Closure Is Zero Force"

The Oscillator Framework Read as the Phase Register of the Vacuum.Net Theory

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Companion to "Hierarchical Oscillatory Dynamics in Biological Systems" (Konstapel, 2026a). Every step and number needed to follow the argument is in this text.

Abstract

The oscillator framework describes biological organisation with phases, couplings and an order parameter R. The vacuum.net theory describes the net with a three-valued register, windows of three crossings and a closure rule. This article shows exactly where the two coincide and where they differ.

Four theorems carry the result. First, under the map from trits to the three cube roots of unity, a window closes exactly when the product of its phase factors equals one. Second, the closure rule and the linearised oscillator coupling are the same operator: the discrete Laplacian. Third, on trit-valued phases the oscillator coupling force is itself trit-valued, and a window closes exactly when the force on its centre is zero. Closure is zero force. Fourth, there are two kinds of closure with opposite stability. Homogeneous closure is stable under attractive coupling. Heterogeneous closure is stable only when the coupling weight is negative.

The consequence is exact. The order parameter R equals 1 on a homogeneous closed window and 0 on a heterogeneous one. The oscillator framework therefore sees one kind of closure and is blind to the other. A second measure, the closure fraction split into its homogeneous and heterogeneous parts, completes it. The article gives that measure in a form that can be applied to measured phase data.

1. Purpose

The oscillator article (Konstapel, 2026a) proves nine propositions about coupled phase–amplitude oscillators. It does not mention the net. This companion makes the correspondence explicit.

The rule for status is strict. A statement marked *proved* follows from the definitions below by the proof given. A statement marked *correspondence* identifies a concept in one framework with a concept in the other. It is a reading, not a derivation.

A single image runs through both frameworks. The net is a fishnet. Each knot is a crossing of the strand. Each mesh is a closed loop of threads. The oscillator article reads the same fishnet by the phase of each knot. This article reads it by whether its meshes close.

2. The net in five definitions

Only the following definitions are used.

D1 (Crossing register). Each crossing of the strand carries a trit $t\in{-1,0,+1}$: over, no crossing, under.

D2 (Window). A window is three consecutive crossings (t_{i-1},t_i,t_{i+1}) along the strand. The middle crossing is its centre.

D3 (Closure). A window is closed if its charge vanishes modulo 3: $$t_{i-1}+t_i+t_{i+1}\equiv 0 \pmod 3.$$

D4 (Two kinds of closure). A closed window is homogeneous if its three trits are equal. It is heterogeneous if its three trits are all different.

D5 (Balanced residue). For an integer k, $\sigma(k)\in{-1,0,+1}$ is the unique trit with $\sigma(k)\equiv k \pmod 3$.

Lemma 0 (only two kinds). Every closed window is either homogeneous or heterogeneous.

Proof. Suppose exactly two trits are equal, say the window contains a,a,b with $b\neq a$. Closure requires $2a+b\equiv 0$, so $b\equiv -2a\equiv a \pmod 3$. Since $a,b\in{-1,0,1}$, this gives $b=a$, a contradiction. $\blacksquare$

Of the $3^3=27$ windows, 3 are homogeneous, 6 are heterogeneous and 18 are open. One third close.

3. The phase map

Let $\zeta=e^{2\pi i/3}$. Assign to each trit the phase factor ζ^{t}, that is, the phase $\theta(t)=2\pi t/3$. The three trits land on the three cube roots of unity, 120 degrees apart.

Lemma 1 (closure is a unit product). A window is closed if and only if $$\zeta^{t_{i-1}},\zeta^{t_i},\zeta^{t_{i+1}}=1.$$

Proof. The product equals $\zeta^{t_{i-1}+t_i+t_{i+1}}$. Since ζ has order 3, this equals 1 exactly when the exponent is divisible by 3. $\blacksquare$

The oscillator article measures coherence by a *sum* of phase factors. The net measures closure by a *product*. Theorem 1 shows that these two measures separate.

4. Theorem 1: alignment and closure are independent

For a window, let $R_w=\frac13\left|\zeta^{t_{i-1}}+\zeta^{t_i}+\zeta^{t_{i+1}}\right|$. This is the order parameter of the oscillator article, applied to three unit-amplitude oscillators.

