A Hierarchy of Symmetries

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Symmetry

We are not very good observers – science is a struggle for us. But we have developed one particular talent along with our evolution that serves us well. This is pattern recognition.

This is fortunate, for, everywhere in Nature, and especially in physics, there are hints that symmetry is the key to deeper understanding.

And physics has shown that the symmetries are often ‘broken’, that is disguised or hidden. A classic example is that between space and time, which are combined in relativity, but which remain obstinately different.

Some questions

Which are the most fundamental symmetries?

Where does symmetry come from?

How do the most fundamental symmetries help to explain the subject?

Why are some symmetries broken and what does broken symmetry really mean?

Many symmetries are expressed in some way using integers. Which are the most important?

I would like to propose here that there is a hierarchy of symmetries, emerging at a very fundamental level, all of which are interlinked.

The Origin of Symmetry

A philosophical starting-point. The ultimate origin of symmetry in physics is zero totality.

The sum of every single thing in the universe is precisely nothing.

Nature as a whole has no definable characteristic.

Zero is the only logical starting-point. If we start from anywhere else we have to explain it. Zero is the only idea we couldn't conceivably explain.

Duality and anticommutativity

Where do we go from zero?

I will give a semi-empirical answer, though it is possible to do it more fundamentally.

The major symmetries in physics begin with just two ideas:
duality and anticommutativity

There are only two fundamental numbers or integers:
2 and 3

Everything else is a variation of these.

Anticommutativity is like creation, duality is like conservation.

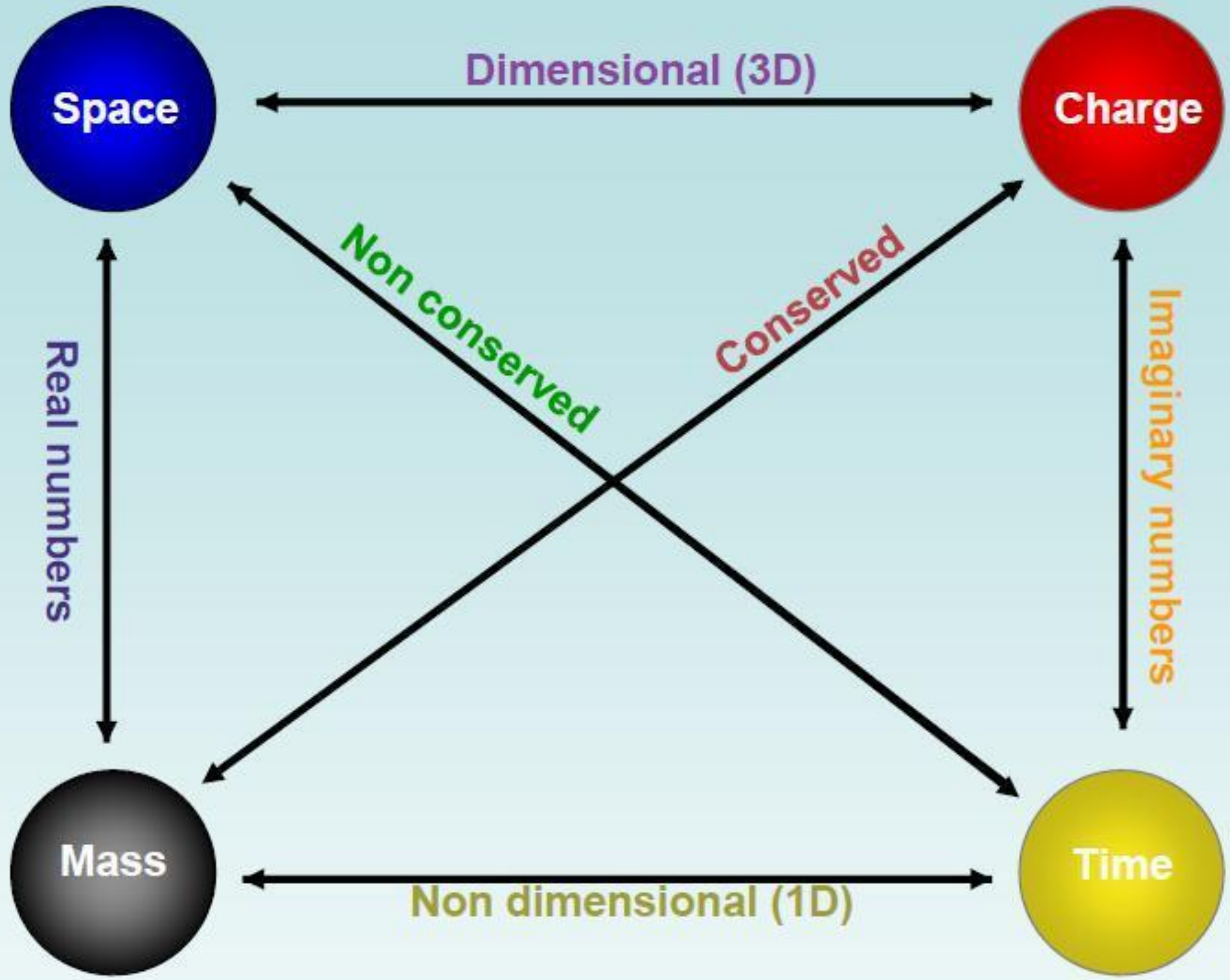
Space, time, mass and charge

Let's start with a symmetry that is not well known, but which I believe to be foundational to physics. This is between the four fundamental parameters

SPACE	TIME	MASS	CHARGE

Here, mass has the more expansive meaning incorporating energy, and charge incorporates the sources of all 3 gauge interactions (electric, strong and weak). The symmetry-breaking between the charges is an emergent property, which we will show later emerges from algebra.

It is possible to represent the properties of these parameters symmetrically in terms of a Klein-4 group.



Klein 4 Group D2

Space	real	nonconserved	dimensional
Time	imaginary	nonconserved	nondimensional
Mass	real	conserved	nondimensional
Charge	imaginary	conserved	dimensional

Some consequences of the symmetry

Many physical, and even some mathematical, facts, not fully understood, may be seen principally as consequences of this symmetry.

Conservation laws and Noether's theorem
Irreversibility of time
The unipolarity of mass
Why like charges repel but masses attract
The need for antistates
Lepton and baryon conservation – nondecay of the proton
Standard and nonstandard analysis, arithmetic, geometry
Zeno's paradox
The irreversibility paradox
Gauge invariance, translation and rotation symmetry

…

Clifford algebra of 3-D space

One of the key aspects of the exactness of the symmetry between the parameters is that space, to be truly symmetrical to charge in its 3-dimensionality, is not just an ordinary vector, but one which has the properties of a Clifford algebra:

i **j** **k**	vector		
*i***i** *i***j** *i***k**	bivector	pseudovector	quaternion
i	trivector	pseudoscalar	complex
1	scalar		

The space-time and charge-mass groupings then become exact mirror images, 3 real + 1 imaginary against 3 imaginary + 1 real.

Clifford algebra of 3-D space

The vectors of physics are what Hestenes called multivariate vectors, isomorphic to Pauli matrices and complexified quaternions, with a full product

$$\mathbf{ab} = \mathbf{a.b} + i\,\mathbf{a} \times \mathbf{b}$$

and a built-in concept of spin (which comes from the $i\,\mathbf{a} \times \mathbf{b}$ term).

Hestenes showed, for example, that if we used the full product $\nabla\nabla\psi$ for a multivariate vector ∇ instead of the scalar product $\nabla\mathbf{.}\nabla\psi$ for an ordinary vector ∇, we could obtain spin ½ for an electron in a magnetic field from the nonrelativistic *Schrödinger equation*.

The significance of quaternions

Space and time become a 4-vector with three real parts and one imaginary, by symmetry with the mass and charge quaternion, with three imaginary parts and one real.

space	time	charge	mass
$\mathbf{i}x\ \mathbf{j}y\ \mathbf{k}z$	it	$\boldsymbol{i}s\ \boldsymbol{j}e\ \boldsymbol{k}w$	$1m$

Vectors, like quaternions, are also anticommutative.

Representation in algebraic symbols

The group properties can be represented very simply using algebraic symbols for the properties / antiproperties:

mass	x	y	z
time	$-x$	$-y$	z
charge	x	$-y$	$-z$
space	$-x$	y	$-z$

In algebraic terms, this is a conceptual zero.

