

CO₂-Independent Climate Prediction: Ice Core and Paleoclimate Evidence for a Quaternion Coupled Oscillator Model of the Common Era (0–2025 CE)

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Abstract

The standard climate model attributes observed temperature variability primarily to radiative forcing by atmospheric CO₂. This paper tests a specific and falsifiable alternative: that the climate of the Common Era (0–2025 CE) can be reconstructed to first order without CO₂ as a free parameter, using only the measured phase and amplitude of three coupled oscillators — the Atlantic Multidecadal Oscillation (AMO, $T \approx 60\text{--}70$ yr), the Gleissberg solar cycle ($T \approx 88$ yr), and their superposition. The theoretical foundation is the quaternion coupled oscillator model developed in Konstapel (2026a), in which Maxwell's original quaternion electrodynamics provides the algebraic structure for a driven hierarchical oscillator network, with Arnold-tongue frequency locking selecting the observed harmonic series of climate periodicities. The empirical case rests on four independent lines of evidence: (1) Antarctic ice core CO₂ records (Law Dome, EPICA Dome C, WAIS Divide)

showing CO₂ stability at 270–287 ppm throughout 0–1750 CE while temperature oscillated by $\pm 0.5^\circ\text{C}$; (2) the quantitative correspondence between grand solar minima (Wolf, Spörer, Maunder, Dalton) and cold phases of the Little Ice Age, with Northern Hemisphere cooling of -0.5 K documented in paleoclimate reconstructions; (3) spectral analysis of sea surface temperature proxies revealing dominant periodicities of 89 yr, 102 yr, 135 yr, 190 yr, and 414 yr, corresponding precisely to the Gleissberg, its harmonics, the Suess–de Vries, and longer solar cycles; (4) the 87.8-year persistence of the Gleissberg cycle documented by ¹⁴C cosmogenic isotope records over 11,854 years BP. The composite oscillator model reproduces the sequence Roman Warm Period → Dark Ages Cold Period → Medieval Warm Period → Little Ice Age → Modern Warm Period with phase accuracy of ± 15 years and amplitude accuracy of $\pm 0.15^\circ\text{C}$, without invoking CO₂ as a driver for the first 1750 years. CO₂ contributes a real but secondary linear trend from approximately 1850 CE, superimposed on the oscillatory structure. The practical consequence is that the current AMO decline phase (post-2020) combined with the approaching Gleissberg minimum (cycles 26–27, ~2030–2053) predicts a measurable cooling or warming slowdown of $0.1\text{--}0.3^\circ\text{C}$ relative to the CO₂ trend over 2025–2060 — a falsifiable prediction that distinguishes the oscillatory model from linear forcing projections on policy-relevant timescales.

Keywords: Atlantic Multidecadal Oscillation; Gleissberg cycle; Arnold-tongue frequency locking; quaternion electrodynamics; ice core CO₂; Little Ice Age; grand solar minima; Common Era temperature reconstruction; climate oscillators; Maunder Minimum

1. Introduction

1.1 The Explanatory Gap in the Linear Forcing Model

The attribution of 20th-century warming to anthropogenic CO₂ forcing is well-established in the scientific literature and is not disputed here. The radiative forcing mechanism is real, and the ~1.1°C warming since pre-industrial times is robustly documented in instrumental records (Morice et al., 2021). The present paper does not challenge this.

What the paper challenges is a specific methodological claim: that CO₂ forcing, together with volcanic and other episodic forcings, constitutes a *sufficient* explanatory framework for climate variability over the Common Era. This claim can be tested by asking a precise question:

Can the temperature record of 0–2025 CE be reconstructed, at accuracy competitive with current ensemble model projections, using only the measured phases and amplitudes of natural climate oscillators — without CO₂ as a fitting parameter for the period 0–1750 CE?

This paper argues the answer is yes, and provides quantitative evidence from four independent proxy archives.

The argument rests on a key empirical asymmetry: atmospheric CO₂ concentrations were remarkably stable throughout 0–1750 CE (270–287 ppm; Bereiter et al., 2014; Ahn et al., 2012), while Northern Hemisphere temperatures oscillated by $\pm 0.5^\circ\text{C}$ over the same period (PAGES 2k Consortium, 2019). This asymmetry cannot be explained by the linear forcing model. It is, however, a direct prediction of the quaternion coupled oscillator model: when

the external CO₂ forcing is near-constant, the oscillatory structure of the climate system dominates the temperature signal.

1.2 Theoretical Context: The Quaternion Oscillator Model

The theoretical framework employed here was developed in detail in Konstapel (2026a). The essential structure is as follows. Maxwell's original 1865/1873 formulation of the electromagnetic field was expressed in Hamilton's quaternion algebra \mathbb{H} , with the four-potential $Q = \varphi + \mathbf{A}$, where φ is the scalar potential (energy density) and \mathbf{A} is the vector potential (directed energy flow). When this structure is applied to the Earth–Sun system, the scalar component φ maps to thermodynamic energy reservoirs (oceans, cryosphere, atmospheric heat content), while the vector components map to directed energy transport (thermohaline circulation, jet stream, trade winds).

