Toroidal Electron: A Bridge Between Quantum Mechanics and Relativity

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Electron with toroidal structure and helical motion are revisited. The helical motion is described by two independent wave-functions, one due to the circular motion of the electron charge and other related to the transversal motion, which acts on the center of mass of the electron. The helical motion is composed by the interference of a pair of left-hand and right-hand circularly polarized waves running at the speed of light, which led to the correct values of the electron spin ($S = \hbar/2$) and the *zitterbewegung* frequency ($\omega = 2mc^2/\hbar$). Application of the transversal wave-function in the Schrödinger equation shows that it is related to the motion of the center of mass and has nothing to do with the electron structure, showing that the toroidal structure respects Quantum Mechanics.

Keywords: *Zitterbewegung*, Electron structure, Schrödinger equation

One century ago, the Quantum Mechanics (QM) was firmly established after the very precise description of the emission spectrum of hydrogen atoms by Niels Bohr [1], who postulated that the electron travels in quantized orbits around the proton. Thanks to Arnold Sommerfeld [2], Louis de Broglie [3], Erwin Schrödinger [4], Paul Dirac [5], Clinton Davisson and Lester Germer [6], Werner Heisenberg [7] and Wolfgang Pauli [8], Max Born [9], and many others, who established the bases of wave-particle duality of the matter after the Bohr model. The undulatory behavior, the wave-function and its relation with average value of the physical properties have led to the statistical probability to find the quantum particle at a place, leading to the uncertainty principle [10].

In QM and its modern theory versions, such as Quantum Field Theory (QFT) and Quantum Electrodynamics (QED), the quantum particles are considered as point like-particles and their sizes and structures are not taken into account. In fact, this has shown to be difficult to understand how point like-particles show some physical properties, such as spin, for instance.

On the other hand, for a long time, even before QM to be settled, there was an enormous effort to stablish the structure of fundamental particles. One of the most noticeable efforts has to do with the structure of the electron, which has been studied since the beginning of the nineties [11–15].

One of the ideas is to look at the particles within semiclassical view considering their electromagnetic properties such as orbital current, electron charge, and spin as related to the particle structures.

The most important structural models for the electron has to do with the ring or toroidal geometries [11–29], in which electron charge travels at the speed light in a circumference of Compton radius ($r_c = \lambda_c/2\pi$). This has been related to the *zitterbewegung* effect predicted by Schrödinger as a solution of the Dirac's relativistic version of the Schrödinger equation [30], which was reinterpreted by David Hestenes [31].

The modern interpretation has led many researchers [11–29] to consider semiclassical theories to describe the physical properties of the electron. One of the most important aspects of these models has to do with the separation of the center of mass and the center of charge of the electron, which has been described by Martin Rivas during the last decades [32, 33]. This has shown that the motion of the electron must consider two motions, which must be related to the center of mass and the charge.

Independently, our group has devoted efforts to show the importance of the *zitterbewegung* physics and the separated motions of the mass and the charge of the electron [34–37]. Using the equivalent Bohr atom, we have proposed that the mean free path of conventional and non-conventional conducting metals is related to the *zitterbewegung* effect, providing a description for the electron-electron scattering at low temperatures, which led to a model in excellent agreement with the Rice's plot [34, 38]. In order to better understand this behavior, the toroidal structure of the electron was revisited taking into account an electromagnetic wave which considers the Schwinger limits [35]. The solution of the four Maxwell equations has provided support for the toroidal structure, leading to an orbital current which is consistent with the electron charge traveling at the speed of light along a circumference of Compton radius, in agreement with the prediction by Dirac [39] and several other works [11–29, 32, 33]. This description led us to find naturally the Bohr postulates [36], which is in excellent agreement with relativistic effects on the helical motion [37].

In this letter, the toroidal structure and the helical motion are revisited in order understand the behavior of both orbital and transversal motions of the electron, as well as to study the interference effects provided by their wave-functions.

In previous works [35–37], it was shown that the toroidal structure and helical motion of the electron can be well described by Figure 1, which provides the Bohr postulates as a consequence [36].