The dual group

There is also a dual version of this group, which reverses one set of properties / antiproperties, say the first:

mass*	$-x$	y	z
time*	x	$-y$	z
charge*	$-x$	$-y$	$-z$
space*	x	y	$-z$

The physical meaning of this will become clear later.

Combining the group and dual group

There is something like a C_2 symmetry between the dual D_2 structures, and the $C_2 \times D_2$ of order 8 creates a larger structure of the form:

*	*M*	*C*	*S*	*T*	*M**	*C**	*S**	*T**
M	*M*	*C*	*S*	*T*	*M**	*C**	*S**	*T**
C	*C*	*M**	*T*	*S**	*C**	*M*	*T**	*S*
S	*S*	*T**	*M**	*C*	*S**	*T*	*M*	*C**
T	*T*	*S*	*C**	*M**	*T**	*S**	*C*	*M*
*M**	*M**	*C**	*S**	*T**	*M*	*C*	*S*	*T*
*C**	*C**	*M*	*T**	*S*	*C*	*M**	*T*	*S**
*S**	*S**	*T*	*M*	*C**	*S*	*T**	*M**	*C*
*T**	*T**	*S**	*C*	*M*	*T*	*S*	*C**	*M**

Combining the group and dual group

Remarkably, this structure is identical to that of the quaternion group (*Q*):

*	1	***i***	***j***	***k***	–1	–***i***	–***j***	–***k***
1	1	***i***	***j***	***k***	–1	–***i***	–***j***	–***k***
i	***i***	–1	***k***	–***j***	–***i***	1	–***k***	***j***
j	***j***	–***k***	–1	***i***	–***j***	***k***	1	–***i***
k	***k***	***j***	–***i***	–1	–***k***	–***j***	***i***	1
–1	–1	–***i***	–***j***	–***k***	1	***i***	***j***	***k***
–***i***	–***i***	1	–***k***	***j***	***i***	–1	***k***	–***j***
–***j***	–***j***	***k***	1	–***i***	***j***	–***k***	–1	***i***
–***k***	–***k***	–***j***	***i***	1	***k***	***j***	–***i***	–1

The exactness of the symmetry

The symmetry may be assumed to be absolutely exact – no exception to this rule has ever been found in forty years. And this condition can be used to put constraints on physics to derive laws and states of matter.

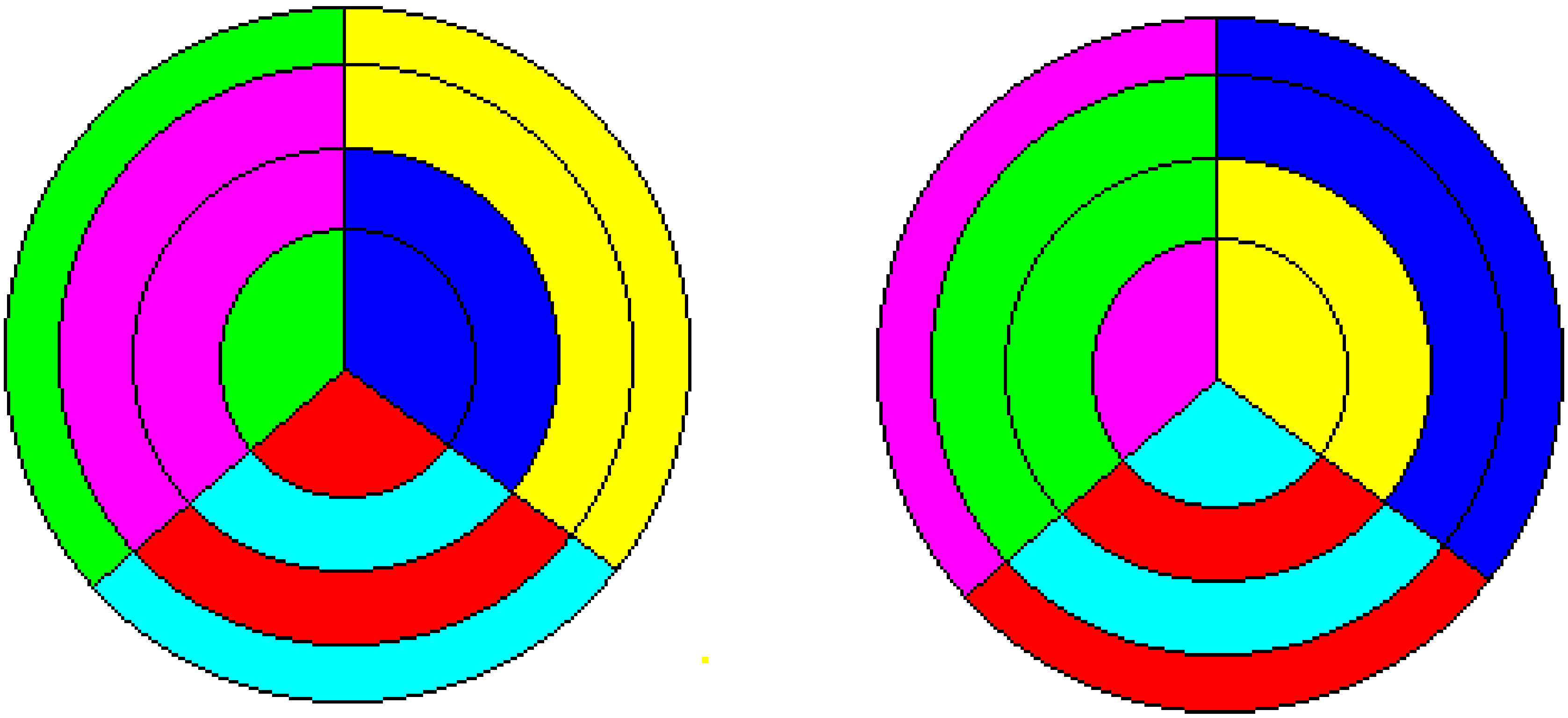
We can also develop a number of representations, which not only show the absoluteness of the symmetry, but also the centrality to the whole concept of the idea of 3-dimensionality.

The exactness of the symmetry

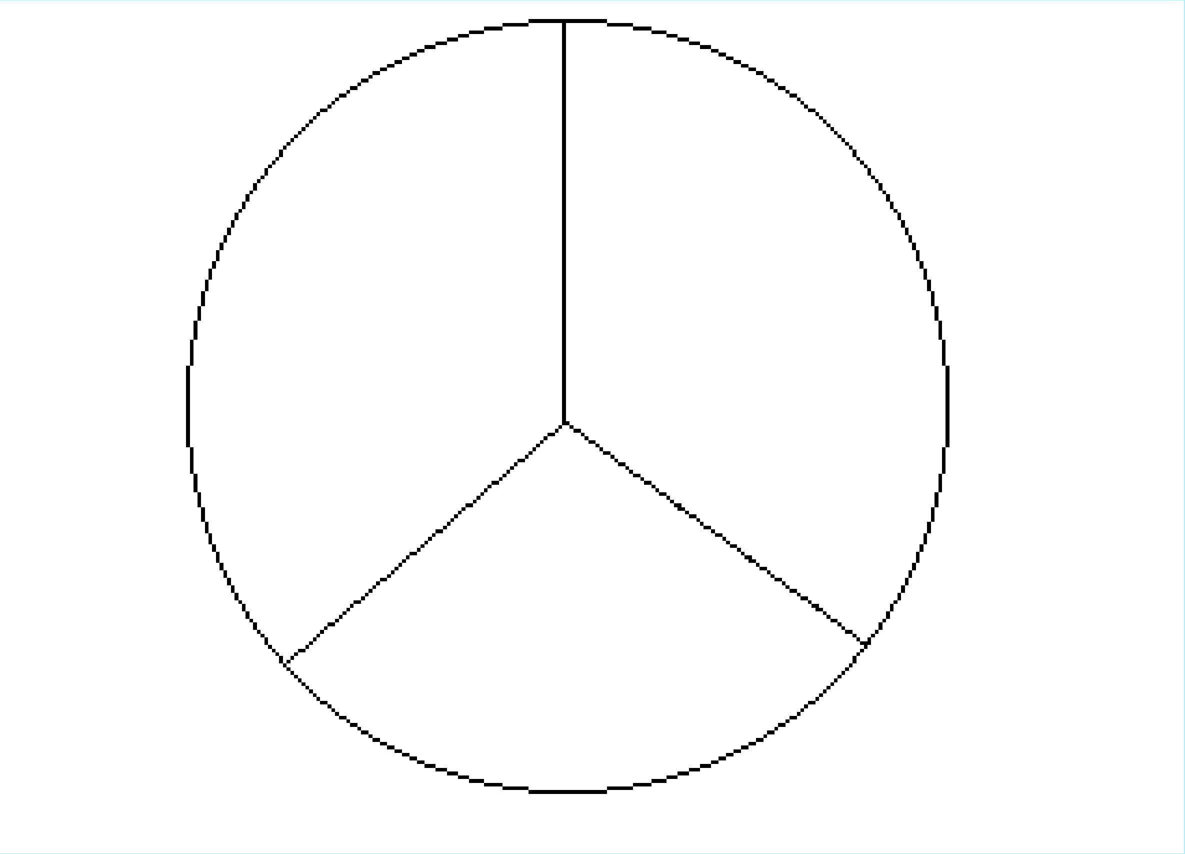
A perfect symmetry between 4 parameters means that only the properties of one parameter need be assumed. The others then emerge automatically like kaleidoscopic images. It is, in principle, arbitrary which parameter we assume to begin with, as the following visual representations will show.