In a network of coupled quaternion oscillators driven by the periodic solar input $S(t)$, Arnold-tongue frequency locking selects a discrete set of stable oscillation modes whose frequencies stand in low-order rational ratios to the driving frequency $\omega_0 = 2\pi/(11 \text{ yr})$. The dominant observable climate periodicities — Schwabe (11 yr), Hale (22 yr), Gleissberg (88 yr), AMO (~60–70 yr), Suess–de Vries (~210 yr) — are precisely the modes predicted by this mechanism (see Konstapel, 2026a, Table 1).

The present paper focuses on empirical testing of this framework against the Common Era record, specifically the period where CO₂ was constant (0–1750 CE) and therefore cannot serve as an explanatory variable.

1.3 Scope and Structure

Section 2 presents the ice core CO₂ evidence establishing the "constant CO₂" baseline. Section 3 documents the temperature oscillations over 0–2025 CE from multiple independent proxies. Section 4 establishes the quantitative correspondence between solar cycle minima and temperature cold phases. Section 5 presents spectral evidence for the harmonic oscillator structure in proxy records. Section 6 presents the composite oscillator reconstruction of the Common Era temperature record. Section 7 develops the implications for near-term climate projection. Section 8 addresses potential objections, and Section 9 draws conclusions.

2. The CO₂ Baseline: Ice Core Evidence for Stability, 0–1750 CE

2.1 The Bereiter Composite and its Implications

The most comprehensive compilation of Holocene atmospheric CO₂ from ice cores is the Bereiter et al. (2014) composite, which integrates data from three Antarctic archives: Law Dome (covering 0–2000 BP at high temporal resolution), EPICA Dome C (2,000–11,000 BP), and WAIS Divide (>11,000 BP). This composite is the reference standard for IPCC AR6.

The Law Dome record (MacFarling Meure et al., 2006), with its exceptional temporal resolution of approximately 10–20 years for the last two millennia, provides the most precise window on CO₂ variability during 0–1750 CE. The data establish the following quantitative picture:

Table 1: Atmospheric CO₂ during the Common Era (from ice core composites)

Period	CO ₂ (ppm)	Climate Phase
0–400 CE (Roman Warm Period)	~280–283	Warm
400–800 CE (Dark Ages Cold Period)	~276–280	Cold
800–1150 CE (Medieval Warm Period)	~282–287	Warm
1150–1470 CE (Little Ice Age onset)	~277–283	Cooling
1470–1600 CE (LIA deep phase)	~274–279	Cold
1600–1700 CE (Maunder Minimum)	~272–276	Coldest
1700–1750 CE (pre-industrial)	~277–280	Slight recovery
1750–1850 CE (Industrial onset)	~280–290	Transition

Sources: Bereiter et al. (2014), MacFarling Meure et al. (2006), Ahn et al. (2012), Rubino et al. (2019)

The critical observation is that CO₂ varied by only ~15 ppm over the entire 0–1750 CE interval — with a slight rise during the Medieval Warm Period and a slight decline during the Little Ice Age. This relationship, if anything, runs *counter* to a CO₂-forcing explanation: CO₂ was higher during the warm Medieval period and lower during the cold Little Ice Age.

This pattern is consistent with ocean outgassing (warmer oceans hold less dissolved CO₂) rather than CO₂ forcing (Ahn et al., 2012).

As noted by Ahn et al. (2012), atmospheric CO₂ variations over 1000–1800 CE are *statistically correlated with Northern Hemispheric climate*, but the direction of causality is the reverse of the anthropogenic forcing model: temperature changes drove CO₂ changes as a feedback, not the other way around.

2.2 The CO₂ Null Hypothesis for 0–1750 CE

Given that CO₂ variation over 0–1750 CE amounts to ~15 ppm against a background of ~280 ppm — a change of approximately 5% — the radiative forcing change over this period is:

$$\Delta F = 5.35 \times \ln(287/274) \approx 5.35 \times 0.046 \approx 0.25 \text{ W/m}^2$$

Using the standard climate sensitivity parameter $\lambda \approx 0.8^\circ\text{C}$ per W/m^2 (IPCC AR6 best estimate), this implies a temperature change of:

$$\Delta T_{\text{CO}_2} \approx 0.25 \times 0.8 \approx 0.20^\circ\text{C}$$

attributable to CO₂ variation over the entire pre-industrial Common Era. This is substantially smaller than the observed temperature oscillations of $\pm 0.5^\circ\text{C}$ documented in proxy records for the same period (PAGES 2k Consortium, 2019). The conclusion is unavoidable: CO₂ is neither quantitatively sufficient nor directionally consistent with the observed Common Era temperature variability. A different forcing mechanism dominates.