Taking into account the relativistic effects related to the helical motion [36] of the Figure 1(b), it is possible to show that

$$v_r = c\sqrt{1 - v^2/c^2}, \tag{1}$$

$$\lambda = \lambda_c\sqrt{1 - v^2/c^2}, \tag{2}$$

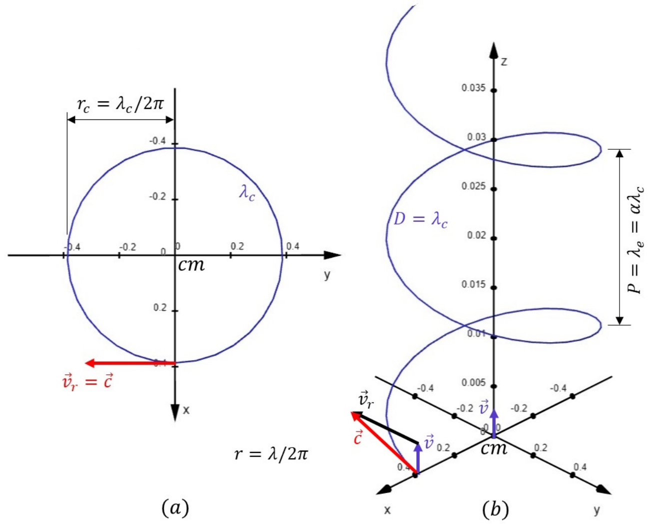

FIG. 1. (a) Toroidal structure of the electron, in which the equivalent center of mass (cm) is at x and $y = 0$ and the charge is at the circumference of Compton wavelength ($r_c = \lambda_c/2\pi$), as predicted by the Schwinger electromagnetic wave. In (b) is displayed the helical motion of the electron, which is composed by the circular motion ($r = \lambda/2\pi$) in the x-y plane and a transversal motion along z direction. The charge moves along $D = \sqrt{4\pi^2 r^2 + P^2} = \lambda_c$ at the speed of light, which has two components related to the orbital (v_r) and transversal (v) velocities, in the way that $c^2 = v_r^2 + v^2$. The helical pitch is given by $P = (v/c)\lambda_c$. The center of mass moves along z direction with the velocity v.

$$r = r_c\sqrt{1 - v^2/c^2}, \tag{3}$$

$$P = \frac{v}{c}\lambda_c, \tag{4}$$

and

$$D = \sqrt{4\pi^2 r^2 + P^2}, \tag{5}$$

where $\lambda_c = 2\pi r_c$ and $\lambda = 2\pi r$.

In such a case, the linear momentum in the helical and circular motions can be calculated, respectively, by

$$p_c = \frac{h}{\lambda_c}, \tag{6}$$

and

$$p_r = \frac{h}{\lambda}, \tag{7}$$

where h is the Planck constant, p_c is the linear momentum of the free electron at rest due to the Schwinger electromagnetic wave [36], which is related to the motion of the electron charge, and p_r is the circular component of linear momentum of the electron, when it is under the influence of an external force, which acts on the center of mass (cm) of the electron and is responsible for the transversal motion along z direction [35], that happens because the Schwinger wave is already at speed of light [36].

Furthermore, the corresponding energies are

$$E_c = p_c c = \frac{hc}{\lambda_c}, \tag{8}$$

and

$$E_r = p_r c = \frac{hc}{\lambda}. \tag{9}$$

Thus, taking into account the linear momentum due to the transversal motion (p_z), one can assume that p_r has two components, one related to p_c and another due to the p_z, in way that

$$p_r = p_c - ip_z, \tag{10}$$

and

$$p_r^2 = p_c^2 + p_z^2, \tag{11}$$

which are justified by the fact the energy-momentum relation is given by [37]

$$(mc^2)^2 = (m_0c^2)^2 + (p_z c)^2, \tag{12}$$

or

$$E_r^2 = E_c^2 + E_z^2, \tag{13}$$

where $E_z = p_z c$, which along with Equations (8) and (9) provide

$$E_r = E_c - iE_z, \tag{14}$$

that clarifies why the summation for the relativistic energy of a quantum particle is quadratic.

Taking Equations (2), (6), and (7) along with (10) and (11), it is possible to show that

$$p_z = \frac{v}{c}\frac{h}{\lambda}, \tag{15}$$

and

$$p_r = \frac{h}{\lambda_c} - i\frac{v}{c}\frac{h}{\lambda}, \tag{16}$$

which respects the fact that $p_r \geq p_c$ and $E_r \geq E_c$, $\lambda \leq \lambda_c$.