The representations also suggest that 3-dimensionality or anticommutativity is a fundamental component of the symmetry.

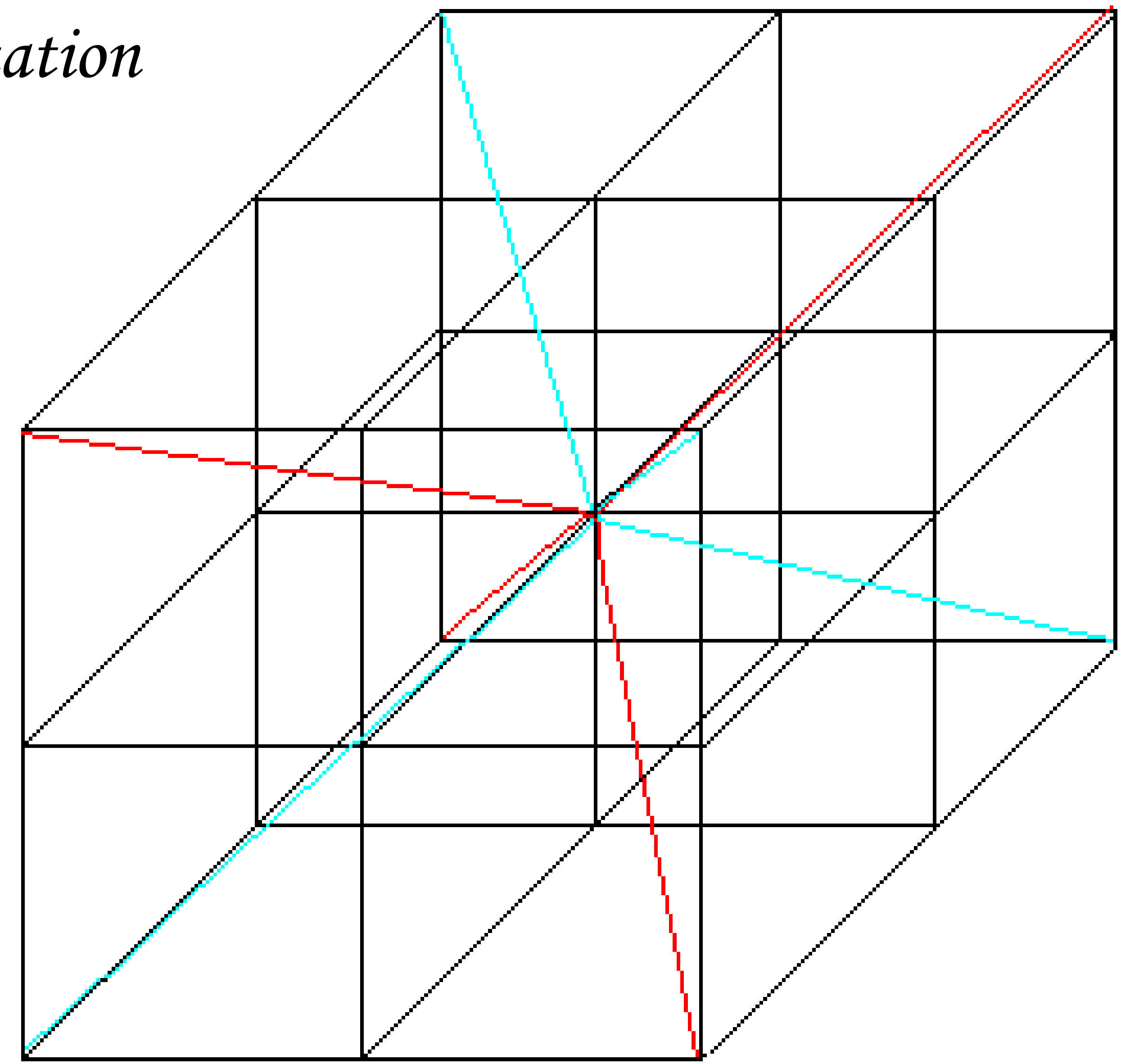
Colour representation



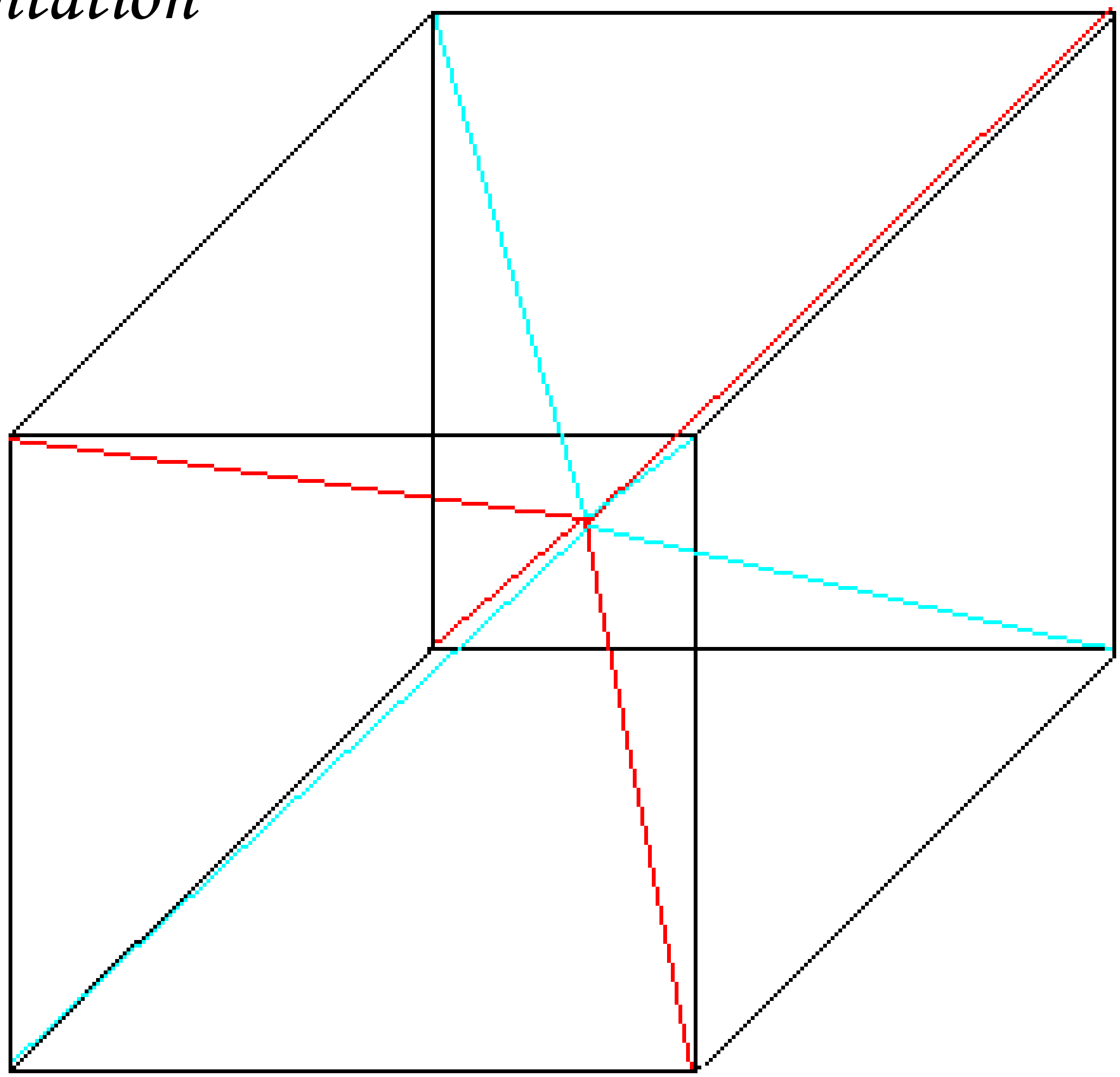
Summation of the colour sectors



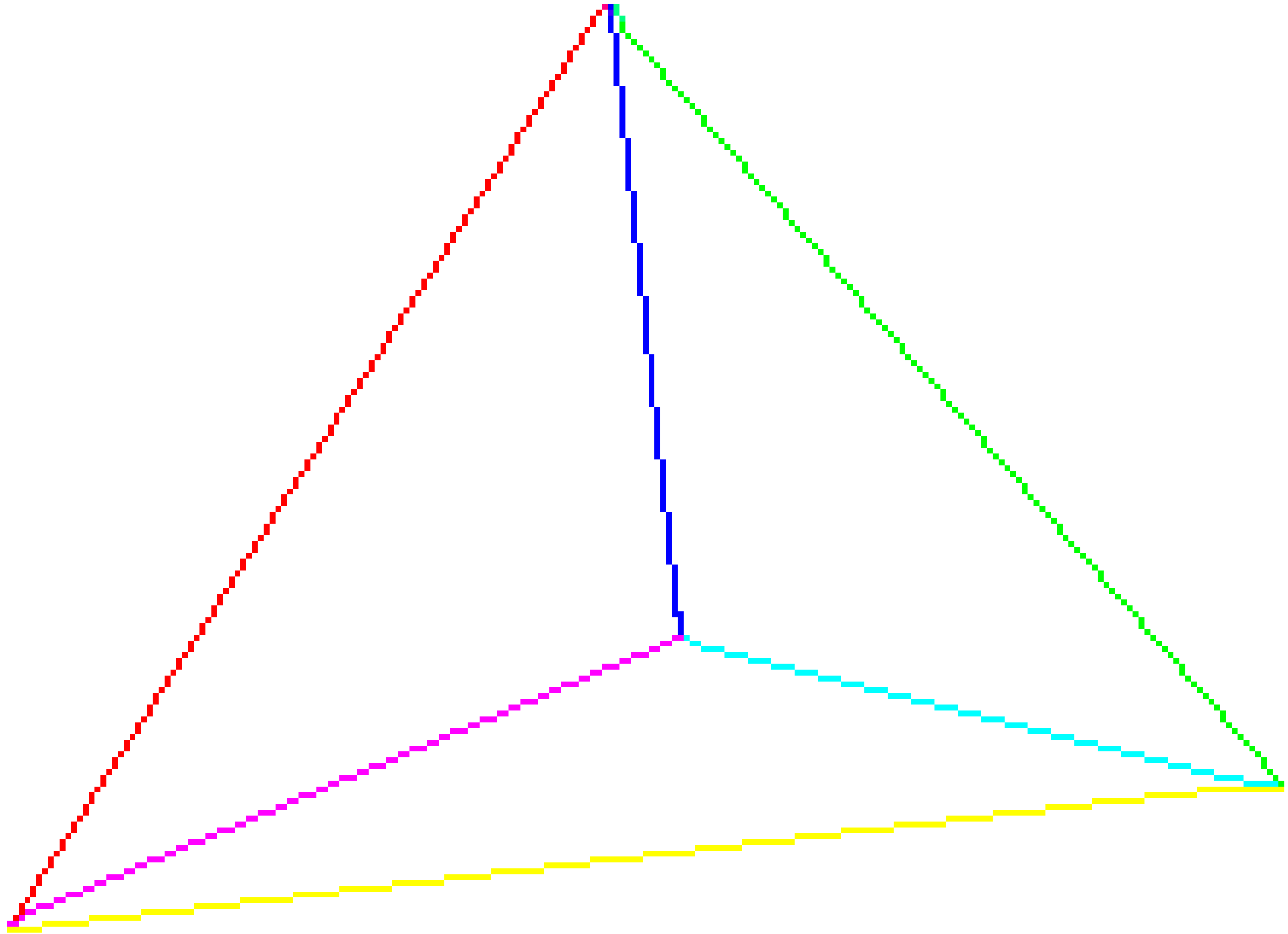
3-D Representation



3-D Representation



Tetrahedral Representation



The four algebras

What is striking about the parameters and their properties is that they are *purely abstract*. They can be reduced, in effect, to pure algebra.

Real / Imaginary and Commutative / Anticommutative are obviously so.

But Conserved / Nonconserved can also be shown to be purely algebraic.

They also each have their *own* algebra, which serves to define them. Their ‘physical’ properties come solely from this algebra.

The four algebras

Mass	1	scalar
Time	*i*	pseudoscalar
Charge	***i j k***	quaternion
Space	**i j k**	vector

The first three are subalgebras of the last, and combine to produce a version of it, let's say **i j k**. In other words they are equivalent to a 'vector space', an 'antispace' to counter **i j k**.

We see why space appears to have a privileged status.

Clifford algebra of 3-D space

It has 3 subalgebras: bivector / pseudovector / quaternion, composed of:

*i***i** *i***j** *i***k**	bivector	pseudovector	quaternion
1	scalar		

trivector / pseudoscalar / complex, composed of:

i	trivector	pseudoscalar	complex
1	scalar		

and scalar, with just a single unit:

1	scalar

An alternative space

The three parameters other than space produce a combined vector-like structure, even though there is no physical vector quantity associated with them.

mass	scalar	1	
time	pseudoscalar	*i*	1
charge	quaternion	***i j k***	1
	pseudovector	*i***i** *i***j** *i***k**	1
	bivector		
COMBINED	vector	**i j k**	***i j k*** *i* 1
STRUCTURE		**i j k**	*i***i** *i***j** *i***k** *i* 1

This is what we will call vacuum space.

An alternative space

We now have another symmetry, leading to zero totality:

Space				Everything else		
				Mass	1	scalar
				Time	*i*	pseudoscalar
				Charge	***i j k***	quaternion
				--		
Space	**i j k**	vector		Antispace	**i j k**	vector
