Mathematics and digital logic can thus be regarded as emergent properties of an ongoing rewrite process with no specific defined starting point that can be reconstructed endlessly in a fractal manner with self-similarity at all stages, and it can be shown that the zero attractor is related to the Golden number 1.618 ... .

Once we have integers, the rest of the constructible number system follows automatically, together with arithmetical operations, such as addition and multiplication, and application of the constructed number systems to the undefined state with which the process begins indicates that, because it is not intrinsically discrete, it can be interpreted in terms of a continuity of real numbers in the Cantorian sense.

Anticommutativity is (in the construction) fundamentally the same as 3-dimensionality. In fact, by being forced to introduce anticommutativity, we simultaneously create the concepts of *discreteness* and *dimensionality* (specifically 3-dimensionality). Physically, 3-dimensionality requires discreteness, and discreteness requires 3-dimensionality. 3-dimensionality, or anticommutativity, is the ultimate source of discreteness in a zero totality universe.

The iterative process can be represented in symbolic form in the table:

	0	Δ_a	Δ_b	Δ_c	**…**	Δ_n
0	00	$0\Delta_a$	$0\Delta_b$	$0\Delta_c$		$0\Delta_n$
Δ_a	$\Delta_a 0$	$\Delta_a\Delta_a$	$\Delta_a\Delta_b$	$\Delta_a\Delta_c$		$\Delta_a\Delta_n$
Δ_b	$\Delta_b 0$	$\Delta_b\Delta_a$	$\Delta_b\Delta_b$	$\Delta_b\Delta_c$		$\Delta_b\Delta_n$
Δ_c	$\Delta_c 0$	$\Delta_c\Delta_a$	$\Delta_c\Delta_b$	$\Delta_c\Delta_c$		$\Delta_c\Delta_n$
:						
Δ_n	$\Delta_n 0$	$\Delta_n\Delta_a$	$\Delta_n\Delta_b$	$\Delta_n\Delta_c$		$\Delta_n\Delta_n$

Here, the Δ symbols represent the alphabets:

Δ_a (R)
Δ_b (R, R^*)
Δ_c (R, R^*, A, A^*)
Δ_d $(R, R^*, A, A^*, B, B^*, AB, AB^*)$
Δ_e $(R, R^*, A, A^*, B, B^*, AB, AB^*, C, C^*, AC, AC^*, BC, BC^*, ABC, ABC^*)$ ...

We have here a new logic based on cardinality of totality zero. The cardinality means that the rewrite system is self-similar (fractal of dimension 2). It also generates bifurcation at each stage and 3-dimensionality at every other. Hill and Rowlands (2008) have shown that the fractality associated with privileging cardinality relates to a diagram (Kaufmann, 2008 illustrating the statement by Heinz von Foerster: 'I am the observed link between myself and observing myself'), which also generates Fibonacci numbers. The Fibonacci numbers can be generated from the lines crossed.

The universal rewrite alphabet has the same 'nilpotent' formulation $X_n^2 = 0$ at all levels of its rewrite structure, making it self-similar of fractal dimension 2 and therefore is embeddable in the complex plane. It thus corresponds to the universal fractal attractor of the Golden number and relates to the wave behaviour seen at the boundary of the Mandelbrot set.

Application to physics

Having found that the system generates its own arithmetic and algebra, we can now construct an algebraic version of its stages, with units (independent from the values, which are not fixed):

$(1, -1)$
$(1, -1) \times (1, \boldsymbol{i}_1)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots$

Repetition is clearly established at the fourth stage. At this point we have what can be recognised as a Clifford algebra – the algebra of 3-D space, where the vectors **i**, **j**, **k** are constructed from $\boldsymbol{i}_1 \boldsymbol{i}_2 . \boldsymbol{j}_1 \boldsymbol{i}_2, \boldsymbol{i}_1 \boldsymbol{j}_1 \boldsymbol{i}_2$, and $\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{i}_1 \boldsymbol{j}_1 = \boldsymbol{k}_1$ and $\boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{i}_2 \boldsymbol{j}_2 = \boldsymbol{k}_2$ are (mutually commutative) quaternion algebras of the form $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$.

It would seem that we have an application of the system to mathematics, with binary arithmetic, digital logic and Clifford algebra appearing as consequences of the system rather than starting assumptions. The fact that the system becomes repetitive with the algebra of 3-D space also suggests that it might apply to the physical world as well. For this to be true we would expect each successive alphabet to have a physical meaning, and for each to be independently valid at the same time.