Furthermore, the transversal component of the linear momentum acting on the equivalent electron mass can be rewritten by

$$p_z = mv = \frac{v}{c}\frac{h}{\lambda} = \frac{h}{\lambda_{dB}}, \tag{17}$$

where

$$\lambda_{dB} = \frac{h}{mv} \tag{18}$$

is the well-known de Broglie wavelength [10], which is related to the contracted Compton wavelength (λ), after remember that the equivalent mass of the electron is $m = (h/c)\lambda$, which has been considered as related to the Higgs mechanism [27, 40].

In order to define the wave-function of the helical motion described in Figure 1, one can assume two wave-functions related to orbital and transverse motions, in way that the orbital motion (Ψ_r) travels the distance Δr along λ in a time interval t, and another motion, which is related to the transversal component along z direction (Ψ_z), travels the distance z with velocity v in the same interval, in the way that

$$\Psi_r = e^{i\omega_r t} e^{-i2\pi\Delta r/\lambda} \tag{19}$$

and

$$\Psi_z = e^{-i\omega_z t} e^{i2\pi(v/c)z/\lambda}, \tag{20}$$

where ω_r and ω_z are related to the frequencies of both motions, Δr and z are the distances traveled along the orbital and the transversal directions, and λ and $(c/v)\lambda$ are the wavelengths defined in Equations (2) and (18).

Additionally, it is important to notice that the distance traveled in the circular orbit is proportional to the distance z, during the same time interval, since the distance traveled along the circumference λ (see Figure 1 again) is proportional to the distance $(c/v)\lambda$ traveled along the z direction. In fact, this is simply due to the effect of the speed of light (c) decomposed along the orbital (v_r) and the transversal (v) directions. For instance, when $\Delta r = \lambda = 2\pi r$, $z = (v/c)\lambda$, which is related to one pitch P in Figure 1(b). So, when the transversal direction changes the distance $z = \lambda$, Δr must travel the distance $(c/v)\lambda = \lambda_{dB}$. If the transversal velocity is the Bohr velocity, then when $z = \lambda_e$, $\Delta r = \lambda_c = \lambda_e/\alpha$, or when $z = \lambda_c$, $\Delta r = \lambda_B = \lambda_c/\alpha = \lambda_e/\alpha^2$, or even when $z = \lambda_B$ (Bohr orbit), $\Delta r = l_B = \lambda_B/\alpha = \lambda_c/\alpha^2 = \lambda_e/\alpha^3$ (for detail see reference 34). Thus, one can rewrite Equation (19) as

$$\Psi_r = e^{i\omega_r t} e^{-i2\pi z/\lambda}. \tag{21}$$

Taking the Ψ_r and Ψ_z wave-functions, the corresponding wave-function for the helical motion of the electron (Ψ_e) can be accounted by the product of them, by

$$\Psi_e = \Psi_r\Psi_z = e^{i(\omega_r-\omega_z)t} e^{-i2\pi(1-v/c)z/\lambda}. \tag{22}$$

which should take into account interference effects between both wave-functions [10].

Taking

$$\omega_r = \frac{v_r}{r} = \frac{c}{r_c} = \omega_c = \frac{2\pi}{\lambda_c}c, \tag{23}$$

$$\omega_z = 2\pi\frac{v}{c}\frac{v}{\lambda} = \frac{v^2}{c^2}\sqrt{1-v^2/c^2}\,\omega_c, \tag{24}$$

$$k_r = \frac{2\pi}{\lambda} = \frac{1}{\sqrt{1-v^2/c^2}}k_c, \tag{25}$$

$$k_z = \frac{2\pi}{\lambda}\frac{v}{c} = \frac{v/c}{\sqrt{1-v^2/c^2}}k_c, \tag{26}$$

and

$$k_c = \frac{2\pi}{\lambda_c}, \tag{27}$$

one can write the helical electron wave-function as

$$\Psi_e = e^{-i(1-\frac{v^2/c^2}{\sqrt{1-v^2/c^2}})\omega_c t} e^{-i\frac{1-v/c}{\sqrt{1-v^2/c^2}}k_c z}. \tag{28}$$

One of the most important aspect of the Equation (28) is that the real and imaginary components Ψ_e should show constructive and destructive interferences, leading to the group and phase wave propagation.

