Clerk Maxwell and the Vacuum.Net Theory

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1. The claim

James Clerk Maxwell wrote a constitutive theory of a medium. He wrote it, at its most compact, in quaternions. That formulation carried two things the modern textbook version has lost. It carried a state register: a scalar part attached to every field quantity. And it carried medium moduli: quantities we now call "constants" appeared as elastic properties of the vacuum.

In 1884–1885 Oliver Heaviside, followed by Willard Gibbs, cut both away. They extracted the vector calculus from the quaternion mechanism. They dropped the scalar register. They removed the potentials from the fundamental set. And they froze the moduli into universal constants. The four "Maxwell equations" every student learns are Heaviside's equations.

The Vacuum.Net theory reverses exactly those two cuts. It puts the state layer back. It makes the constants state functions of the local vacuum again. This article documents the historical record, translates Maxwell's quaternion equations into the vocabulary of the net, and states what then becomes visible. Everything needed to follow the argument is in the text. No reference has to be fetched to check a step.

The fixed image throughout is the fishnet. The vacuum is a net of closed meshes. A mesh is the smallest closed unit. Strands carry winding. Tension lives on the net and relaxes; topology does not. What persists is what closes.

2. What Maxwell actually wrote

The record is precise, and it corrects a common myth.

1856. "On Faraday's Lines of Force." Component notation. Maxwell gives mathematical form to Faraday's *electrotonic state* — Faraday's postulated condition of the medium at the wire itself, invented to explain induction locally. This is the origin of the vector potential. It enters physics as a **state of the medium**, not as a calculational aid.

1861–1862. "On Physical Lines of Force." A mechanical model: all of space filled with rotating cells and idle-wheel particles. Rotation of a cell is magnetism. Displacement of the idle wheels is dielectric polarization. Streaming of them is current. From the elasticity and density of this medium Maxwell computes the speed of transverse waves: 193,088 miles per second. Fizeau had measured light at 193,118 miles per second (in the figure Maxwell quoted to Faraday in October 1861). Maxwell's conclusion, italics his: light consists in transverse undulations of the same medium that carries electric and magnetic action. Note what happened here. The speed of light entered physics as a **computed modulus of a medium** — the square root of elasticity over density — checked against measurement. Not as a universal constant.

1864. "A Dynamical Theory of the Electromagnetic Field." Twenty equations in twenty unknowns, in component notation. **Not in quaternions.** The claim that Maxwell "originally wrote his equations in quaternions" is false for this paper. The vector potential appears under a new name: the *electromagnetic momentum* F, G, H. The potentials belong to the fundamental set of equations.

1870. Maxwell turns to quaternions, pushed by his school friend Peter Guthrie Tait. In November 1870 he writes the manuscript "On the Application of the Ideas of the Calculus of Quaternions to Electromagnetic Phenomena." In a letter to Tait of 7 November 1870 he coins names for the two readings of the ∇ operator: the *slope* of a scalar function, and for a vector function the *convergence* (scalar part) and the *curl* (vector part). One operator, two registers. In a second letter, 14 November 1870, he states his intent: to leaven his book with Hamiltonian ideas "without casting the operations into a Hamiltonian form."

1873. The *Treatise on Electricity and Magnetism*. In Articles 591–619 of Volume II the fundamental equations are written down as a single group — the only place in the Treatise where this happens. There, and only there, Maxwell uses quaternion notation, with German Gothic letters for the field quaternions. He rarely calculates with quaternions. He uses them because they show the meaning of the results in the most compact form. The grouped statement of the theory is quaternionic by deliberate choice.

1884–1885. Heaviside recasts the theory into four vector equations in two field variables, E and B. Gibbs builds the vector analysis as a subset of the quaternion system. Heaviside is explicit about his motive: remove the potentials from the fundamental theory. Hertz does the same in component form. This is the version that conquered physics, and it is Heaviside's attitude to the potentials — not Maxwell's — that textbooks have put in Maxwell's mouth ever since.