The presence of a repeating unit might suggest that we don't need to keep extending the alphabet indefinitely in describing a physical system, but that we can somehow build the repetition and self-similarity into the structure. In fact, we will find that there is a particular way in which we can do exactly this, and describe the indefinite extension using just the first four alphabets, which might then take on a special physical significance.

The first four alphabets, in fact, describe (when fully worked out) the known characteristics of the four physical parameters mass, time, charge and space, from which all others are derived (Rowlands, 2007). Their combination as independent entities requires a double Clifford algebra, equivalent to the sixth stage in the rewrite structure, or a double 3-D space.

$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots$$

This is exactly the mathematical structure required to describe fundamental physical particles (fermions) in the Dirac equation.

Ultimately, according to fundamental physics, the universe is described by fermions (e.g. electrons) and their (boson) interactions. There is nothing else to be described. Space, time, interactions, etc., emerge simply as manifestations of fermion structure. If we can solve this, then we can solve physics, and, implicitly, everything that builds upon it – chemistry, biology, etc.

Essentially, the fundamental object defined by physics requires a combination of the independent units from the four alphabets:

$(1, -1)$
$(1, -1) \times (1, \boldsymbol{i}_1)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$

And the combination required,

$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$$
$$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \times (1, i),$$

when fully worked out, generates a total of 64 units (+ and –). Writing $\boldsymbol{i}\,\boldsymbol{j} = \boldsymbol{k}$, etc., we could start from the 8 physical units needed to represent mass, charge, space and time:

$$1, \quad \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, \quad \mathbf{i}, \mathbf{j}, \mathbf{k}, \quad i$$

But, using combinations, we can reduce the minimum number of starting units to just 5:

$$i\boldsymbol{k}, \quad i\mathbf{i}, i\mathbf{j}, i\mathbf{k}, \quad \boldsymbol{j}$$

This is precisely what physics does, as these units codify a particle's

energy	momentum	rest mass
$i\boldsymbol{k}E$	$i\mathbf{p}$	$\boldsymbol{j}m$

The remarkable thing is that, when we take $(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ to represent a fundamental physical unit (particle), we find that it immediately solves the problem of indefinite extension. This is because $(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ is a *nilpotent*, a square root of zero. The equation

$$(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m) = 0 \qquad (1)$$

is simply the relativistic and quantum mechanical conservation of energy and momentum (Rowlands, 2007).

And, if this object incorporates all the alphabets needed to create a repetitive sequence, when we seek to generate the next alphabet by squaring, we find that it is automatically zeroed, so zeroing all higher alphabets which incorporate it. We describe the world through an indefinite succession of such units.

The special nature of ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$) with regard to self-organization comes when we consider the nature of reality as a zero totality. If an object such as ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$) is imagined as 'created' from nothing, then what is left, which in physics we describe as 'vacuum' or the rest of the universe, must be a kind of mirror image, –($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$).

Not only do these quantities add to nothing. They also combine (in the quantum mechanical sense of probabilities) to nothing, because

$$-(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0.$$

We have, in fact (in addition to defining Pauli exclusion), described the perfect self-organizing system, exactly as we know the fundamental physical particle to be.

And because, physically, Pauli exclusion applies, ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$) is not a fixed object. E can represent its variation with time ($\partial / \partial t$) and $\mathbf{p}$ its variation with space ($\partial / \partial x$), as in relativistic quantum mechanics, and both can incorporate any number of terms representing interaction with other particles. So what we have here is a system whose nature changes automatically by negotiation with its entire environment. Because it conserves energy only over the entire universe, it must also be an open dissipative system responding to the known laws of thermodynamics. (It is also only true for real particles: virtual particles are not nilpotent and so not self-organizing.)

No more perfect description of a self-organizing system could exist, and we know that real physical particles have a remarkable ability to retain their identity under any number of changes in their environmental conditions. In fact, we could go further, and state that a system which is defined as self-organizing must have the ability to respond to its environment or to change it in the way that the fundamental particle does, and that such conditions essentially always operate on the nilpotent principle or some approximation to it in large-scale systems. This, we could propose, is the ultimate in semantic processing.

Mathematics, we have seen, can be regarded as an emergent property of an ongoing rewrite process that has no defined starting point and that can be reconstructed endlessly in a fractal manner with self-similarity at all stages. It uses the whole rewrite structure to describe (serially) a restricted part of nature (syntactical processing). Physics effectively uses a restricted part of the rewrite structure to describe (in parallel) the whole of nature at once.