In fact, a careful inspection of Ψ_e provides the group velocity (v_g) and phase velocity (v_p) [10] given by

$$v_g = \frac{\omega_z}{k_z} = \frac{v^2/c^2}{\sqrt{1-v^2/c^2}}\omega_c / \frac{v/c}{\sqrt{1-v^2/c^2}}k_c = v \tag{29}$$

and

$$v_p = \frac{\omega_r}{k_r} = \frac{\omega_c}{k_c/\sqrt{1-v^2/c^2}} = c\sqrt{1-v^2/c^2} = v_r, \tag{30}$$

where v_r is the orbital velocity shown in Figure 1 and $\lambda_c f_c = \omega_c/k_c = c$ is due to the Schwinger electromagnetic wave, which travels at the speed of light ($v = 0$ and $v_r = c$) around the toroidal structure.

When $v = 0$, $v_g = 0$, $\omega_z = 0$, $k_z = 0$ ($\lambda_{dB} = \infty$), and $\Psi_z = 1$, which implies that there is no motion in the transversal direction and Ψ_z does not change Ψ_e ($= \Psi_r$). In such a case, electron is a free particle with equivalent center of mass at rest [35, 36], in which electron charge motion is due to Schwinger wave traveling in a circumference of Compton radius at the speed of light c, with frequency f_c and wavelength λ_c.

In order to simplify our view about the Ψ_r and Ψ_z wave-functions, let us consider the instant $t = 0$. This leads to

$$\Psi_r = e^{-i2\pi z/\lambda}, \tag{31}$$

$$\Psi_z = e^{i2\pi(v/c)z/\lambda}, \tag{32}$$

and

$$\Psi_e = e^{-i2\pi(1-v/c)z/\lambda}, \tag{33}$$

providing the real ($\Psi_{e(R)}$) and the imaginary ($\Psi_{e(I)}$) components as

$$\Psi_{e(R)} = cos[(1-v/c)k_r z] \tag{34}$$

and

$$\Psi_{e(I)} = -sin[(1-v/c)k_r z]. \tag{35}$$

Furthermore, the real and the imaginary components can be written by composing the following wave-functions

$$\Psi_1 = \frac{1}{2}e^{-i(1-v/c)k_r z}. \quad (36)$$

$$\Psi_2 = \frac{1}{2}e^{i(1-v/c)k_r z}. \quad (37)$$

in the way that $\Psi_{e(R)} = \Psi_1 + \Psi_2$ and $\Psi_{e(I)} = \Psi_1 - \Psi_2$, which must show the envelope behavior well described in literature [10], but in the present case in tridimensional way, since the terms $k_r z$ and $(v/c)k_r z$ have to do with the orbital and transversal motions, respectively.

Making $k_r z = \theta$, the helical motion can have position vectors computed at instant $t = 0$ by

$$r(\theta) = \Psi_e r = re^{-i(1-v/c)\theta}, \quad (38)$$

$$r_1(\theta) = \Psi_1 r = \frac{r}{2}e^{-i(1-v/c)\theta}, \quad (39)$$

and

$$r_2(\theta) = \Psi_2 r = \frac{r}{2}e^{i(1-v/c)\theta}, \quad (40)$$

where $r_1(\theta)$ and $r_2(\theta)$ represent a pair of circular waves running in opposite directions with $r/2$, which denotes the radius defined by Williamson and der Mark [17] as

$$r_W \equiv \frac{r}{2} = \frac{\lambda}{4\pi} \quad (41)$$

that must be related to the electron structure [17, 27, 40].

At this point, it is important to observe that the expected value of the position vector at any z is given by