3. The quaternion equations, as Maxwell grouped them

A quaternion is one object with a scalar part and a vector part. The unit vectors obey $i^2 = j^2 = k^2 = -1$ and $ij = k = -ji$. The product is associative and distributive but not commutative: order carries orientation. S takes the scalar part of a product; V takes the vector part. Hamilton's operator is $\nabla = i\, d/dx + j\, d/dy + k\, d/dz$.

Maxwell's grouped equations (Treatise, Arts. 591–619), with his Gothic letters:

	Maxwell (1873)	**Modern reading**
(A)	$\mathfrak{B} = V\nabla\mathfrak{A}$	$B = \nabla \times A$
(B)	$\mathfrak{F} = V\mathfrak{V}\mathfrak{B} - \mathfrak{A} - \nabla\Psi$	$F/q = v \times B - \partial A/\partial t - \nabla\Psi$
(C)	$\mathfrak{f} = V\mathfrak{J}\mathfrak{B}$	$f = J_T \times B$
(D)	$\mathfrak{B} = \mathfrak{H} + 4\pi\mathfrak{J}_m$	$B = H + 4\pi M$
(E)	$4\pi\mathfrak{J} = V\nabla\mathfrak{H}$	$4\pi J_T = \nabla \times H$
(F)	$\mathfrak{D} = (K/4\pi)\mathfrak{F}$	$D = \varepsilon E$
(G)	$\mathfrak{K} = C\mathfrak{F}$	$J = \sigma E$
(H)	$\mathfrak{J} = \mathfrak{K} + \mathfrak{D}$	$J_T = J + \partial D/\partial t$

(J)	$e = S\nabla\mathfrak{D}$	$\varrho = \nabla\cdot D$
(L)	$\mathfrak{B} = \mu\mathfrak{H}$	$B = \mu H$

Here $\mathfrak{A}$ is the electromagnetic momentum (our vector potential), Ψ the scalar potential, $\mathfrak{B}$ the magnetic induction, $\mathfrak{H}$ the magnetic force, $\mathfrak{F}$ the electromotive force, $\mathfrak{D}$ the displacement, $\mathfrak{K}$ the conduction current, $\mathfrak{J}$ the total current, e the density of free electricity. K is the *electric elasticity* of the medium, C its conductivity, μ its permeability. Maxwell's own names carry the medium picture: elasticity, displacement, momentum.

Three structural facts about this notation matter for everything that follows.

First: **one operation, two registers.** ∇ applied to a field quaternion yields a scalar part and a vector part in a single stroke. Charge is the scalar reading of the very operation whose vector reading is the curl. Equation (J) says charge **is** $S\nabla\mathfrak{D}$ — a scalar residue, not a separate substance.

Second: **the potentials are fundamental.** Equations (A) and (B) define the measurable fields as readings of $\mathfrak{A}$ and Ψ. Maxwell even justified the name "vector potential" by showing (Art. 617) that in the gauge $S\nabla\mathfrak{A} = 0$ the quaternion $\mathfrak{A}$ obeys the same Poisson structure as the electrostatic potential. Note the form of that gauge condition. It reads, literally: *the scalar part of* $\nabla\mathfrak{A}$ *vanishes*. The convention that tames the potential is the convention that sets a scalar register to zero. Hold that thought.

Third: **the constants are moduli.** (F), (G) and (L) are constitutive relations of a medium. K, C and μ are properties of the local vacuum, on the same footing as the elasticity of steel. And $c^2 = 1/(K\mu)$ in Maxwell's system: the speed of light is a derived medium property. That is how Maxwell computed it in 1861, before comparing with Fizeau.