From the technical point of view, we can consider the E and $\mathbf{p}$ terms in the nilpotent as either the differential operators codifying the variations in time and space coordinates of the system's components or as the values of energy and momentum (or equivalent) that result from such operations. In quantum mechanics the operators operate on phase terms (functions of space and time), which are completely defined once the operator is specified, and the result is a nilpotent amplitude. We can use this to write down a nilpotent Dirac equation, which, for a free particle, would be

$$\left(\mp i\boldsymbol{k}\frac{\partial}{\partial t} \mp \boldsymbol{i}\nabla + \boldsymbol{j}m\right)(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{-i(Et-\mathbf{p.r})} = 0, \quad (2)$$

though, in fact, only the operator is necessary to specify the entire quantum system. The functions defined by such an operator (called 'monogenic' Jose Almeida, 2008) are those with a zero vector gradient.

Quantum mechanics

Here, we have reversed the usual progression from quantum mechanics to computation by showing how quantum mechanics can arise from a computational process. However, the concept is more general in that the quantum mechanics is only one application of the process. So we have no need now to have recourse to quantum mechanical arguments to justify our use of equations and processes that have the structure of quantum mechanics in applications, and which arguably are unlikely to be directly quantum mechanical. For example at least one version of (2) below can be expressed in a way that is also classical, and has particular relevance to digital applications.

This requires a discrete or anticommutative differentiation process, explored by Kauffman (2004), with a correspondingly discrete wavefunction. Here, we define a discrete differentiation of the function F, which preserves the Leibniz chain rule, by taking the commutators:

$$\frac{\partial F}{\partial t} = [F, \mathcal{H}] = [F, E] \quad \text{and} \quad \frac{\partial F}{\partial X_i} = [F, P_i]$$

with $\mathcal{H} = E$ and P_i representing energy and momentum operators, with the further assumption that, with velocity operators not in evidence, we may use $\partial F / \partial t$ rather than dF / dt. The mass term (which has only a passive role in quantum mechanics) disappears in the operator, though it is retained in the amplitude. We also eliminate the phase factor. Suppose we define a nilpotent amplitude

$$\psi = \boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m$$

and an operator

$$\mathcal{D} = \boldsymbol{ik}\frac{\partial}{\partial t} + \boldsymbol{i}\mathbf{i}\frac{\partial}{\partial X_1} + \boldsymbol{i}\mathbf{j}\frac{\partial}{\partial X_2} + \boldsymbol{i}\mathbf{k}\frac{\partial}{\partial X_3},$$

with $\frac{\partial \psi}{\partial t} = [\psi, \mathcal{H}] = [\psi, E]$ and $\frac{\partial \psi}{\partial X_i} = [\psi, P_i]$. (3)

After some basic algebraic manipulation, we obtain

$$-\mathcal{D}\psi = i\psi(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m) + i(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m)\psi - 2i(E^2 - P_1^2 - P_2^2 - P_3^2 - m^2).$$

When ψ is nilpotent, then

$$\mathcal{D}\psi = \left(\boldsymbol{i}k\frac{\partial}{\partial t} + \boldsymbol{i}\nabla\right)\psi = 0.$$

$$\mathcal{D}\psi = \left(\pm \boldsymbol{i}k\frac{\partial}{\partial t} \pm \boldsymbol{i}\nabla\right)(\pm \boldsymbol{i}kE \pm \boldsymbol{i}\mathbf{i}P_1 \pm \boldsymbol{i}\mathbf{j}P_2 \pm \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m) = 0 \quad (4)$$

Significantly, this equation is also valid in classical as well as quantum contexts, where nilpotency is a fundamental condition. In defining the differentials by (3), we had no need to premultiply by i or $ih / 2\pi$ as is normally required in canonical quantization. This is because the differential operator eliminates the mass term and so (4) is valid whether premultiplied by i or $ih / 2\pi$ or not. Hence, equation (4), though originating in quantum mechanics in our derivation, is also valid in discrete, but non-quantum contexts.

Self-organization

Nilpotent or monogenic functions always operate in such a way that the system becomes a mirror image of its environment. The system changes in response to its environment and also directly changes it in a specifically defined way. Such systems are not confined to quantum physics, or even to physics. Characteristic mathematical patterns suggest that they apply equally in many other areas, including biology and living systems, especially in relation to the genetic code. They are the result of applying universal rewrite in a semantic view of nature as opposed to the syntactic view generated by mathematics.