$$|\Psi_e r| = r = r_c\sqrt{1 - v^2/c^2} \quad (42)$$

which is constant for a given transversal velocity. This is important since any physical propriety described by $G(x, y, z, t) = G_0 e^{i(kz-t)} = G_0 e^{i\theta}$ has its instantaneous value given by G_0, due to the modulus of $|e^{i\theta}| = 1$, no matter the position and the time. This is particularly important for the case of circular motion, where the centripetal force acting on the particle and its position vector change along the azimuthal direction, but not in intensity and in distance with regard to the center of the motion. Even more interesting is the fact that this motion is well described by an imaginary plane, where the x and y components of $G(r, t) = G_x(r, t) + iG_y(r, t)$ are due to $G_x(r, t) = G_0 cos\theta$ and $G_y(r, t) = G_0 sin\theta$, where $|G(r, t)| = G_0$. In fact, this can be used for all physical properties which are related to the circular motion. Thus, a wave-function of the type $\Psi = e^{i(kz-t)} = e^{i\theta}$ allows one to determine any physical property in each position at any time by $G(\theta) = G_0\Psi$, which provides an additional interpretation for the wave-function when the motion is orbital. This is the case of $r(x, y, z, t) = r(\theta) = r_0\Psi$ with $|r(x, y, z, t)| = r_0$, which is important for describing the motion of the electron in this work, since equations (38) to (40) have exactly this form. Thus, in order to observe the electron motion in the imaginary x-y plane, one can take into account the behavior of the wave-function along the z-axis, which has to do with the pitch shown in Figure 1(b), given by $P = (v/c)\lambda_c = 2\pi(v/c)r_c$ [35, 36]. In such a way, one can write the transversal component as

$$z(\theta) = \frac{v}{c}r_c\theta \quad (43)$$

which along with Equations (31) to (40) define the components of the position vector of the electron in the helical motion.

Figures 2 and 3 display several plots for the r_1, r_2, and r in the imaginary x-y plane, using $v/c = v_B/c = \alpha = 1/137.036$, $\lambda_c = h/m_0c = 2.426$ pm, $r_c = \lambda_c/2\pi = 0.3861$ pm, $r = r_c\sqrt{1-\alpha^2} \approx 0.3861$ pm, $r_W = r/2 = r_c\sqrt{1-\alpha^2}/2 = \lambda_c\sqrt{1-\alpha^2}/4 \approx 0.1910$ pm, $\lambda_{dB} = (c/v)\lambda = \lambda_c\sqrt{1-\alpha^2}/\alpha = 332.44$ pm $\approx \lambda_B$, and $r_B = \lambda_B/2\pi = 52.911$ pm.

The helical behavior predicted previously is clearly observed. It is important to notice that r_1 and r_2 form a pair of a right-hand and a left-hand circularly polarized waves propagating in the z direction, with velocity v and wavelength given by the pitch $P = (v/c)\lambda_c$, since $x(\theta)$ and $y(\theta)$ components show 90^0 phase difference (see Figure 2(a) and (b) at $z = 0$) [41]. If an electrical current runs along r_1 and r_2 spirals, let us say in the clockwise direction, due to the motion of the electron charge traveling at the speed of light, it will produce electrical and magnetic fields in both spirals.

Even more interesting about the behaviors of r_1 and r_2 have to do with the fact they provide the correct value for the electron spin ($\vec{S}$) [27, 40], since

$$\vec{S} = |\vec{r}_W \times \vec{p}| = |\frac{\lambda}{4\pi}\hat{a}_r \times \frac{h}{\lambda}\hat{a}_\theta| = \frac{\hbar}{2}\hat{a}_z. \quad (44)$$

The interference between the r_1 and r_2 spirals seems to suggest that the electron structure takes into account entangled effects [42] related to the spin of each spiral, which ultimately leads to the helical motion with contracted Compton radius $r = \lambda/2\pi$. The origin of this behavior and its possible relation with Schwinger electromagnetic wave will be subject of future work.

In addition, they also predict the expected *zitterbewegung* frequency (ω_{Zitter}), which is given by

$$\omega_{Zitter} = \frac{c}{r_W} = \frac{4\pi}{\lambda}c = \frac{2mc^2}{\hbar} \quad (45)$$

since the Schwinger electromagnetic wave travels at the speed of light along the distance D (see Figure 1 again), which is in complete agreement with the first prediction of the *zitterbewegung* effect by Schrödinger [30], the speech by Dirac in his Nobel prize [39], and the modern interpretation of the QM by Hestenes [31].