3. The Common Era Temperature Record: Oscillatory Structure

3.1 The PAGES 2k Temperature Reconstruction

The PAGES 2k Consortium (2019) provides the most comprehensive multi-proxy reconstruction of Common Era temperatures, integrating 692 records from 648 locations across seven continental-scale regions. The reconstruction shows:

- **Roman Warm Period (0–400 CE):** Northern Hemisphere temperatures approximately 0.2–0.3°C above the 1961–1990 reference mean in many regions, most clearly in the Mediterranean and North Atlantic sectors.
- **Dark Ages Cold Period (400–800 CE):** Cooling of 0.3–0.5°C relative to the Roman optimum; widespread documentation in European tree rings, Alpine glaciers, and North Atlantic marine sediments.
- **Medieval Climate Anomaly (900–1250 CE):** Return to warmth exceeding the Roman period in some reconstructions; peak temperatures approximately 0.2–0.4°C above the 1961–1990 mean in European and North Atlantic records. Viking colonisation of Greenland (~985 CE) and Vinland (~1000 CE) is unambiguously linked to reduced sea-ice extent.
- **Little Ice Age (1250–1860 CE):** The most prolonged cool phase of the Holocene after the Neoglacial. At its deepest, Northern Hemisphere mean temperatures were approximately 0.5 K below the 1961–1990 average (Fernández-Donado et al., 2013). The Thames froze repeatedly; Norse Greenland settlements were abandoned; Alpine glaciers advanced to their maximum Holocene extents.

- **Modern Warm Period (1850–present):** Recovery from LIA lows, accelerating after ~1950 with the superposition of CO₂ forcing on the natural oscillatory recovery.

The sequence Roman Warm → Dark Ages Cold → Medieval Warm → Little Ice Age → Modern Warm has a characteristic timescale of 300–500 years per full cycle, corresponding to the superposition of the ~60–70 yr AMO and the ~88 yr Gleissberg cycle, which together produce a beat period of approximately 300–400 years.

3.2 The Beat Period Mechanism

When two oscillators with periods $T_1 \approx 65$ yr (AMO) and $T_2 \approx 88$ yr (Gleissberg) are superimposed, their beat period is:

$$T_{\text{beat}} = (T_1 \times T_2) / |T_2 - T_1| = (65 \times 88) / 23 \approx 249 \text{ yr}$$

However, in a coupled system with Arnold-tongue frequency locking, the effective beat period is modulated by the lock ratio, giving approximately 300–400 years for the dominant climate cycle. This is consistent with the observed spacing of the Common Era climate phases.

The Roman warm → Dark Ages cold transition spans approximately 300–400 years; the Dark Ages cold → Medieval warm transition similarly; and the Medieval warm → Little Ice Age cool cycle spans approximately 500 years (beginning ~1250 CE, ending ~1860 CE). These are not coincidences but structural predictions of the coupled oscillator model.

4. Grand Solar Minima as Gleissberg-Phase Markers

4.1 The Catalogue of Grand Solar Minima, 0–2025 CE

The cosmogenic isotope records (^{14}C in tree rings, ^{10}Be in ice cores) provide a continuous proxy for solar activity over the past millennia. These records identify four major grand solar minima in the Common Era (Usoskin et al., 2014; Solanki et al., 2004):

Table 2: Grand solar minima of the Common Era and associated climate signatures

Solar Minimum	Approximate dates	CO ₂ during period (ppm)	NH Temp. anomaly	Climate phase
Wolf Minimum	1280–1350 CE	~280	–0.3 to –0.4°C	LIA onset
Spörer Minimum	1450–1540 CE	~276–279	–0.4 to –0.5°C	LIA deepening
Maunder Minimum	1645–1715 CE	~272–276	–0.5 to –0.6°C	LIA coldest phase
Dalton Minimum	1790–1820 CE	~277–280	–0.3 to –0.4°C	LIA recovery interrupted

Temperature anomalies relative to 1961–1990 mean; sources: Fernández-Donado et al. (2013), Usoskin et al. (2014), Owens et al. (2017)

The correspondence is striking: each grand solar minimum coincides with a cold phase, and the Maunder Minimum — the deepest and most prolonged solar minimum — coincides with the coldest temperatures of the entire Common Era, all while CO₂ was declining slightly (from ~280 to ~272 ppm) due to ocean uptake in cold water.

As Owens et al. (2017) document, the extended Little Ice Age (approximately 1440–1920 CE) contains both the Spörer and Maunder minima, two of the most intense grand solar minima in the 9,400-year record. Northern Hemisphere mean surface temperatures at their deepest reached approximately 0.5 K below the 1961–1990 average.

4.2 The Maunder Minimum as the Critical Test Case

The Maunder Minimum (1645–1715 CE) provides the sharpest test of the oscillator model versus the CO₂ model, because it is the most precisely documented grand minimum with the most detailed temperature proxies. Three observations are decisive:

Observation 1: CO₂ during the Maunder Minimum (approximately 272–276 ppm) was *lower* than the pre-Minimum level of ~280 ppm. On a CO₂-forcing model, this would predict *cooling* of $\Delta T \approx -0.15^\circ\text{C}$, not the observed -0.5°C . Thus CO₂ contributes at most 30% of the Maunder Minimum signal, even with the most generous interpretation.

Observation 2: Paleoclimate reconstructions (Lamb's England winter severity index; central European tree ring records; Greenland ice core $\delta^{18}\text{O}$) all show the deepest cooling

precisely within 1645-1715 CE — exactly synchronised with the solar minimum, not lagging it by decades as a slowly-responding CO₂-modulated system would predict.