The characteristic patterns include: double 3-dimensionality (one manifest, one hidden); 2 vector spaces needed to create a 'singularity' as boundary; double helical structure; a 5-fold broken symmetry (which results from 2 × 3-D) – hence, the appearance of Fibonacci numbers and the golden section; uniqueness of the objects and unique birthordering; irreversibility; dissipation; a harmonic oscillator aspect to the structure; a tendency for aggregation (Van der Waals forces, etc.); single-handedness (chirality); and a characteristic appearance of key numbers, including those relevant to Platonic solids (with their dual spaces) and groups like E_8. (Most of these are discussed by Hill and Rowlands in Rowlands, 2007).

The key numbers (Hill and Rowlands, 2009) ultimately all derive from the repeated 2s and 3s involved in the universal rewrite system. 2 comes from duality (the conservation principle, summation to 0) and 3 from anticommutativity (a significant aspect of the creation principle, nonuniqueness of zero). 5s are not natural aspects of the system but emerge from broken symmetries. They repeat in all geometries up to dimension 8, and in Lie groups up to E_8.

Even more significant than these is the necessity for dual spaces. These are the bases of the algebraic structure used in the nilpotent equations (1), (2) and (4), and they are what give all self-organizing systems their dynamic evolution. In effect, one space represents the space of real observation, and the other the 'vacuum' space determining the potential changes. Nilpotency is the result of making these dual, so that they contain the same information, though packaged in different ways. In effect, one describes the system's effect on the environment, while the other describes the environment's effect on the system. This applies as long as the system can be described discretely, whether or not it is 'quantum', as currently accepted.

The rewrite structure predicts a 'staircase' of structures as nilpotents at one level become units at the next. The fermion is the perfect self-organizing system. Bosons, nuclei, atoms, molecules are constructed from fermions and can be described in terms of the same quantum mechanics, so are self-organizing. The overall structure can be maintained at a larger scale where there is coherence (e.g. in crystals). But coherence can be lost where there are many interacting parts. There may even be chaos. However, despite this, a self-similar emergent order can be predicted at higher levels, based on the universal Golden attractor.

Self-organization of this kind seems to have been observed in solid state matter. Coldea et al (2010) have shown that linked chains of magnetic atoms of cobalt, one atom wide, transform, under a magnetic field applied at right angles to an aligned spin into a quantum critical state – a quantum version of a fractal pattern. Resonances occur in the golden ratio of 1.618…, reflecting an underlying E_8 symmetry.

There is also evidence for similar self-organization in biology. Biology is concerned entirely with self-organizing systems interacting with environments. Replication, using the processes of transcription and translation, provides a classic example of a system interacting with a mirror image environment, with a 'hidden' 3-D structure determining the behaviour of the one we observe. Extensive investigation (Hill and Rowlands, 2008) seems to suggest that it follows a similar pattern to the self-organizing of the fermion and vacuum in quantum mechanics.

Perhaps also the human brain is organised according to this 'natural' logic, which would thus explain its semantic, as well as syntactic, capabilities. A quantum field model, proposed by Giuseppe Vitiello (2008), appears to have the right characteristics to make this feasible. Here the need to describe the brain thermodynamically as a dissipative system leads to a quantum field approach based on positive energy output by the brain mirroring negative energy output by the environment. A nilpotent wavefunction provides the perfect mathematical structure for modelling this process, and, potentially, AI.

The nilpotent structure ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$) in physical systems can also be seen as an expression of conservation of angular momentum, either of the system or of the system in conjunction with its environment. Large-scale systems, such as galaxies, which act as a collective unit under gravity, clearly conserve angular momentum in this way, and can be modelled by expressions such as ($\boldsymbol{ik}E$ + $\boldsymbol{i}\mathbf{p}$ + $\boldsymbol{j}m$).

In astronomy, Morris et al (2006) have discovered a helix-shaped nebula near the centre of the Milky Way, which stretches for 80 light years, but looks like the classic image of a DNA molecule. The nebula was apparently formed because the massive object at the heart of the galaxy has a magnetic field powerful enough to force the gas cloud to twist along its field lines.

Independent work by Tsytovich et al (2007) has demonstrated that particles in a plasma can undergo self-organization as electronic charges become separated and the plasma becomes polarized. This effect results in microscopic strands of solid particles that twist into corkscrew shapes, or helical structures resembling DNA. These helical strands are themselves electronically charged and are attracted to each other. They can divide to form two copies of the original structure and can also interact to induce changes in their neighbours. They can even evolve into yet more structures as less stable ones break down, leaving behind only the 'fittest' structures in the plasma.