				Vacuum space		

Mathematical hierarchy

We note that the algebras of charge, time, mass are subalgebras of vector algebra.

It seems that, though all the parameters are equivalent in the group structure, they also produce a mathematical hierarchy, which suggests an ‘evolutionary’ structure in a logical, not a time sequence.

This evolution can, in fact, be derived, and applied much more generally as a fundamental information process.

It seems to operate in mathematics, computer science, chemistry and biology, as well as in more complex aspects of physics.

We can also derive many aspects of the complexity directly.

Packaging physical information: the combined algebra

Time	Space	Mass	Charge
i	**i j k**	1	***i j k***
pseudoscalar	vector	scalar	quaternion

Working out every possible combination of the four requires 64 units.

This turns out to be the algebra of the Dirac equation, the relativistic quantum mechanical equation of the fermion, the only true fundamental object that we know must exist.

There are 64 possible products of the 8 units

$(\pm 1, \pm i)$	4	units
$(\pm 1, \pm i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k})$	12	units
$(\pm 1, \pm i) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	12	units
$(\pm 1, \pm i) \times (\mathbf{i}, \mathbf{j}, \mathbf{k}) \times (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	36	units

The algebra of the Dirac equation

The + and – versions of the units:

$\boldsymbol{i}$	$\boldsymbol{j}*$	$\boldsymbol{k}$	$i\boldsymbol{i}$	$i\boldsymbol{j}$	$i\boldsymbol{k}*$	i	1
$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$	$i\mathbf{i}$	$i\mathbf{i}$	$i\mathbf{k}$		
$\mathbf{i}\boldsymbol{i}*$	$\mathbf{i}\boldsymbol{j}$	$\mathbf{i}\boldsymbol{k}$	$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$		
$\mathbf{j}\boldsymbol{i}*$	$\mathbf{j}\boldsymbol{j}$	$\mathbf{j}\boldsymbol{k}$	$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$		
$\mathbf{k}\boldsymbol{i}*$	$\mathbf{k}\boldsymbol{j}$	$\mathbf{k}\boldsymbol{k}$	$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{i}$		

form a *group*. The simplest starting point for a group is to find the *generators*.