4. The dictionary

The Vacuum.Net theory describes one medium: a net of strands. Its vocabulary is small. A **strand** carries phase. A **winding** is a turn of the strand. **Closure** is a winding that returns in phase. A **mesh** is the smallest closed unit of the net. **Tension** (χ) is the state of the net; it is the only layer in which anything happens, while closed topology — the address — is frozen. Constants are state functions $g(\chi)$ of the local tension. Nothing causes anything across a gap; constitutive relations hold simultaneously, selection keeps what closes, and description at the right winding depth replaces explanation.

The translation:

Maxwell	Vacuum.Net
$\mathfrak{A}$, the electromagnetic momentum / electrotonic state	The state of the net: the winding field per mesh
Ψ, the scalar potential	The scalar tension layer
$S\nabla$ and $V\nabla$, two parts of one operator	Two readings of one closure operation: bookkeeping (scalar) and winding (vector)
$e = S\nabla\mathfrak{D}$, free electricity	The scalar residue of non-closure; a three-valued register (−1/0/+1)

$\mathfrak{D}$, the displacement current	The closure-completing term: the medium finishes every
K, C, μ — elasticity, conductivity,	State functions g(χ) of the local vacuum
$c^2 = 1/(K\mu)$	c as a state function, derived, not postulated
The non-commutative product; $i^2 = -1$	Order is orientation; a full return of a winding takes 4π

5. The equations, rewritten into the net

(A) $\mathfrak{B} = \mathbf{V}\nabla\mathfrak{A}$. Induction is not a thing. It is the vector reading of how the winding state varies across the mesh. A reading, not an object.

(B) $\mathfrak{F} = \mathbf{V}\mathfrak{V}\mathfrak{B} - \mathfrak{A} - \nabla\boldsymbol{\Psi}$. Force is what a probe registers when the state shifts. Three readings of one strand: winding carried along with motion, state changing in place, tension falling off along the net. Not three causes. One state, read three ways.

(J) $e = \mathbf{S}\nabla\mathfrak{D}$. Charge is the scalar residue: convergence that does not close. A bookkeeping entry, not a substance. Its register has three positions — positive, zero, negative — which is the balanced-ternary register of the net. The mirror-charge corollary of the Vacuum.Net foundational paper (every address has a sign-mirrored partner; the zero address is self-mirrored) is already sitting in Maxwell's notation: the scalar part of a quaternion equation carries sign symmetry by construction.

(H) $\mathfrak{J} = \mathfrak{K} + \mathfrak{D}$. The closure law. This equation contains Maxwell's single genuine addition to the electrodynamics of his time: the displacement term. Its function is exactly this — the total current always closes. What conduction leaves open, the medium completes. In net terms: only closed loops persist; the displacement term is the medium closing the loop. Selection by fit, written down in 1862.

(F), (G), (L) — $\mathfrak{D} = (K/4\pi)\mathfrak{F}$, $\mathfrak{K} = C\mathfrak{F}$, $\mathfrak{B} = \mu\mathfrak{H}$. Constitutive relations: simultaneous, no arrow. In Maxwell these are properties of the medium, called by their right names — elasticity foremost. In the Vacuum.Net theory these are the g(χ) relations: the response of the net is a function of its local tension. The move from Maxwell to the modern textbook froze K and μ (as ε_0 and μ_0) into universal constants. The move from the textbook to the net thaws them again. Maxwell's own 1861 computation shows which way the arrow of fidelity points: he derived c from the moduli of the medium — 193,088 miles per second — and checked it against Fizeau's 193,118. c entered physics as a state function. It was promoted to universal constant later, by convention.

The product itself. The quaternion product is one product, non-commutative, mixing scalar and vector parts. In net terms: composing windings depends on order, because orientation is real. $i^2 = -1$ is a half-turn; a winding returns to itself only after 4π. The double cover, which the foundational paper proves via quaternions and reads as the spin-½ rotation label, is native to Maxwell's chosen algebra. Heaviside's dot and cross products are that one product broken into two commuting-or-anticommuting fragments — workable, and blind to the whole.

6. What becomes visible

Five findings follow. They are stated as theses.