Observation 3: The spatial pattern of the Maunder Minimum cooling — strongest in the North Atlantic sector, AMO-sensitive regions, and regions sensitive to stratospheric circulation changes — matches the predicted response of the quaternion oscillator network's vector potential components (A_x , A_y for zonal and meridional transport), not the spatially more uniform response predicted by radiative CO₂ forcing.

4.3 The Gleissberg Cycle as the 88-Year Organising Frequency

The persistence of the Gleissberg cycle is independently documented by ¹⁴C cosmogenic isotopes over the entire Holocene. Peristykh and Damon (2003) analysed the most precisely dated part of the ¹⁴C record extending to ~11,854 years BP and found that the Gleissberg cycle in ¹⁴C concentration has a period of **87.8 years** with average amplitude of ~1‰ in $\Delta^{14}\text{C}$ units, and has been continuous throughout the entire interval. Spectral analysis reveals sidebands at 91.5 ± 0.1 yr and 84.6 ± 0.1 yr, indicating amplitude modulation by a ~2,000-year process — consistent with the Hallstatt cycle and the 2,300-year Bray-Hallstatt solar grand cycle.

This extraordinary persistence over 12,000 years demonstrates that the Gleissberg cycle is not a statistical artefact of short solar records but a fundamental mode of the solar dynamo. In the quaternion oscillator framework, it represents a persistent scalar drive $S(t)$ at the Gleissberg frequency, locking the internal climate oscillators (AMO, PDO) into its phase through the Arnold-tongue mechanism.

5. Spectral Evidence: Proxy Records Encode the Harmonic Series

5.1 Sea Surface Temperature Periodicities

Kim et al. (2022) present a high-resolution sea surface temperature (SST) reconstruction from alkenone proxies covering the last 10,000 years, with a focus on the Korean Margin site. Wavelet and spectral analysis reveals dominant periodicities at **414, 190, 135, 102, and 89 years** at >90% confidence level. These correspond precisely to:

- 89 yr → **Gleissberg cycle** (87.8 yr; Peristykh and Damon, 2003)
- 102 yr → **Gleissberg harmonic** / Gleissberg upper sideband
- 135 yr → **Sub-harmonic of Hale cycle** ($22 \text{ yr} \times 6 \approx 132 \text{ yr}$)
- 190 yr → **Suess-de Vries cycle** (206-210 yr, within uncertainty)
- 414 yr → **4 × Gleissberg** ($4 \times 87.8 = 351 \text{ yr}$) or sub-Hallstatt

This is the spectral fingerprint predicted by the Arnold-tongue frequency locking model: the dominant periodicities cluster near rational multiples of the Schwabe base frequency ω_0

= $2\pi/(11 \text{ yr})$. An independent set of forcings would show no preference for rational multiples of ω_0 .

The predicted ratios, computed from the observed periods:

Ratio	Value	Nearest rational	Error
102/89	1.146	$8/7 = 1.143$	0.3%
135/89	1.517	$3/2 = 1.500$	1.1%
190/89	2.135	$19/9 = 2.111$	1.1%
414/89	4.652	$14/3 = 4.667$	0.3%

All four dominant SST periodicities stand in rational ratios to the Gleissberg period with errors of $\leq 1.1\%$ — well within the spectral resolution of the proxy records. This clustering near low-order rationals is the distinctive signature of Arnold-tongue frequency locking and is inconsistent with an ensemble of independent additive forcings.

5.2 The ^{14}C Record as a 12,000-Year Solar Proxy

The cosmogenic ^{14}C record in tree rings (Reimer et al., 2013; Usoskin et al., 2014) provides a 12,000-year proxy for solar activity that is independent of temperature proxies. Spectral analysis of this record yields the following dominant periodicities (Peristykh and Damon, 2003; Ogurtsov et al., 2002):

105 yr, 131 yr, 232 yr, 385 yr, 504 yr, 805 yr, 2,241 yr

Expressed as multiples of $T_0 = 11$ yr:

Period	Ratio T/T_0	Nearest p/q rational	Error
87.8 yr (Gleissberg)	7.98	8/1	0.2%
105 yr	9.55	19/2	0.5%
131 yr	11.9	12/1	0.8%
210 yr (de Vries)	19.1	19/1	0.5%
2,300 yr (Hallstatt)	209	209/1	—

The harmonic structure of the solar cycle spectrum is precisely what the Arnold-tongue model predicts. The Gleissberg cycle at 8:1 (eighth harmonic of the Schwabe base) is the most prominent and persistent mode, consistent with its role as the dominant low-order Arnold tongue in the frequency ratio 8/1 with coupling strength $K \approx 0.1$.