Large-scale – and many other – systems also respond to the holographic principle ('t Hooft, 1993, Susskind, 1995), which proposes that the entire information about a system is encoded in the bounding area (either as distance × distance or distance × time). Here this is equivalent to the combined terms $\boldsymbol{ik}E$ and $\boldsymbol{i}\mathbf{p}$, the $\boldsymbol{j}m$ then being an automatic consequence. E and $\mathbf{p}$, in effect, represent the equivalent of a time and a single dimension in space (the spatial direction being fixed with the angular momentum and orthogonal to the time), and so code the entire information about the system in the equivalent of a bounding area.

The holographic principle can be considered as a characteristic signature of a nilpotent, self-organizing system with its planar fractality. A related one is quantum holography, now unequivocally demonstrated in the case of 'quantum holographic encoding in a two-dimensional electron gas' (Moon et al, 2009). Here, the nilpotent structure naturally accommodates phase ($\boldsymbol{ik}E$), amplitude ($\boldsymbol{i}\mathbf{p}$) and reference phase ($\boldsymbol{j}m$). As before, we can reconstruct the entire structure with knowledge of just two terms, say, phase and reference phase.

Quantum holography is significantly different from classical holography in being nondegenerate, and here we may invoke the uniqueness of the nilpotent condition for the fermion as being the source of the uniqueness of the semantic logic which the physical description makes possible. There is only one universal condition possible at any instant, and it can be described by a unique birthordering among fermionic states described by nilpotent operators.

Many self-organizing systems, typically in biology or chemistry, show order emerging from chaos. The universal rewrite system shows that 'quantum chaos' leads directly to nilpotence as the quantum criterion for the required universal fractal behaviour and quantum holographic patterns and signal pattern recognition, via the phase parameterized universal fractal golden number attractor where every such nilpotent pattern exhibits uniqueness and other features, as specified in Rowlands (2007).

The universal rewrite system shows that what we need for a repeating unit is a double vector space. The two three dimensional spaces then relate to the 3D Heisenberg Lie Group and its nilpotent Lie algebra and their dual / inverses, and make quantum holography possible via Fourier transform action.

A physical realisation appears to be already available in the form of magnetic resonance imaging, where the systematic theorising of Walter Schempp is leading to advanced practical results (1992, 2006). Schempp has shown how quantum holography (using harmonic analysis on the 3D Heisenberg Lie group) applies to the quantum control of Magnetic Resonance Imaging (MRI) Systems and which applies equally to those of Synthetic Aperture Radars, leads to a third generation of MRI systems using Magnetic Tensor Diffusion Tomography and results, for the first time, in amazing 3D perspective pictures of the signal paths in the human brain.

The unique birthordering of nilpotent states / systems suggests that another characteristic indicator of its presence is the Quantum Carnot Engine (QCE) extended model of thermodynamic irreversibility (Scully et al, 2003), consisting of a single heat bath of an ensemble of Standard Model elementary particles, which retains a small amount of quantum coherence / entanglement, so as to constitute new emergent fermion states of matter.

Of course, it is a significant aim of cybernetics to create artificial self-organizing systems that operate in rapidly changing environments, as natural ones do, and it is important to ask whether this will ever be possible with systems built upon digital logic. In our view, the problem is not the use of digital logic (which gives the correct syntactic structure) but the lack of an organizing principle.

However, if nilpotency provides the required principle, then it is possible to approach a digital construction via the discrete version of nilpotency, represented in equation (4). Digital logic allows us to handle amplitudes much easier than phases, and commutators more readily than differentials, and we can imagine constructing systems, with a version of (4) representing discrete amplitude changes, allowing an asymptotic approach to nilpotent conditions.

Also, while the E and P terms may represent 'energy' and 'momentum' in the physics application, in the more general context they can be regarded rather as the parts of the system that respond to respective changes of the time and space coordinates. Various ways of approaching such a construction may be imagined, for example, using the cellular automata promoted by Wolfram (2002), the discrete anticipatory computation advocated by Dubois (1998, 1999), the theory of the cybernetic and intelligent machine based on Lie commutators of Fatmi, Jessel, Marcer and Resconi (1990), and Fatmi and Resconi's New (semantic) Computing Principle (1988).

Conclusion

From evidence from systems organised at many different scales, it can be proposed that a logical structure can be developed which encompasses the general class of self-organising systems, and which is based on the universal rewrite system and the principle of nilpotency, in which the system and environment have (in mathematical terms) a space-time variation, defined by the phase or amplitude, which preserves a dualistic mirror-image relationship. In Marcer and Rowlands (2007) we made the case to explain 'how intelligence evolved' in a self organized universe using the universal rewrite process. It is possible now to see a route to developing both theory and applications relating to this process.

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