These are the set of elements within the group that are sufficient to generate it by multiplication. Here they are marked *.

The algebra of the Dirac equation

Since vectors are complexified quaternions and quaternions are complexified vectors, we obtain an identical algebra if we use complexified double quaternions:

i	***j****	***k***	*i**i***	*i**j***	*i**k****	*i*	1
i	***j***	***k***	*i**i***	*i**i***	*i**k***		
ii*	***ij***	***ik***	*i**ii***	*i**ij***	*i**ik***		
ji*	***jj***	***jk***	*i**ji***	*i**jj***	*i**jk***		
ki*	***kj***	***kk***	*i**ki***	*i**kj***	*i**kk***		

The algebra of the Dirac equation

There is also a double vector version:

i	**j***	**k**	*i***i**	*i***j**	*i***k***	*i*	1
i	**j**	**k**	*i***i**	*i***i**	*i***k**		
ii*	**ij**	**ik**	*i***ii**	*i***ij**	*i***ik**		
ji*	**jj**	**jk**	*i***ji**	*i***jj**	*i***jk**		
ki*	**kj**	**kk**	*i***ki**	*i***kj**	*i***ki**		

form a *group*. The simplest starting point for a group is to find the *generators*.

These are the set of elements within the group that are sufficient to generate it by multiplication. Here they are marked *.

Generators of the Dirac algebra

We started with eight basic units, but, by the time that we have worked out all the possible combinations of vectors, scalars, pseudoscalars and quaternions, we find that the Dirac algebra has 32 possible units or 64 if you have + and – signs.

This group of order 64 requires only 5 generators. There are many ways of selecting these, but all such pentad sets have the same overall structure.

However, the most efficient way of generating the 2×32 is to start with five *composites*, rather than eight primitives.

Generators of the algebra

All the sets of 5 generators have the same pattern, as we can see by splitting up the 64 units into 1, –1, *i* and – *i*, and 12 sets of 5 generators, each of which generates the entire group:

1	*i*				–1	–*i*			
***i*i**	***i*j**	***i*k**	*i**k***	***j***	–***i*i**	–***i*j**	–***i*k**	–*i**k***	–***j***
***j*i**	***j*j**	***j*k**	*i**i***	***k***	–***j*i**	–***j*j**	–***j*k**	–*i**j***	–***i***
***k*i**	***k*j**	***k*k**	*i**j***	***i***	–***k*i**	–***k*j**	–***k*k**	–*i**j***	–***i***
*i***i*i***	*i***i*j***	*i***i*k***	*i***k**	**j**	–*i***i*i***	–*i***i*j***	–*i***i*k***	–*i***k**	–**j**
*i***j*i***	*i***j*j***	*i***j*k***	*i***i**	**k**	–*i***j*i***	–*i***j*j***	–*i***j*k***	–*i***i**	–**k**
*i***k*i***	*i***k*j***	*i***k*k***	*i***j**	**i**	–*i***k*i***	–*i***k*j***	–*i***k*k***	–*i***j**	–**i**

Symmetry-breaking introduced

The creation of any set of 5 generators requires symmetry-breaking of one 3-D quantity. From the perfect symmetry of

$$i \qquad \mathbf{i} \qquad \mathbf{j} \qquad \mathbf{k} \qquad 1 \qquad \boldsymbol{i} \qquad \boldsymbol{j} \qquad \boldsymbol{k}$$

we rearrange to produce:

i	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$	1
$\boldsymbol{k}$		$\boldsymbol{i}$		$\boldsymbol{j}$

and finally:

$$i\boldsymbol{k} \qquad \boldsymbol{i}\mathbf{i} \qquad \boldsymbol{i}\mathbf{j} \qquad \boldsymbol{i}\mathbf{k} \qquad 1\boldsymbol{j}$$

Symmetry-breaking introduced

The symmetry-breaking has an impact on the nature of the parameters involved:

Time	Space	Mass	Charge
i	**i j k**	1	***i j k***

Take one of each of ***i j k*** on to each of the other three.

*i**k***	**i*i* j*i* k*i***	1***j***	

You have to break the symmetry **i j k** of one space or the other ***i j k***.

Symmetry-breaking introduced

Physically, to create the generators we have to distribute the charge units onto the other parameters:

Time	Space	Mass	Charge
i	**i j k**	1	***i j k***

This creates new ‘compound’ (and ‘quantized’) physical quantities:

i***k***	**i*i* j*i* k*i***	1***j***
Energy	Momentum	Rest mass
E	$p_x\ p_y\ p_z$	m

The combined object as nilpotent

The combined object is *nilpotent*, squaring to zero, because

$$(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)\,(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m) = 0$$

and we can identify this as Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0$$

The Dirac equation simply quantizes the nilpotent equation, using differentials in time and space for E and p.

The Dirac equation

Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0$$

or

$$(i\mathbf{k}E + \mathbf{i}\mathbf{i}p_x + \mathbf{j}\mathbf{i}p_y + \mathbf{k}\mathbf{i}p_y + \mathbf{j}m)\;(i\mathbf{k}E + \mathbf{i}\mathbf{i}p_x + \mathbf{j}\mathbf{i}p_y + \mathbf{k}\mathbf{i}p_y + \mathbf{j}m) = 0$$

becomes

$$\left(\mp \boldsymbol{k}\frac{\partial}{\partial t} \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m\right)\left(\pm i\boldsymbol{k}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m\right)e^{-i(Et - \mathbf{p.r})} = 0$$

by simultaneously applying nonconservation and conservation. Here, we note there are four sign variations in E and **p**. The fact that this is reduced by nilpotency from eight leads to another symmetry-breaking. We lose a degree of freedom, leading to chirality.

The nilpotent Dirac equation

Written out in full the four components are:

$(ikE + i\mathbf{p} + jm)$	fermion spin up
$(ikE - i\mathbf{p} + jm)$	fermion spin down
$(-ikE + i\mathbf{p} + jm)$	antifermion spin down
$(-ikE - i\mathbf{p} + jm)$	antifermion spin up

The signs are, of course, intrinsically arbitrary, but it is convenient to identify the four states by adopting a convention.

The spinor structure

The spinor properties of the algebra still hold, even when we don't use a matrix representation, and ψ is a 4-component spinor, incorporating fermion / antifermion and spin up / down states.

We can easily identify these with the arbitrary sign options for the iE and **p** (or **σ.p**) terms. This is accommodated in the nilpotent formalism by transforming ($i\mathbf{k}E$ + $\mathbf{i}\mathbf{p}$ + $\mathbf{j}m$) into a column vector with four sign combinations of iE and **p**,

which may be written in abbreviated form as ($\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m$). Using an accepted convention, this can be either operator or amplitude. The symmetry between operator and amplitude is another leading to 0.

The spinor structure

$$(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \rightarrow 0$$

gives us both relativity and quantum mechanics – a version which is much simpler and seemingly more powerful than conventional QM.

In QM we take the first bracket as an operator acting on a phase factor. The E and $\mathbf{p}$ terms can include any number of potentials or interactions with other particles.

Squaring to 0 gives us the Pauli exclusion principle, because if any 2 particles are the same, their combination is 0.

Fermions with interactions

In this form, we don't even need an equation, just an operator of the form $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$ because the operator will uniquely determine the phase factor needed to produce a nilpotent amplitude. Rather than using a conventional form of the Dirac equation, we find the phase factor such that, using the defined operator,

$$(\text{operator acting on phase factor})^2 = \text{amplitude}^2 = 0.$$

If the operator has a more complicated form than that of the free particle, the phase factor will, of course, be no longer a simple exponential but the amplitude will still be a nilpotent.

The broken symmetry between charges

$i\mathbf{k}$	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$1\boldsymbol{j}$

also gives us the broken symmetry between the 3 charges

weak	strong	electric

which now adopt the characteristics of the mathematical objects they are connected to:

pseudoscalar	vector	scalar
$SU(2)$	$SU(3)$	$U(1)$

The connections can be demonstrated with full rigour.