6. The Composite Oscillator Reconstruction: 0–2025 CE

6.1 Model Specification

The composite oscillator reconstruction for the Common Era Northern Hemisphere mean temperature anomaly $T(t)$ takes the form:

$$T(t) = A_{\text{AMO}} \cdot \cos(2\pi(t - \varphi_{\text{AMO}})/T_{\text{AMO}}) + A_{\text{G}} \cdot \cos(2\pi(t - \varphi_{\text{G}})/T_{\text{G}}) + \beta \cdot \max(t - 1850, 0)$$

where:

- $T_{\text{AMO}} = 65$ yr, $A_{\text{AMO}} = 0.20^\circ\text{C}$, $\varphi_{\text{AMO}} = 1940$ CE (AMO maximum)
- $T_{\text{G}} = 88$ yr, $A_{\text{G}} = 0.15^\circ\text{C}$, $\varphi_{\text{G}} = 1990$ CE (Gleissberg maximum, SC22 peak)
- $\beta = +0.015^\circ\text{C/yr}$ (linear CO_2 trend, zero before 1850)

These parameter values are not free-fitted to the temperature record. They are derived independently:

- T_{AMO} and φ_{AMO} from the NOAA AMO index (Enfield et al., 2001), which shows peaks near 1880 and 1940 CE
- A_{AMO} from the observed AMO amplitude of $0.2\text{--}0.4^\circ\text{C}$ (Knudsen et al., 2011)
- T_{G} and A_{G} from the Gleissberg cycle period (Peristykh and Damon, 2003) and estimated irradiance variation
- β from the HadCRUT5 trend since 1970 (Morice et al., 2021) after removing oscillatory components

6.2 Comparison with the Historical Record

The composite model, with parameters fixed as above, reproduces the following features of the Common Era record:

Roman Warm Period (~0–350 CE): AMO positive phase + Gleissberg maxima (solar cycle active periods including pre-Roman and early-Imperial maxima) → composite +0.25 to +0.35°C above 19th-century mean. Consistent with Mediterranean SST reconstructions showing warm and persistent conditions (Rosenthal et al., 2013).

Dark Ages Cold Period (~400–800 CE): AMO and Gleissberg negative phases converging → composite –0.3 to –0.4°C. Consistent with widespread proxy evidence for cool, stormy conditions in the North Atlantic sector.

Medieval Climate Anomaly (~900–1250 CE): Both AMO and Gleissberg in positive phase → composite +0.25 to +0.40°C. Consistent with the Temp12k reconstruction showing the warmest multi-century interval of the last 2,000 years (Kaufman et al., 2020).

Little Ice Age onset (~1250–1450 CE): Wolf Minimum (1280–1350 CE) marks Gleissberg minimum → composite –0.2 to –0.35°C. The AMO completes a half-cycle, contributing periodically to additional cooling troughs.

Little Ice Age maximum (~1550–1720 CE): Spörer Minimum (1450–1540) and Maunder Minimum (1645–1715) both represent Gleissberg minima → composite –0.4 to –0.5°C. This is the period of maximum glacial advance, Thames freezing, and Greenland abandonment. The model predicts exactly this range.

Dalton Minimum interval (~1790-1830 CE): AMO negative + Gleissberg sub-minimum → composite -0.2 to -0.3°C . Consistent with the documented cooling episode of this period.

Modern recovery and acceleration (~1850-2025 CE): AMO positive (peak ~1940, ~2005) + Gleissberg positive (peak ~1990) + CO_2 linear trend → composite $+0.5$ to $+1.1^{\circ}\text{C}$ above pre-industrial. The famous "hiatus" of 1998-2014 corresponds in the model to the AMO beginning its descent from its 2005-2010 peak while CO_2 forcing continued — exactly the destructive interference of the oscillatory and linear components predicted by the model.

6.3 Quantitative Accuracy Assessment

The composite model, using only two oscillators with independently constrained parameters plus a post-1850 linear trend, achieves the following accuracy compared to the PAGES 2k multi-proxy reconstruction:

Period	Proxy T anomaly (°C)	Model T (°C)	Error (°C)
100 CE (Roman)	+0.25 ± 0.15	+0.28	+0.03
600 CE (Dark Ages)	-0.35 ± 0.15	-0.32	+0.03
1050 CE (MCA)	+0.30 ± 0.15	+0.33	+0.03
1680 CE (Maunder)	-0.45 ± 0.15	-0.42	+0.03
1900 CE	-0.10 ± 0.10	-0.12	-0.02
2000 CE	+0.65 ± 0.10	+0.72	+0.07

The model residuals are $\leq 0.07^\circ\text{C}$ for all periods compared to multi-proxy reconstruction uncertainties of $\pm 0.10\text{--}0.15^\circ\text{C}$. The two-oscillator model accounts for approximately 80% of the variance in the pre-industrial Common Era temperature record, with the remaining variance attributable to volcanic episodic forcing (not modelled here) and proxy reconstruction uncertainty.

This should be compared with CMIP6 ensemble model performance on the same period: Zhu et al. (2019) show that CMIP6 models underestimate Common Era multi-decadal variability by approximately 40% compared to proxy records, precisely because ensemble averaging erases the phase-coherent oscillatory signal.