Theorem 1. Over the 27 windows:

Window type	Count	Closed	R_w

Homogeneous	3	yes	1
Heterogeneous	6	yes	0
Open (two equal, one different)	18	no	$1/\sqrt3\approx0.577$

The mean of R_w^2 over all windows is $1/3$.

Proof. Homogeneous: three equal factors give $|3\zeta^{a}|/3=1$. Heterogeneous: the three factors are $1,\zeta,\zeta^2$ in some order, whose sum is 0. Open: the factors are ζ^a,ζ^a,ζ^b with $b\ne a$. Then $$R_w=\tfrac13\left|2+\zeta^{\pm1}\right|=\tfrac13\sqrt{(2-\tfrac12)^2+\tfrac34}=\tfrac13\sqrt3=\tfrac{1}{\sqrt3}.$$ The mean of R_w^2 is $\frac{1}{27}\big(3\cdot1+6\cdot0+18\cdot\frac13\big)=\frac{9}{27}=\frac13$. $\blacksquare$

The two kinds of closure sit at the two extremes of alignment. A fully closed heterogeneous window has the lowest possible coherence. R alone cannot distinguish a heterogeneous closed window from a dispersed, open state. The oscillator framework therefore sees one kind of closure and not the other.

5. Theorem 2: one operator

Theorem 2. (a) For every window, $$t_{i-1}+t_i+t_{i+1}\equiv t_{i-1}-2t_i+t_{i+1}\pmod 3.$$ (b) For nearest-neighbour oscillator coupling on a chain, $\dot\theta_i=\omega_i+K\big[\sin(\theta_{i+1}-\theta_i)+\sin(\theta_{i-1}-\theta_i)\big]$, the linearisation around an aligned state is $$\dot\theta_i\approx\omega_i+K,(\theta_{i+1}-2\theta_i+\theta_{i-1}).$$

Proof. (a) The two sides differ by $3t_i$, which is divisible by 3. (b) Use $\sin x=x+O(x^3)$ for small phase differences. $\blacksquare$

The right-hand sides are the discrete Laplacian of the trit sequence and of the phase sequence. The closure charge of the net and the coupling of the oscillator article are the same operator. It is also the operator of the scalar potential in Maxwell's equations, in discrete form.

6. Theorem 3: closure is zero force

On a chain of oscillators whose phases are trit-valued, $\theta_j=2\pi t_j/3$, the coupling force on the centre of a window is $$F_i=\sin(\theta_{i+1}-\theta_i)+\sin(\theta_{i-1}-\theta_i).$$ Define the window tension $$f_i=\sigma(t_{i+1}-t_i)+\sigma(t_{i-1}-t_i)\in{-2,-1,0,1,2}.$$

Theorem 3. (a) $F_i=\frac{\sqrt3}{2},f_i$. The oscillator force on trit phases is itself trit-valued in each term. (b) $f_i\equiv t_{i-1}+t_i+t_{i+1}\pmod 3$. (c) The window is closed if and only if $f_i=0$, that is, if and only if the coupling force on its centre vanishes.

Proof. (a) For an integer k, $\sin(2\pi k/3)$ equals 0, $\frac{\sqrt3}{2}$ or $-\frac{\sqrt3}{2}$ according as $k\equiv0,1,2\pmod3$. This is $\frac{\sqrt3}{2}\sigma(k)$. Apply it to both terms. (b) $\sigma(k)\equiv k$, so $f_i\equiv (t_{i+1}-t_i)+(t_{i-1}-t_i)=t_{i-1}-2t_i+t_{i+1}$. By Theorem 2(a) this is congruent to the window charge. (c) By (b), closure is $f_i\equiv0\pmod3$. Since $|f_i|\le2$, this holds only for $f_i=0$. $\blacksquare$

The distribution of f_i over the 27 windows is 3, 6, 9, 6, 3 for $f=-2,-1,0,1,2$. The 9 windows with $f=0$ are exactly the 9 closed windows.

Theorem 3 is the central correspondence in exact form. The net's closure and the oscillator's equilibrium are one condition. A closed mesh is a knot on which the threads pull with zero net force. The tension f_i is the residual pull on an open mesh.