The quantized phase space of the fermion

The meaning of the dual group has also now become clear. By attaching quaternion operators to the time, space and mass terms, we have effectively exchanged real and imaginary terms, and the fourth term is provided by the spin angular momentum, which provides the same role in the quantized system as the overall charge structure.

The first group effectively provides the entire ontology of physics, the second the means of observing it. The two groups together give us the quantized phase space of the fermion.

The quantized phase space of the fermion

In quantum mechanics, however, the two groups are not independent, since the second is derived from the first, and they do not commute. Ontology and epistemology are not independent.

Also, the fourth term in the dual group (angular momentum) is not independent of the others.

Ultimately the parameter group and its dual form ‘cancel’, not to zero, but to h.

The octonion mapping

There is also an octonion mapping of the 8 algebraic units of the 4 parameters, and to make it more exact we can take imaginary values of the spatial coordinates.

Here, we see that the antiassociative parts of the multiplication table are those which have no physical meaning. It is as though antiassociativity were actually created to define the boundaries.

In addition, group structure plays a key and defining role in both physics and the universal rewrite structure which we have described for all information systems, and antiassociativity prevents the octonions from being defined as a group.

The parameters arranged in algebraic units

*	m	s	e	w	t	x	y	z
m	m	s	e	w	t	x	y	z
s	s	–m	w	–e	x	t	–z	y
e	e	–w	–m	s	y	z	–t	–x
w	w	e	–s	–m	z	–y	x	–t
t	t	–x	–y	–z	–m	s	e	w
x	x	t	–z	y	–s	–m	–w	e
y	y	z	t	–x	–e	w	–m	–s
z	z	–y	x	t	–w	–e	s	–m

The octonion mapping

*	1	i	j	k	e	f	g	h
1	1	i	j	k	e	f	g	h
i	i	–1	k	–j	f	–e	–h	g
j	j	–k	–1	i	g	h	–e	–f
k	k	j	–i	–1	h	–g	f	–e
e	e	–f	–g	–h	–1	i	j	k
f	f	e	–h	g	–i	–1	–k	j
g	g	h	e	–f	–j	k	–1	–i
h	h	–g	f	e	–k	–j	i	–1

The H4 algebra

A particular subalgebra of the 64-part algebra creates a symmetry between the two spaces which remains unbroken.

This is the H4 algebra, which can be obtained using *coupled* quaternions, with units 1, ***ii***, ***jj***, ***kk***.

The result is a cyclic but commutative algebra with multiplication rules

$$\mathbf{ii}\ \mathbf{ii} = \mathbf{jj}\ \mathbf{jj} = \mathbf{kk}\ \mathbf{kk} = 1$$
$$\mathbf{ii}\ \mathbf{jj} = \mathbf{jj}\ \mathbf{ii} = \mathbf{kk}$$
$$\mathbf{jj}\ \mathbf{kk} = \mathbf{kk}\ \mathbf{jj} = \mathbf{ii}$$
$$\mathbf{kk}\ \mathbf{ii} = \mathbf{ii}\ \mathbf{kk} = \mathbf{jj}$$

The H4 algebra

The same algebra can be achieved with the *negative* values of the paired vector units 1, –**ii**, –**jj**, –**kk**. (1 is equivalent here to –*ii*.)

This time we have:

(–**ii**) (–**ii**) = (–**jj**) (–**jj**) = (–**kk**) (–**kk**) = 1
(–**ii**) (–**jj**) = (–**jj**) (–**ii**) = (–**kk**)
(–**jj**) (–**kk**) = (–**kk**) (–**kk**) = (–**ii**)
(–**kk**) (–**ii**) = (–**ii**) (–**kk**) = (–**jj**)

The H4 algebra

If we use the symbols **I** = ***ii*** = –**ii**, **J** = ***jj*** = –**jj**, **K** = ***kk*** = –**kk**, 1, to represent this algebra, we can structure the relationships in a group table:

*****	1	**I**	**J**	**K**
1	1	**I**	**J**	**K**
I	**I**	1	**K**	**J**
J	**J**	**K**	1	**I**
K	**K**	**I**	**J**	1

The group is a Klein-4 group, exactly like the parameter group.

Defining a dual space spinor

All the standard aspects of spin and helicity are easily recovered with nilpotent quantum mechanics. This means that it is possible to find a spinor structure which will generate the NQM state vector. A set of primitive idempotents constructing a spinor can be defined in terms of the H4 algebra, constructed from the dual vector spaces:

$$(1 - \mathbf{ii} - \mathbf{jj} - \mathbf{kk}) / 4$$
$$(1 - \mathbf{ii} + \mathbf{jj} + \mathbf{kk}) / 4$$
$$(1 + \mathbf{ii} - \mathbf{jj} + \mathbf{kk}) / 4$$
$$(1 + \mathbf{ii} + \mathbf{jj} - \mathbf{kk}) / 4$$

As required the 4 terms add up to 1, and are orthogonal as well as idempotent, all products between them being 0.

Defining a dual space spinor

The same terms can be generated using coupled quaternions rather than vectors:

$$(1 + \boldsymbol{i}\boldsymbol{i} + \boldsymbol{j}\boldsymbol{j} + \boldsymbol{k}\boldsymbol{k}) / 4$$
$$(1 + \boldsymbol{i}\boldsymbol{i} - \boldsymbol{j}\boldsymbol{j} - \boldsymbol{k}\boldsymbol{k}) / 4$$
$$(1 - \boldsymbol{i}\boldsymbol{i} + \boldsymbol{j}\boldsymbol{j} - \boldsymbol{k}\boldsymbol{k}) / 4$$
$$(1 - \boldsymbol{i}\boldsymbol{i} - \boldsymbol{j}\boldsymbol{j} + \boldsymbol{k}\boldsymbol{k}) / 4$$

The ‘spaces’ in the spinor structure are notably completely dual. The orthogonality condition effectively creates a quartic space structure with zero size, a point-particle. There is a notable chirality, however, in that the signs cannot be completely reversed. Ultimately, when the spinors are applied to constructing the Dirac wavefunction, this manifests itself in the positive sign of the *m* term.

Defining a dual space spinor

We can reduce the 4-spinor expressions to a 2-spinor form, which does not have the chirality. The chirality emerges from introducing the symmetry of 3-dimensionality (with its inherent anticommutativity) into a system based on the commutative symmetry of 2-dimensionality (based on complex numbers or equivalent). So the non-chiral 2-spinor form is essentially (using the quaternion version) something like

$$(1 + \mathbf{i}\mathbf{i}) / 2$$
$$(1 - \mathbf{i}\mathbf{i}) / 2$$

which can be reduced to the more familiar projection operators

$$(1 - i\mathbf{i}) / 2$$
$$(1 + i\mathbf{i}) / 2$$

Defining a dual space spinor

With ***ij*** for ***ii***, these are equivalent to $(1 - \gamma^5) / 2$ and $(1 + \gamma^5) / 2$. But, if ***i*** and ***i*** are each part of a 3-D structure (as, ultimately, is required by γ^5), then 'doubling' the complexity through dimensionalization and application to **p** (as in the universal rewrite system) makes 2 × 2 into 3 + 1 and so introduces the chirality that we see in (1) and (2), as

$(1 - \boldsymbol{jj}) / 2$ and $(1 + \boldsymbol{jj}) / 2$

necessarily presuppose the existence of

$(1 - \boldsymbol{kk}) / 2$ and $(1 + \boldsymbol{kk}) / 2$

Defining a dual space spinor

The result shows the fundamental difference between symmetries based on the number 2 and those based on the number 3. In many respects, uniqueness in Nature (which presupposes symmetry-breaking) comes about through a ‘competition’ between these symmetries.

We may note that the Weyl equation (Dirac equation for massless particles, which applies to condensed matter pseudoparticles, but not fundamental ones) effectively ‘halves the wavefunction’, eliminating the RH fermion and LH antifermion using these projection operators.

Geometrically, the 2-component Pauli spinor is specified by a Möbius band, requiring a spatial twist; the 4-component Dirac spinor by a Klein bottle which is made of two Möbius bands.

Defining a dual space spinor

By making the massless fermion one-handed, chirality is introduced even with the 2-spinor structure, but the chirality is yet another broken symmetry, because introducing a mass term also introduces a degree of the other handedness.

The structure of the Dirac operator makes the chirality left-handed as opposed to the right-handed chirality of human beings (demonstrated by words such as *L* sinister for left-handed and *OE* widdershins for anticlockwise).