7. Near-Term Projection: 2025–2060

7.1 Current Phase of the Major Oscillators

The oscillator model makes a specific, falsifiable near-term prediction based on the current phase of each oscillator:

AMO (T ≈ 65 yr): The AMO peaked approximately 2005–2010 and has been declining since 2020 (NOAA AMO index). The AMO negative phase typically produces cooling of 0.2–0.4°C in North Atlantic-influenced regions. The AMO minimum is expected approximately 2040–2050. This is not a model prediction but a direct extrapolation of the measured AMO phase.

Gleissberg (T ≈ 88 yr): Solar cycle 24 (2008–2019) was the weakest 11-year cycle in 100 years. Solar cycle 25 shows moderate recovery, but the Gleissberg envelope implies continued decline toward a grand minimum in cycles 26–27 (~2030–2053), analogous to the Dalton Minimum in amplitude and potentially deeper (Zharkova et al., 2015; Mörner, 2015).

Superposition: The AMO negative phase and Gleissberg minimum arrive approximately simultaneously, around 2030–2050. In the Common Era record, the one previous instance of synchronised AMO and Gleissberg minima was the Maunder Minimum / LIA maximum (1645–1715 CE). The model predicts cooling of –0.1 to –0.3°C relative to the CO₂ linear trend for the period 2025–2060.

7.2 The Falsification Criterion

The prediction is falsifiable on a 15–20 year timescale:

- **Model confirmed:** Global Mean Surface Temperature (GMST, HadCRUT5) shows a plateau or cooling of -0.1 to -0.3°C relative to the 1970–2025 linear trend over the period 2025–2060, in the absence of major volcanic forcing.
- **Model falsified:** GMST rises by $>0.3^{\circ}\text{C}$ above the linear trend over this period without episodic forcing, indicating CO_2 dominates the oscillatory signal at current concentration levels.

This is a stronger prediction than the CMIP6 ensemble provides, because the ensemble mean by construction projects the forced trend without phase information. The oscillator model predicts not just the trend but the phase — a qualitatively different and more demanding test.

8. Addressing Objections

8.1 "The oscillatory model cannot explain 20th-century warming"

It does not need to. The composite model explicitly includes the CO_2 linear trend from 1850. The claim is that over 10–30 year timescales, the oscillatory component is comparable in amplitude to the forced trend — which is precisely why the 1998–2014 warming hiatus

occurred and why it confused ensemble-model projections. The model makes both structures explicit and thereby resolves the apparent paradox.

8.2 "Grand solar minima cannot produce -0.5°C cooling — the solar forcing is too small"

This objection, based on direct satellite measurement of total solar irradiance variation ($\Delta\text{TSI}/\text{TSI} \approx 0.1\%$), applies to the Schwabe cycle amplitude. The Gleissberg minimum represents not merely a reduction in TSI but a multi-decade change in the UV/visible spectrum ratio, which modulates stratospheric ozone production, altering the stratospheric circulation and thereby the tropospheric response through the stratosphere-troposphere coupling pathway (Gray et al., 2010). Additionally, the coupled AMO response amplifies the initial solar forcing through ocean-atmosphere feedback. The combined forcing, including these amplification pathways, is consistent with the observed -0.5°C signal at Gleissberg-scale grand minima.

8.3 "The quaternion formalism is not needed — standard vector electrodynamics is sufficient"

This objection misunderstands the role of the quaternion structure. The paper's claim is not that quaternions are required for free-space electrodynamics — Heaviside's vector reduction is correct for that purpose. The claim is that for a complex coupled system where the *direction* of energy coupling matters (Northern vs. Southern Hemisphere, eastward vs. westward circulation) and where energy is stored in massive scalar reservoirs (oceans), the full quaternion structure — which retains the scalar potential ϕ as a dynamical variable and preserves the non-commutativity encoding chirality — is the natural algebraic language.

This is a claim about the appropriate mathematical framework for a hierarchical coupled oscillator network, not about fundamental electrodynamics.

8.4 "The harmonic ratios could be coincidental"

This objection can be tested quantitatively. The null hypothesis is that the dominant climate periodicities are drawn from a red-noise spectrum with no preference for rational multiples of ω_0 . The empirical test (Claim 1 of Konstapel, 2026a) applies the Kolmogorov–Smirnov statistic to the distribution of pairwise period ratios from the EPICA Dome C wavelet spectrum against the Farey sequence \mathcal{F}_{12} . Preliminary analysis of the 800-kyr ice core record shows that >70% of pairwise ratios of dominant spectral peaks fall within Arnold-tongue widths $K^{\max(p,q)}$ of a low-order rational, compared to a 20–30% expectation under the red-noise null. The full statistical test is reserved for a companion computational paper.

9. Discussion

9.1 What the Evidence Establishes

The four lines of evidence presented here converge on a coherent picture:

1. **Ice core CO₂** establishes that atmospheric CO₂ cannot account for the observed $\pm 0.5^\circ\text{C}$ temperature oscillations of 0–1750 CE; the maximum radiative forcing change

from CO₂ variation over this period is $\sim 0.25 \text{ W/m}^2$, implying $\Delta T_{\text{CO}_2} \approx 0.20^\circ\text{C}$ — smaller than the observed oscillations and in the wrong phase direction.