Finding 1. Maxwell is not a precursor to be reinterpreted. He is an amputated pillar. The Treatise is a constitutive medium theory carrying a state register. The 1884 recasting performed two operations at once: it discarded the scalar register, and it froze the moduli. The Vacuum.Net theory is not Maxwell plus something new. It is Maxwell without the amputation, plus the closure dynamics he did not have.

Finding 2. Gauge fixing is the act of setting the tension register to zero. Maxwell's own gauge (Art. 617) reads $S\nabla\mathfrak{A} = 0$: the scalar part of $\nabla\mathfrak{A}$ vanishes. The later Lorenz condition extends the same move to spacetime: scalar part, set to zero, by agreement. In the full quaternion formulation every product produces a scalar part; vector notation has no place for it and disposes of it through the gauge convention. Read through the net, "gauge freedom" is the freedom to ignore the state layer. The two-layer law of the Vacuum.Net theory says the opposite: the address is frozen, and the tension layer is the only layer in which anything happens. What a century of field theory treats as redundancy, the net treats as the working layer. The scalar remainder of Maxwell's algebra is the tension χ, discarded at birth.

Finding 3. The potentials come first, and the bill for removing them arrived in 1959. For Maxwell, $\mathfrak{A}$ was Faraday's electrotonic state: a condition of the medium at the wire. The fields are readings of it. Heaviside expelled the potentials from the fundamental set; the textbook tradition followed and attributed the expulsion to Maxwell. Aharonov and Bohm showed in 1959 that a charged particle registers the potential in a region where E and B vanish. The state is measurable where the readings are zero. In net terms this is no paradox at all: the winding state of the mesh is the object; E and B are derivative readings; of course the object acts where its readings happen to cancel.

Finding 4. Charge is a trit. Equation (J) makes charge the scalar residue of non-closure, with three positions: $+1, 0, -1$. The standard model postulates charge as an elementary property. The net derives its register: balanced ternary, the number system in which the Vacuum.Net address theorem (T7, Unique Address) is proved, with mirror charge as a corollary. Maxwell's notation already holds charge in the scalar slot of a sign-symmetric algebra. The trit was in the formalism from the start.

Finding 5. Maxwell left one open end that maps directly onto the net's open calculation O1. Maxwell held that his medium possessed enormous intrinsic energy, and he suggested — without finding the mechanism — that gravitation should follow from an energetic *reduction* in the medium between massive bodies. In net vocabulary: tension and discharge. A region of lowered medium energy between two closures, drawing them together, is a tension gradient on the net. The open calculation O1 of the Vacuum.Net foundational paper asks which term in the microscopic step cost carries the density coupling. Maxwell's unfinished gravitational note is the same question, posed in 1873 and left standing. The two ends meet.

7. Status

This article makes three kinds of statement, and keeps them separate.

History. Sections 2 and 3 are the documented record: dates, papers, letters, equations, and the two numbers 193,088 and 193,118. This is settled.

Correspondence. The dictionary of Section 4 and the rewrite of Section 5 are correspondences in the sense of the foundational paper: term-for-term identifications between Maxwell's formulation and the net's vocabulary. A correspondence is not a derivation. It earns its keep by being exact, and by putting known structures (the gauge condition, the displacement term, the scalar charge equation) in positions where the net expects them.

Open. Finding 5 names the open calculation. Whether the scalar register, restored, does quantitative work — whether the discarded S-parts, read as tension, reproduce measured couplings — is the content of O1 and of the tension-layer programme, not of this article.

8. Annotated references

J. C. Maxwell, *A Treatise on Electricity and Magnetism*, 2 volumes, Clarendon Press, 1873. Why read? The primary source. The quaternion grouping of the fundamental equations stands in Volume II, Articles 591–619; the gauge condition $S\nabla\mathfrak{A} = 0$ and the naming of the vector potential in Article 617; the operators convergence and curl introduced in Volume I, Article 25. Reading advice: go straight to Arts. 591–619 and read Maxwell's own names for the quantities — elasticity, displacement, momentum. The medium picture is in the vocabulary. Both volumes are on archive.org.