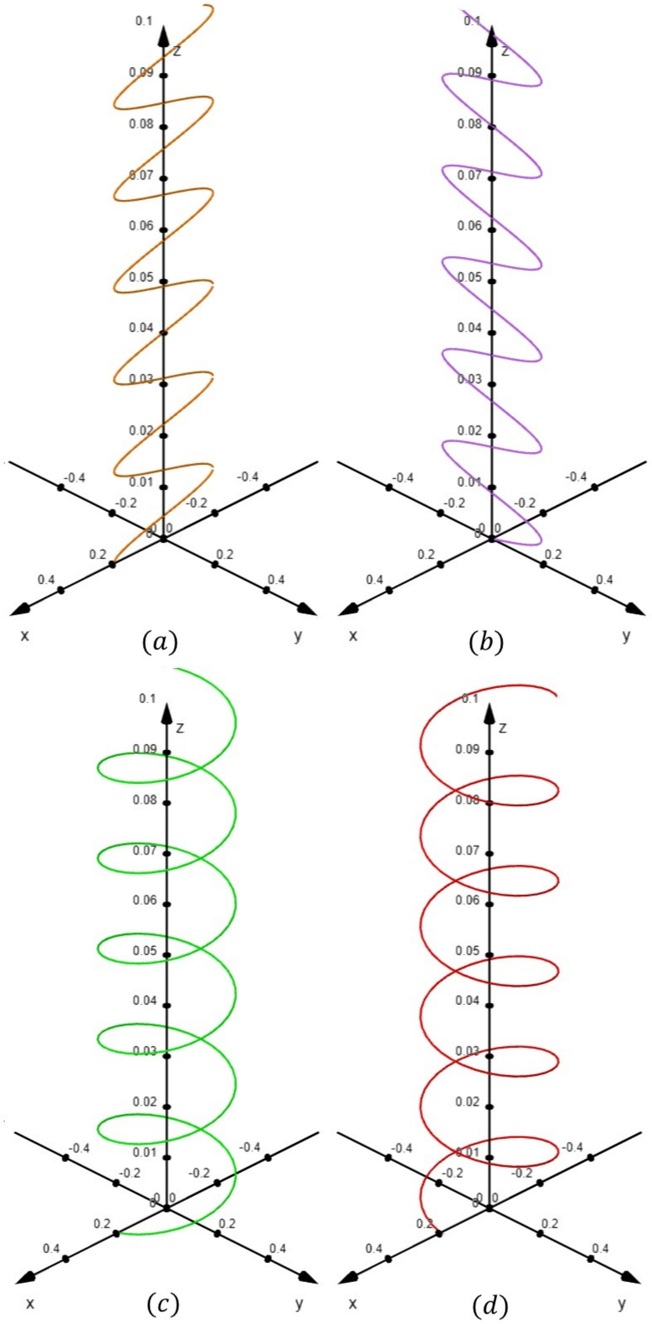

FIG. 2. (a) and (b) Plane waves related to the $x(\theta)$ and $y(\theta)$ components of r_1 and r_2 running along z direction with velocity v and wavelength given by the pitch $P = (v_B/c)\lambda_c = \alpha\lambda_c = \lambda_e$. In (c) and (d) are displayed the helical motions related to a right-hand and a left-hand circularly polarized waves running along z direction.

Finally, let us see whether the wave-function due to the de Broglie ($\Psi_{dB} = \Psi_z$), which is responsible for the transverse motion and must be related to the force that acts on the electron, respects the Schrödinger equation. Taking the de Broglie wave-function

$$\Psi_{dB} = \Psi_z = \Psi = e^{-i\omega_z t} e^{i2\pi(v/c)z/\lambda} \tag{46}$$

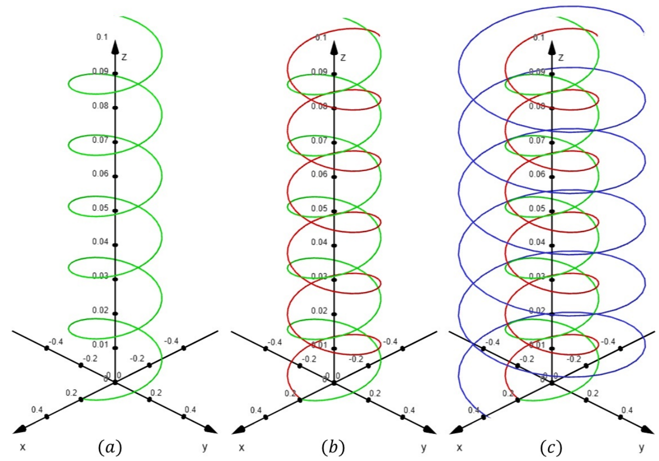

FIG. 3. (a) It is displayed the helical motion of $r_1(\theta)$. In (b) is shown the helical motion of $r_1(\theta)$ and $r_2(\theta)$. (c) It is shown the helical motion of $r(\theta)$, which takes into account the interference effects of r_1 and r_2. The result is the helical motion of the electron with radius $r = r_c\sqrt{1 - v^2/c^2}$, pitch $P = (v/c)\lambda_c$, and $D = \sqrt{4\pi^2 r^2 + P^2} = \lambda_c$, in agreement with the previous report [37].