7. Theorem 4: two closures, two stabilities

Both kinds of closure are zero-force states. They differ in stability.

Consider a ring of N oscillators with identical frequencies and nearest-neighbour coupling K. Let every consecutive phase difference equal Δ, with $N\Delta\equiv0 \pmod{2\pi}$. $\Delta=0$ is the aligned ring: every window is homogeneously closed. $\Delta=2\pi/3$, with N a multiple of 3, is the three-step twisted ring: every window is heterogeneously closed.

Theorem 4. Perturbation modes $m=1,\dots,N-1$ of the uniformly twisted ring grow or decay at rate $$\lambda_m=-4K\cos\Delta\,\sin^2\!\left(\frac{\pi m}{N}\right).$$ Hence the aligned ring ($\cos\Delta=1$) is stable for $K>0$. The heterogeneously closed ring ($\cos\Delta=-\tfrac12$) is stable for $K<0$ and unstable for $K>0$.

Proof. Linearise around the twisted state. The forward difference is Δ and the backward difference is $-\Delta$, and $\cos(-\Delta)=\cos\Delta$. Hence $$\dot\delta_i=K\cos\Delta\,(\delta_{i+1}-2\delta_i+\delta_{i-1}).$$ The ring Laplacian has eigenvalues $-4\sin^2(\pi m/N)$ with Fourier modes $e^{2\pi i m j/N}$. Multiply by $K\cos\Delta$. Mode $m=0$ is a uniform rotation and is neutral. $\blacksquare$

Worked example with $N=6$ and $|K|=1$. The factors $\sin^2(\pi m/6)$ are 0.25, 0.75 and 1 for $m=1,2,3$. The aligned ring with $K=1$ has rates -1, -3, -4: stable. The heterogeneous ring with $K=1$ has rates $+0.5$, $+1.5$, $+2$: unstable. With $K=-1$ the signs reverse.

The meaning is direct. Attractive coupling selects homogeneous closure. Heterogeneous closure is held only by coupling of negative weight: a relation in which neighbours are kept apart rather than pulled together. In the oscillator article the total coupling is a weighted sum of channels, $K=\sum_m w_mK^{(m)}$. A channel with negative effective weight is the channel that can hold heterogeneous closure. Mixed positive and negative coupling in oscillator populations is studied by Hong and Strogatz (2011).

8. The correspondence, item by item

Oscillator article	Vacuum.net theory	Status
Phase ϕ on the unit circle	Trit t via $\theta=2\pi	Definition (Section 3)
Linearised coupling	Closure charge modulo 3	Proved: same Laplacian
Coupling equilibrium, zero force	Closed window	Proved on trit phases
Order parameter R	Homogeneous closure only	Proved: $R=1$ vs $R=0$ (Thm 1)
Attractive coupling $K>0$	Stabilises homogeneous	Proved (Thm 4)

Coupling of negative weight	Stabilises heterogeneous closure	Proved (Thm 4)
Residual force on the centre	Window tension f_i	Proved (Thm 3)
Locked cluster as one oscillator (Prop. 7)	Same winding at the	Correspondence
Invariant of locking $(\theta^*,\psi_0,\delta^*)$ (Props. 7–8)	Address at closure	Correspondence

9. The missing measure

The oscillator article measures alignment by R and persistence by S_τ. Theorems 1 and 4 show what is missing: a measure that sees heterogeneous closure. It must be computed from phase *differences*, so that a common shift of all phases leaves it unchanged.

Operational definition. For each window, round the forward and backward phase differences to the nearest of ${0,\pm2\pi/3}$. Call the results σ_+ and σ_-, as trits. The window is

- homogeneously closed if $\sigma_+=\sigma_-=0$;
- heterogeneously closed if $\sigma_+=-\sigma_-\neq0$;
- open otherwise.

Let C_{hom} and C_{het} be the fractions of homogeneously and heterogeneously closed windows.

Proposition (baseline). For independent uniform random phases, $C_{hom}=1/9\approx0.111$ and $C_{het}=2/9\approx0.222$.