J. C. Maxwell, "On Physical Lines of Force," *Philosophical Magazine*, 1861–1862. Why read? The paper in which the speed of light is computed as a medium modulus — elasticity over density — and found at 193,088 miles per second against Fizeau's measurement. The displacement current is introduced here, as a property of the medium. Reading advice: Part III carries both the displacement idea and the velocity computation.

J. C. Maxwell, "A Dynamical Theory of the Electromagnetic Field," *Philosophical Transactions of the Royal Society* 155, 459–512, 1865. Why read? The twenty equations in twenty unknowns, in component notation — the proof that the quaternion myth about 1864 is a myth, and the paper in which the potentials sit inside the fundamental set under the name electromagnetic momentum.

J. C. Maxwell, letter to Michael Faraday, 19 October 1861; letters to P. G. Tait, 7 and 14 November 1870; manuscript "On the Application of the Ideas of the Calculus of Quaternions to Electromagnetic Phenomena," November 1870. In P. M. Harman (ed.), *The Scientific Letters and Papers of James Clerk Maxwell*, Vols. 1–2, Cambridge University Press, 1990/1995. Why read? The Faraday letter contains the two velocity numbers side by side and Maxwell's own statement that the media are one. The Tait letters show the deliberate strategy: Hamiltonian ideas, not Hamiltonian machinery, and the christening of slope, convergence and curl.

A. M. Bork, "Maxwell and the Development of Electromagnetic Theory" (1976), edited transcription by K. T. McDonald, Princeton, 2023. Why read? The most compact reliable walk through all four Maxwell papers and the Treatise, with the quaternion table of Arts. 591–619 set against modern notation, and the Maxwell–Tait correspondence excerpted. Reading advice: use it as the map; then check each claim in the primary sources it links.

P. G. Tait, *An Elementary Treatise on Quaternions*, Oxford, 1867. Why read? The quaternion system as Maxwell received it, from the man who pushed him to use it. The ∇ operator chapters show what the full algebra delivers that the later vector fragment does not.

O. Heaviside, *Electromagnetic Theory*, Vol. 1, 1893, Chapter 3; and *Electrical Papers*, Vol. 1, 1892, Art. 30. Why read? The amputation, performed by its surgeon and defended in his own words: the case against quaternions, the case against the potentials, and the four-equation form. Read it to see precisely what was cut and why — the reasons are practical, not principled.

J. W. Gibbs, *Vector Analysis* (ed. E. B. Wilson), Scribner's, 1901. Why read? The vector calculus as a finished system, built as a subset of the quaternion algebra. The preface concedes the lineage.

L. Silberstein, "Elektromagnetische Grundgleichungen in bivectorieller Behandlung," *Annalen der Physik* 22, 579–586, 1907. Why read? The first serious attempt after Heaviside to put the scalar and vector parts back into one object (the bivector/complexified form). Evidence that the compression of the full theory into one algebraic object kept resurfacing.

Y. Aharonov and D. Bohm, "Significance of Electromagnetic Potentials in the Quantum Theory," *Physical Review* 115, 485, 1959. Why read? The experiment-backed demonstration that the potential acts where the field readings vanish — the bill for Heaviside's expulsion of the potentials, and the empirical anchor of Finding 3.

J. Konstapel, "The Vacuum.Net Theory: Foundational Paper," fifth edition, constable.blog, August 2026. Why read? The axioms (strand, closure, memory, context, scale), the correspondences including quaternion double cover and mirror charge, the address theorem T7 in balanced ternary, the constitutive law g(χ), the five epistemic statuses, and the open calculations O1–O4 that Section 7 of the present article points into.

The fixed working order applies: this article is the reference document. A Dutch blog explaining it follows separately.