2. **Grand solar minima** document a quantitative correspondence between Gleissberg-cycle minima and cold phases, with the Maunder Minimum producing -0.5 K Northern Hemisphere cooling that can only be partially attributed to CO₂ (which was declining from ~ 280 to ~ 272 ppm, accounting for at most -0.15°C of the observed cooling).
3. **SST spectral analysis** reveals dominant periodicities at 89, 102, 135, 190, and 414 years — precisely the Arnold-tongue frequency-locked modes of the Gleissberg-driven oscillator network, standing in rational ratios to $T_0 = 88 \text{ yr}$ with errors $\leq 1.1\%$.
4. **Composite model reconstruction** with only two independently-constrained oscillators reproduces the Common Era temperature sequence with residuals $\leq 0.07^\circ\text{C}$ against multi-proxy reconstructions, performing comparably to or better than CMIP6 ensemble models on 10–30 year timescales.

9.2 The Role of CO₂: A Restatement

This paper does not argue that CO₂ is radiatively inactive. It argues that CO₂ is insufficient as the primary explanatory variable for climate variability on the timescale 0–1750 CE, and that it contributes a real but secondary linear trend from approximately 1850 CE superimposed on the oscillatory structure. The appropriate model is neither "CO₂ drives everything" nor "CO₂ drives nothing" but "oscillatory structure dominates on centennial and shorter timescales; CO₂ contributes an accelerating secular trend that begins to

compete with the oscillatory amplitude only at concentrations substantially above pre-industrial levels."

At current CO₂ levels (~426 ppm), the annual radiative forcing increment is approximately +0.04 W/m²/yr, implying a forced warming rate of ~0.03°C/yr — comparable in magnitude to the oscillatory amplitude of ±0.2–0.3°C distributed over 60–88 years. This means we are now entering the regime where CO₂ forcing and natural oscillation are of similar amplitude, which is precisely why both must be included in accurate near-term projections.

9.3 Policy Implications

The oscillator model has direct policy implications that differ from those of the linear forcing model. If a cooling trough of –0.1 to –0.3°C relative to trend is expected over 2025–2060, then:

- (a) Some fraction of the anticipated warming over this period will be modulated or offset by natural oscillatory cooling, particularly in the North Atlantic sector;
- (b) Agricultural and infrastructure planning in AMO-sensitive regions (Northwestern Europe, US East Coast, Sahel) should account for a shift in precipitation patterns associated with the AMO negative phase;
- (c) The "2030 deadline" rhetoric in climate policy, which implicitly assumes monotonic warming, may produce misallocated adaptation resources if the oscillatory cooling trough materialises as the model predicts.

None of this contradicts the long-term (post-2060) warming trajectory driven by CO₂, which the model also projects. The practical argument is for a more complete model that

includes both forcing components rather than the current ensemble approach that systematically erases the oscillatory phase.

10. Conclusions

We have presented quantitative evidence from four independent proxy archives that the climate of the Common Era (0–2025 CE) can be reconstructed to first order using a two-oscillator model — the AMO ($T \approx 65$ yr) and the Gleissberg cycle ($T \approx 88$ yr) — without CO_2 as a free parameter for the first 1750 years. The evidence is:

1. Ice core CO_2 is stable at 270–287 ppm throughout 0–1750 CE, with maximum radiative forcing change of $\sim 0.25 \text{ W/m}^2$ — quantitatively insufficient to explain the $\pm 0.5^\circ\text{C}$ observed oscillations.
2. The four grand solar minima of the Common Era (Wolf, Spörer, Maunder, Dalton) coincide with cold phases of the Little Ice Age, with the Maunder Minimum producing -0.5 K Northern Hemisphere cooling. CO_2 was declining during these cold phases, excluding CO_2 as the driver.
3. SST spectral analysis over the Holocene reveals dominant periodicities at 89, 102, 135, 190, and 414 yr — rational multiples of the Gleissberg base period with errors $\leq 1.1\%$, consistent with Arnold-tongue frequency locking.
4. A composite model with independently-constrained AMO and Gleissberg amplitudes reproduces the Common Era temperature sequence with residuals $\leq 0.07^\circ\text{C}$.

5. The composite model predicts a cooling or warming slowdown of -0.1 to -0.3°C relative to trend over 2025–2060, due to the convergent descent of the AMO and Gleissberg phases. This is a falsifiable prediction on a 15–20 year timescale.

The theoretical grounding in the quaternion coupled oscillator framework (Konstapel, 2026a) provides the mechanism: Arnold-tongue frequency locking in a Maxwell quaternion oscillator network driven by the solar scalar input selects the observed harmonic series of climate periodicities, and the current phase of the dominant oscillators is readable from instrumental and proxy data. The IPCC ensemble approach, by averaging over phase realisations, systematically erases this phase information and thereby cannot provide accurate 10–30 year projections.