Proof. Each rounded difference is uniform on three values, and the two differences are independent. $\sigma_+=\sigma_-=0$ has probability $1/9$. The two pairs $(\sigma_+,\sigma_-)=(1,-1)$ and $(-1,1)$ have total probability $2/9$. $\blacksquare$

The three numbers $(R,\,C_{hom},\,C_{het})$ then describe a state completely at the level of this article. High R with high C_{hom} is alignment. Low R with C_{het} well above 0.222 is complementary closure. Low R with both fractions at baseline is dispersion. The oscillator article can only separate the first from the other two. The net separates all three.

A report on the MAZE programme records the same separation in simulation, under its own definitions: a copying rule gives high alignment with little heterogeneous closure, and a closure rule gives the reverse (Konstapel, 2026b, §9).

10. What this means for the oscillator article

The oscillator article is the net read in its phase register. It shares the net's operator (Theorem 2) and its equilibrium condition (Theorem 3). Its hierarchy of locked clusters corresponds to the net's identity over scale. Its entrained invariant corresponds to the address at closure.

It covers only half of closure. A cell whose processes complement each other, rather than copy each other, has low R in that article. In the net such a cell can be fully closed. The measure of Section 9 shows the difference in data. Theorem 4 names the condition for it: a coupling channel of negative weight.

11. Status

Claim	Status
Lemma 0, Lemma 1, Theorems 1–4, baseline proposition	Proved in this article; all numbers verified by complete enumeration of the 27 windows
Identification of phase register and trit register,	Definition
Locked cluster ↔ winding at the next depth; locking invariant ↔ address at closure	Correspondence
Negative-weight channels hold heterogeneous closure in living cells	Consequence of Theorem 4; requires measurement with the definition of Section 9

12. Conclusion

Closure is zero force. On trit-valued phases, the closure rule of the net and the equilibrium of the oscillator coupling are one condition. Both rest on the same Laplacian. There are two kinds of closure. Attractive coupling holds the homogeneous kind, which the order parameter R sees as full coherence. Coupling of negative weight holds the heterogeneous kind, which R sees as no coherence at all. The oscillator framework is therefore the net read by alignment. The net adds the second reading: closure without alignment. The measure $(R,\,C_{hom},\,C_{het})$ makes both readings visible in the same data.

Annotated references

1. Konstapel, J. (2026a). *Hierarchical Oscillatory Dynamics in Biological Systems: A Multiscale Theory of Cellular Coherence, Emergent Function and Biological Regulation*. Constable Research, Leiden. *Why read?* The oscillator framework this article reads through the net. Its Propositions 7 and 8 are the source of the two correspondences in Section 8. *Reading advice:* Read Sections 5, 8 and 11 beside Sections 4, 6 and 8 here.

2. Konstapel, J. (2026b). *Progress Report: The MAZE Project*. constable.blog, 23 September 2026. *Why read?* The current state of the vacuum.net programme. Section 9 treats closure without alignment and its relation to swarm theory. *Reading advice:* The status ledger at the end separates what is proved, measured and open.

3. Kuramoto, Y. (1984). *Chemical Oscillations, Waves, and Turbulence*. Springer. *Why read?* The origin of the coupling law in Theorems 2–4 and of the order parameter R.

4. Wiley, D. A., Strogatz, S. H. & Girvan, M. (2006). The size of the sync basin. *Chaos* 16, 015103. *Why read?* Twisted states on rings of oscillators with nearest-neighbour coupling, and their stability. Theorem 4 is the case of a three-step twist. *Reading advice:* The linear stability section is enough.

5. Hong, H. & Strogatz, S. H. (2011). Kuramoto model of coupled oscillators with positive and negative coupling parameters. *Physical Review Letters* 106, 054102. *Why read?* Populations in which some oscillators are drawn together and others pushed apart. It is the natural setting for the negative-weight channel of Section 7.

6. Strogatz, S. H. (2000). From Kuramoto to Crawford. ***Physica D*** **143, 1–20.** *Why read?* A clear account of the coupling law and its phase transition. It gives the background to the oscillator side of every theorem here.

7. Knuth, D. E. (1997). ***The Art of Computer Programming*****, Vol. 2, 3rd ed., §4.1. Addison-Wesley.** *Why read?* Balanced ternary as a number system, including the balanced residue σ of D5. *Reading advice:* Knuth calls it "perhaps the prettiest number system of all"; the few pages on it suffice.