Vacuum

Another way of looking at Pauli exclusion leads to another symmetry. Here, we say that Nature represents a totality of zero, and if you imagine creating a particle (with all the potentials representing its interactions) in the form

$$(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

then you must structure the rest of the universe, so that it can be represented by

$$-\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

Vacuum

The nilpotent formalism indicates that a fermion ‘constructs’ its own vacuum, or the entire ‘universe’ in which it operates, and we can consider the vacuum to be ‘delocalised’ to the extent that the fermion is ‘localised’.

We can consider the nilpotency as defining the interaction between the localised fermionic state and the delocalised vacuum, with which it is uniquely self-dual, the phase being the mechanism through which this is accomplished.

We can also consider Pauli exclusion as saying that no two fermions can share the same vacuum.

Vacuum

The ‘hole’ left by creating the particle from nothing is the rest of the universe needed to maintain it in that state. We give it the name vacuum.

So the vacuum for one particle cannot be the vacuum for any other.

We can also think of the dual ‘spaces’ represented by **i j k** and **i j k** as combining together to produce *zero totality* in a point particle with zero size. It is the only way we can produce discrete points in space.

Boundary of a boundary

We can additionally understand the behaviour of fermion and vacuum in terms of more abstract mathematics. Set boundaries themselves have vanishing boundaries. The boundary of a boundary is zero:

$$\partial\partial = \partial^2 = 0$$

For A as subspace of the entire space X, then the boundary ∂A is the intersection of the closures of A and of the complement of A or X – A, the closure being the union of the set and its boundary.

Here the universe is X, the fermion A, the rest of the universe X – A.

The point-fermion is itself a boundary. The boundary of the fermion is 0. This is nilpotency.

Vacuum space

If we look at the four components of the fermion:

$(ikE + \mathbf{i}\mathbf{p} + \mathbf{j}m)$	fermion spin up
$(ikE - \mathbf{i}\mathbf{p} + \mathbf{j}m)$	fermion spin down
$(-ikE + \mathbf{i}\mathbf{p} + \mathbf{j}m)$	antifermion spin down
$(-ikE - \mathbf{i}\mathbf{p} + \mathbf{j}m)$	antifermion spin up

We see that 2 have $+E$ and two have $-E$. Where are those with $-E$?
The answer is that they are in the vacuum space. There are as many antifermions as fermions. However, the chirality we have built into the structure (and that we can derive conventionally from the Dirac equation) means that only those in real space are observable.

One-fermion theory of the universe

This view of vacuum suggests many new ideas. The one-fermion theory of the universe (a modification of the one-electron theory of Stueckelberg-Wheeler-Feynman) becomes an increasingly attractive option. Here, the whole structure of the universe can be represented by a fermion in an endless succession of backward and forward time states. The entire forward history of the universe will be contained in the fermion's vacuum or the rest of the universe associated with it. We can avoid determinism, however, because the fermion state can never be exactly defined.

The one fermion theory may be interestingly compared with a computer program. One fermion in many different states, one symbol 1 in many different states.

Negative energy

The idea that negative energy is essentially that of vacuum links it with gravity, which produces negative energy between identical masses compared with positive energy between identical charges. This means that gravitational energy can be considered as a kind of cancellation of the energies of the three gauge interactions, a gravity-gauge theory correspondence which was inherent in the present author's work long before it was adopted by string theorists.

There is a further link, via the Dirac filled vacuum and the Higgs field, through the weak interaction. Because of the complex nature of the $i\mathbf{k}E$ term, the weak charge forms an effective dipole with the vacuum of opposite energy, with the spin of the fermion acting as a weak dipole moment. Dipoles, unlike monopoles, are attractive and so create negative energy.

Local antifermions

The symbol 1 for the fermion parallels 111111111111111… for vacuum (–1), or vacuum space, alongside –*E* and –*t*.

This reflects the built-in bias for the fermion to be local and the antifermion nonlocal, though there are equal numbers of each.

How, then can antifermions be local? This comes about because there are ways of *apparently* reversing time or making negative energy positive, which relate to the fact that, although real space and vacuum (charge space) are totally dual, neither of these space is totally dual with energy-momentum space, because the latter is partly constructed from each.

Real space, vacuum space and momentum space

real space (x, y, z) DUAL vacuum / charge space (w, s, e)

UNCERTAINTY

momentum space (**p**)

Real or local antifermions and vacuum antifermions will then be a measure of how much this uncertainty affects the duality of E, t and **p**, **r**.

Real antifermions

To determine how many real, i.e. local, antifermions there are as a fraction compared to real fermions, we should look at processes such as *CP* violation, and the creation of neutrino masses, which would be unexpected in terms of pure charge considerations, but are certainly needed for neutrinos to be fermions. All the processes are related to the peculiarities of the weak interaction.

The ratio of neutrino mass to the electroweak energy scale is about 10^{-12}. Something of a similar proportion occurs in $CP = T$ violation.

In beta decay the mass factor disparity between nucleon and antineutrino is of order 7 billion. Baryon / antibaryon asymmetry is estimated from the photon / proton ratio of 10^9.

CPT Symmetry

If the lead term in the fermionic column vector, defines the fermion type, then we can show that the remaining terms are equivalent to the lead term, subjected to the respective symmetry transformations, P, T and C, by pre- and post-multiplication by the quaternion units $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ defining the *vacuum space*:

Parity	P	$\boldsymbol{i}\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i} = (\pm\, i\boldsymbol{k}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
Time reversal	T	$\boldsymbol{k}\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k} = (\mp\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
Charge conjugation	C	$-\boldsymbol{j}\,(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j} = (\mp\, i\boldsymbol{k}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$

We can easily show that $CP \equiv T$, $PT \equiv C$, and $CT \equiv P$ also apply, and that $TCP \equiv CPT \equiv$ identity as

$$\boldsymbol{k}(-\boldsymbol{j}(\boldsymbol{i}(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\boldsymbol{k})\boldsymbol{j})\boldsymbol{j} = -\boldsymbol{kji}(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\boldsymbol{ijk} = (\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

Partitioning the vacuum

The nilpotent formalism defines a continuous vacuum $-(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$ to each fermion state $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$, and this vacuum expresses the nonlocal aspect of the state.

However, the use of the operators $\mathbf{k}$, $\mathbf{i}$, $\mathbf{j}$ suggests that we can partition this state into discrete components with a dimensional structure. In fact, this is where the idempotents become relevant. If we postmultiply $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$ by the idempotent $\mathbf{k}(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$ any number of times, the only change is to introduce a scalar multiple, which can be normalized away.

$$(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)\ \mathbf{k}(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)\ \mathbf{k}(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)\ \dots \rightarrow (\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$$

Partitioning the vacuum

The identification of $\mathbf{i}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$, $\mathbf{k}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ and $\mathbf{j}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ as vacuum operators and $(i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m)$, $(-i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ and $(-i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m)$ as their respective vacuum 'reflections' at interfaces provided by P, T and C transformations suggests a new insight into the meaning of the Dirac 4-spinor.

We can now interpret the three terms other than the lead term *in the spinor* as the vacuum 'reflections' that are created with the particle. We can regard the existence of three vacuum operators as a result of a partitioning of the vacuum as a result of quantization and as a consequence of the 3-part structure observed in the nilpotent fermionic state, while the *zitterbewegung* can be taken as an indication that the vacuum is active in defining the fermionic state.

The duality of real and vacuum spaces

i, ***j***, ***k*** have many fundamental roles. They are charges, *C*, *P*, *T* transformation operators, vacuum projections onto 3 axes, indicators of fermion / antifermion / spin up / down in the Dirac spinor …

They constitute the dimensions of vacuum space, dual to real space.

The fermion has a half-integral spin because it requires simultaneously splitting the universe into two halves which are mirror images of each other at a fundamental level, but which appear asymmetric at the observational level because observation privileges the fermion singularity.

Zitterbewegung is an obvious manifestation of the duality, but, in observational terms, it privileges the creation of positive rest mass.

Reflection in a mirror

Reflection in a real mirror is due to an aspect of the electric force. The mirror produces a laterally-inverted virtual image.

The mirror reflection is actually due to the rest of the universe (‘vacuum’) of which the mirror is a component.

The virtual image is the reflection due to one component force. The mirror is constructed to concentrate the resources of vacuum almost entirely on this single force.