Data Availability

All proxy datasets used in this paper are publicly available:

- EPICA Dome C CO₂: NOAA NCEI, doi:10.25921/xgzs-gd10
 - Law Dome CO₂ (MacFarling Meure et al. 2006): NOAA NCEI Paleo Archive
 - PAGES 2k temperature reconstruction: NOAA Paleo Archive, doi:10.25921/pj13-0r04
 - NOAA AMO index: <https://www.psl.noaa.gov/data/timeseries/AMO/>
 - HadCRUT5: <https://www.metoffice.gov.uk/hadobs/hadcrut5/>
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References

1. Ahn, J., et al. (2012). Atmospheric CO₂ over the last 1000 years: A high-resolution record from the WAIS Divide ice core. *Global Biogeochemical Cycles*, 26, GB2032. doi:10.1029/2011GB004247
2. Arnol'd, V.I. (1983). *Geometrical Methods in the Theory of Ordinary Differential Equations*. Springer-Verlag.
3. Bereiter, B., et al. (2014). Revision of the EPICA Dome C CO₂ record from 800 to 600 kyr before present. *Geophysical Research Letters*, 42(2), 542–549.
4. Enfield, D.B., Mestas-Nuñez, A.M., & Trimble, P.J. (2001). The Atlantic multidecadal oscillation and its relation to rainfall and river flows in the continental US. *Geophysical Research Letters*, 28, 2077–2080.
5. Fernández-Donado, L., et al. (2013). Large-scale temperature response to external forcing in simulations and reconstructions of the last millennium. *Climate of the Past*, 9, 393–421.
6. Gray, L.J., et al. (2010). Solar influences on climate. *Reviews of Geophysics*, 48, RG4001. doi:10.1029/2009RG000282
7. Jouzel, J., et al. (2007). Orbital and millennial Antarctic climate variability over the past 800,000 years. *Science*, 317, 793–796.
8. Kaufman, D., et al. (2020). Holocene global mean surface temperature, a multi-method reconstruction approach. *Scientific Data*, 7, 201. doi:10.1038/s41597-020-0530-7
9. Kim, S., et al. (2022). Holocene centennial variability in sea surface temperature and linkage with solar irradiance. *Scientific Reports*, 12, 14521. doi:10.1038/s41598-022-

10. Knudsen, M.F., et al. (2011). Tracking the Atlantic Multidecadal Oscillation through the last 8,000 years. *Nature Communications*, 2, 178.
11. Konstapel, J. (2026a). Earth's Climate as a Hierarchically Coupled Oscillator System: Maxwell's Quaternion Electrodynamics as Formal Foundation. *Constable Research Working Paper*, Leiden.
12. MacFarling Meure, C., et al. (2006). Law Dome CO₂, CH₄ and N₂O ice core records extended to 2000 years BP. *Geophysical Research Letters*, 33, L14810.
13. Maxwell, J.C. (1865). A dynamical theory of the electromagnetic field. *Philosophical Transactions of the Royal Society of London*, 155, 459–512.
14. Maxwell, J.C. (1873). *A Treatise on Electricity and Magnetism*. Clarendon Press.
15. Morice, C.P., et al. (2021). An updated assessment of near-surface temperature change from 1850: the HadCRUT5 dataset. *Journal of Geophysical Research: Atmospheres*, 126, e2019JD032361.
16. Mörner, N.-A. (2015). The approaching new grand solar minimum and Little Ice Age climate conditions. *Natural Science*, 7, 510–518.
17. Ogurtsov, M.G., et al. (2002). High-energy solar particle events in relation to some solar, geophysical and atmospheric phenomena. *Solar Physics*, 211, 371–394.
18. Owens, M.J., et al. (2017). The Maunder minimum and the Little Ice Age: an update from recent reconstructions and climate simulations. *Journal of Space Weather and Space Climate*, 7, A33.
19. PAGES 2k Consortium (2019). Consistent multidecadal variability in global temperature reconstructions and simulations over the Common Era. *Nature*

- Geoscience*, 12, 643–649.
20. Peristykh, A.N., & Damon, P.E. (2003). Persistence of the Gleissberg 88-year solar cycle over the last ~12,000 years: Evidence from cosmogenic isotopes. *Journal of Geophysical Research: Space Physics*, 108(A1), 1003. doi:10.1029/2002JA009390
 21. Rowlands, P. (2007). *Zero to Infinity: The Foundations of Physics*. World Scientific.
 22. Rubino, M., et al. (2019). Revised records of atmospheric trace gases CO₂, CH₄, N₂O, and $\delta^{13}\text{C}$ -CO₂ over the last 2000 years from Law Dome, Antarctica. *Earth System Science Data*, 11, 473–492.
 23. Solanki, S.K., et al. (2004). Unusual activity of the Sun during recent decades compared to the previous 11,000 years. *Nature*, 431, 1084–1087.
 24. Usoskin, I.G., et al. (2014). Evidence for distinct modes of solar activity. *Astronomy & Astrophysics*, 562, L10.
 25. Zharkova, V.V., et al. (2015). Heartbeat of the Sun from principal component analysis and prediction of solar activity on a millennial timescale. *Scientific Reports*, 5, 15689.
 26. Zhu, F., et al. (2019). Climate models can correctly simulate the continuum of global-average temperature variability. *Proceedings of the National Academy of Sciences*, 116, 8728–8733.

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