and defining the linear momentum along z-axis at each point and any time as

$$-ip_z\Psi = -imv\Psi = -i\frac{hv}{c\lambda}\Psi \tag{47}$$

leads one to define the electrical force acting on the electron as related to the component of the linear momentum given by Equation (10) as

$$F\Psi \equiv \frac{\partial(-ip_z\Psi)}{\partial t}. \tag{48}$$

If one considers that this force is related to a centripetal force with an angular velocity given by $\omega_z = \omega_R = v/R$, where $R = r/\alpha$ is the radius of the circular motion (R contracts in the same way as λ [37]), which is the Bohr radius (r_B) for the case in the first orbit of the Bohr atom [10], one can write the centripetal force as

$$F\Psi = -\frac{mv^2}{R}\Psi. \tag{49}$$

Taking

$$\alpha = \frac{e^2}{2\epsilon_0 ch}, \tag{50}$$

$$v = v_B = \alpha c, \tag{51}$$

and

$$m = \frac{h}{c\lambda} = \frac{h}{c\alpha 2\pi R}, \tag{52}$$

one finds

$$F = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2}, \tag{53}$$

which is the Coulomb force between the electron and the proton in the Bohr atom at any point any time.

In order to calculated the potential energy (V), one can write

$$F\Psi = -\frac{\partial V}{\partial R}\Psi, \tag{54}$$

which after proper integration yields

$$V\Psi = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}\Psi. \tag{55}$$

Furthermore, one can calculate the kinetic energy (K_z) and the total energy (E_z) of the electron along z direction under the centripetal Coulomb potential, with transversal velocity (v), which are given, respectively, by

$$K_z\Psi = -\frac{\hbar}{2m}\frac{\partial^2\Psi}{\partial z^2} = \frac{1}{2}\frac{h}{c\lambda}v^2\Psi = \frac{p_z^2}{2m} = -\frac{1}{2}V\Psi \tag{56}$$

and

$$E_z\Psi = i\hbar\frac{\partial\Psi}{\partial t} = \hbar\omega\Psi = \frac{h}{2\pi}\frac{v}{R}\Psi = \frac{h\alpha c}{2\pi R}\Psi = -V\Psi. \tag{57}$$

Putting equations (55) to (57) together, one is able to show that the energy equation is given by

$$K_z\Psi + V\Psi = E_z\Psi \tag{58}$$

which leads naturally to the well-known Schrödinger equation

$$-\frac{\hbar}{2m_0}\frac{\partial^2\Psi}{\partial z^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{59}$$

with

$$E_z\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{60}$$

and

$$p_z\Psi = -i\hbar\frac{\partial\Psi}{\partial z}, \tag{61}$$

demonstrating that the electron with toroidal structure respects one of the most important fundaments of the Quantum Mechanics. Furthermore, this result shows that the Schrödinger equation describes the motion of the center of mass and has nothing to do with the helical motion of the electron charge, because the electrical force due to it is almost constant, since $R/r_c = r_B/r_c = 1/\alpha = 137.036$. This seems to explain why quantum particles have been considered point-like particles.

In summary, this work shows that the motion of the electron can be properly described by two wave-functions, one related to the orbital motion due to the Schwinger electromagnetic wave and another due to the de Broglie wave-function. The helical motion described by both wave-functions provide support for the recent prediction of the Bohr postulates [36]. The helical motion with contracted Compton radius (r) is composed by interference of a pair of left-hand and right-hand circularly polarized waves with radius $r/2$ propagating in the transversal direction with the group velocity v, which provides naturally the corrected values of the electron spin and the *zitterbewegung* frequency. Furthermore, the work demonstrates that the electron with toroidal structure respects the Schrödinger equation and is not necessary to have a point-particle to attend it. In fact, the results show that the Schrodinger equation tracks the motion of the center of mass of the electron by means of the de Broglie wave-function, which is almost independent of the helical motion of the electron charge, since the electrical force that acts on it is almost constant.

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