The origin of broken symmetries

The pattern of double 3-dimensionality emerging from duality and anticommutativity and leading to broken symmetry at order 5 is very apparent in biology as work done with Vanessa Hill testifies, and work done with Peter Marcer suggests that it underlies self-governing systems in general. We have traced the pattern and its mathematical origin in zero totality and find the same numbers and characteristic consequences repeat for Platonic and Archimidean solids (in any number of dimensions), kissing numbers, algebraic equations, quantum mechanics, fundamental particles, the periodic table, DNA / RNA, higher biological structures, etc. The 5-fold pattern is the one that links lower order systems with higher order ones. This is especially apparent in biology, where the 5-fold structure can be seen emerging in the structures in both downward and upward directions.

The origin of broken symmetries

Multiples of 2 and 3 occur over and over again in these structures, and, where they do, the dualistic and anticommutative origins can be established. Where 5 occurs it is always due to a broken symmetry, and the emergence of 5 can be seen as the key factor in the emergence of something new. Groups like E_8 are entirely constructed from such units.

As illustration we could take some structures related to fundamental particles.

Symmetries of fundamental particles

If we look at the fundamental particles, all the symmetries which apply to them seem to be constructed from smaller symmetries based on these units.

The same also applies to many of the groups thought to be of significance in this area, particularly those based on the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8.

Because the symmetry-breaking is ultimately 3-dimensional in origin (and manifested, for example, in quarks and 3 particle generations), the symmetries involved in particle groupings tend to map naturally onto geometries in 3-dimensional space.

Fermion states from the algebra

generation		*isospin*					
1	up quark	up	$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$
	down quark	down	$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$
2	charm quark	up	$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$
	strange quark	down	$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$
3	top quark	up	$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$
	bottom quark	down	$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$
1	antiup-quark	up	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
	antidown-quark	down	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
2	anticharm-quark	up	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{i}$	$-\boldsymbol{k}$
	antistrange-quark	down	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
3	antitop-quark	up	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
	antibottom-quark	down	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

Larger group structures for fermions and bosons

The second structure is especially interesting as it was created to explain fundamental particles using the E_8 symmetry, and algebra, but, as Vanessa and I showed, clearly applies on a massive scale to geometrical, chemical and biological structures.

If we look at the fundamental particles, all the symmetries which apply to them seem to be constructed from smaller symmetries based on these units. The same also applies to many of the groups thought to be of significance in this area, particularly those based on the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8.

Because the symmetry-breaking is ultimately 3-dimensional in origin (and manifested, for example, in quarks and 3 particle generations), the symmetries involved in particle groupings tend to map naturally onto geometries in 3-dimensional space.

Larger group structures for fermions and bosons

The group E_8 has long been suspected of being a possible unifying group for the fundamental particles, and was discussed as such, among other places, in *Zero to Infinity*.

In 2007, Garrett Lisi proposed that all known fermions and gauge bosons could be fitted into the 240 root vectors of the E_8 group.

The model has been heavily criticized, and doesn’t look right as it stands. Its particles don’t add up to 240, leading to a completely *ad hoc* speculation about particles needed to make up the numbers, the gravity theory is very speculative, the generations don’t arise naturally, etc. Some of the assignments seem very difficult to understand.

The Fundamental particles in E8

	quarks	leptons	bosons		f	b							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	S				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	S	I			
5	18	6	6	=	24	6	=	30	S				G
6	24	8	8	=	32	8	=	40	S	I	A		
7	36	12	12	=	48	12	=	60	S	I			G
8	48	16	16	=	64	16	=	80	S	I	A	V	
9	72	24	24	=	96	24	=	120	S	I	A		G
10	144	48	48	=	192	48	=	240	S	I	A	V	G

4 factor 2 dualities (spin up / down **S**, isospin up / down **I**, fermion / antifermion **A**, particle / vacuum **V**, and relating to space, charge, time and mass) and 1 factor 3 triplet (generations, **G**)

The Fundamental particles in E8

Particles constitute a 5, of 3 quarks + lepton + boson, which multiplies by 4 factor 2 dualities and 1 factor 3 triplet.

All products of 5 are equally artificial constructs, for example, linking fermions with bosons in the exceptional groups $E6$ to $E8$ through the fact that the last term in the 5 can be a scalar.

Notably absent from this structure are the spin 0 Higgs boson (which is not a gauge boson), the spin 2 graviton (which may not exist), and the spin 1 inertial pseudoboson (which is not really a separate particle from the photon, but a special realisation of it at the Planck energy).

The Fundamental particles in E8

In effect, we invert the derivation of 12 structures from a 5-unit pentad, and map the fermions and bosons onto a new pentad structure, of which the pseudoscalar component (the *iE* term) is 24 leptons / antileptons, and the vector component (the **p** term) 72 quarks / antiquarks.

Bosons are scalar particles, and scalars are the squared products of pseudoscalars and vectors, just as bosons are the squared products of fermions / antifermions.

So if the 24 bosons occupy the *scalar* part of the pentad (the *m* term), then we can use nilpotency to group the 96 fermions (24 leptons and 72 quarks) with the 24 bosons into a single structure with 120 fermions plus bosons, and these would seem to be represented by the stages 48 + 12 = 60, 96 + 24 = 120, 192 + 48 = 240.

Appendix: The Coupling Constants

The coupling constants for the 3 gauge interactions 'run' with different energies of interaction (μ). These are given by standard formulae, but I have previously modified the first for quarks with integral charges (Rowlands, 2007, 2014).

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_G} + \frac{3}{\pi} ln \frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G} - \frac{5}{6\pi} ln \frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{7}{4\pi} ln \frac{M_X^2}{\mu^2}$$

$$\sin^2 \theta_W = \frac{\alpha(\mu)}{\alpha_2(\mu)}$$

Appendix: The Coupling Constants

As before, we have 4 equations and 6 unknowns. For each value of μ, we need α, α_3, α_2, the grand unified coupling constant α_G, the grand unified mass M_X, and $\sin^2\theta_W$. Working out the equations, with $\sin^2\theta_W = 0.25$ unifies all three coupling constants exactly at the Planck mass with $\alpha_G =$ 1/52.4. However, it is always worth trying out variations on a theme, even if the alternatives seem less likely. We have observed how a_3 becomes something like 1/8 at m = 60 GeV, and stays reasonably close to this value for energies within the electroweak scale (80.2 to 246 GeV). This would then be comparable with a_2 = 1/32 and a = 1/128 at these energies.

Appendix: The Coupling Constants

These coupling coefficients, when multiplied by the square of the 'Planck charge' ($\hbar c$) give us the charge squared values for the strong, electric and weak interactions at these energies, which we could write as: $(1/2^2)\hbar c/2$, $(1/2^4)\hbar c/2$, $(1/2^6)\hbar c/2$. However, there is no definite reason to choose $\hbar c$ as the fundamental unit of charge squared, and equivalently, G × fundamental unit of mass squared in 'quantum gravity', rather than, say, $\hbar c/2$. If we choose $\hbar c/2$, then we can form a set of ratios for the sources of gravitational, strong, weak and electric interactions approximating to $1/2^0$, $1/2^1$, $1/2^2$ and $1/2^3$.

Appendix : The Fine Structure Constants

Maybe this is no more than numerology, but the reductions seem to follow the degrees of specification which the charges introduce and the progression from vector to pseudoscalar to scalar. They also reflect the same, as applied to angular momentum conservation. The strong charge incorporates information about magnitude, handedness and direction, the weak charge incorporates magnitude and handedness, and the electric charge magnitude only, with corresponding reductions in strength. The source of gravity has no reductions because it has no quaternionic charge structure, with + and – values, and so does not even have the 1/2 reduction of the strong charge.

Appendix : The Fine Structure Constants

If we follow the logic of using $\hbar c\ /2$, and remember that these calculations are only good to first order, then we would need to replace $M_X = M_P$ with $M_X = M_P /\sqrt{2}$.

Essentially, then, a_G becomes 1/52, $a \approx 1/127$ at the electroweak scale, with $a_2 \approx 1/31$,

while $a_3 = 0.5$ (the new ideal value), occurs at 72 MeV, which is close to the assumed 'fundamental mass' $m_f = m_e / a = 70$ MeV;

a_3 becomes 0.33 at 1 GeV, which is close to the observed value.

The End