

Dual Space A Universal Scientific Language for Abstract Configuration Spaces

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Abstract

Modern science has independently developed a remarkable variety of abstract spaces. Physicists work with Hilbert spaces, phase spaces, and configuration spaces. Control engineers formulate systems in state spaces. Artificial intelligence employs latent spaces and embedding spaces. Evolutionary biology uses fitness landscapes. Ecology uses niche hypervolumes. Neuroscience describes neural population activity as attractor manifolds. Economics and the social sciences use agent state spaces and phase portraits. Cognitive science studies conceptual spaces. Linguistics treats meaning as a system of relational differences. Psychology has modeled behavior as movement within a topological life space. Cosmology describes the universe itself as a point moving through an abstract configuration space. Information geometry, semantic spaces, and solution spaces appear throughout mathematics and the natural sciences. In at least three of these disciplines — physics, neuroscience, and ecology — the relevant space additionally supports a dynamic reading in which coherent states arise through synchronization between coupled oscillators.

Although these concepts emerged independently, they perform the same fundamental function: they describe the space of possible relations rather than the objects themselves.

This paper proposes that these apparently different mathematical constructions are specializations of a more general concept, here called Dual Space. Unlike Peter Rowlands' specific algebraic implementation, Dual Space is introduced as a universal scientific

framework describing the relational configuration space underlying every dynamical system.

The purpose of this paper is not to replace existing mathematical formalisms but to demonstrate that modern science already relies upon a common conceptual architecture whose general significance has remained largely unnoticed.

1. The Hidden Consensus of Science

Scientific disciplines appear remarkably different.

Quantum mechanics studies wave functions. Biology studies living organisms. Artificial intelligence studies representations. Psychology studies cognition and behavior. Linguistics studies meaning. Economics studies decisions. Cosmology studies the universe as a whole.

Yet every mature discipline eventually introduces an abstract mathematical space that is not directly observable.

These spaces are not physical objects. They are spaces of possibility.

This observation is so common that it has become almost invisible. The present paper argues that this recurring pattern is not accidental.

2. The Universal Appearance of Abstract Spaces

The following overview illustrates how independently each discipline arrived at the same conceptual move — replacing objects with relational coordinates.

2.1 Physics: Hilbert space, phase space, and dual vector space

Classical mechanics represents a system's possible states not by the objects in it but by points in phase space, where each point encodes position and momentum simultaneously. J. Willard Gibbs formalized this in 1902 with the concept of the statistical ensemble: rather than following one system through its succession of configurations, one determines how a whole population of systems is distributed among the various conceivable configurations and velocities at a given time [1].

Quantum mechanics generalizes this to Hilbert space, where physical states are vectors and observables are operators acting on that vector space.

A more radical version of this idea comes from Peter Rowlands' nilpotent quantum mechanics. Rowlands argues that a single three-dimensional vector space cannot by itself generate the singularities that correspond to fundamental particles; this requires combining two three-dimensional vector spaces that are dual to each other in a physical sense, and their combination produces a nilpotent quantum mechanics that incorporates exact supersymmetry and removes self-interaction anomalies [2]. In his own later formulation, physics cannot be reduced to a single space-time-like structure; instead its true structure requires two vector spaces, or a space and an antispaces, that are dual to each other, with all physical measurements mediated via one of these spaces while the other supplies the structure that makes fermionic singularities possible [3].

Rowlands, together with Vanessa Hill, extended this dual-space construction into biology in a series of papers collectively titled "Nature's Code," and in a dedicated chapter of his book *Zero to Infinity: The Foundations of Physics* [4]. They argue that the Dirac nilpotent fermion-plus-vacuum structure — with its four fundamental components (space, time, mass, charge), the 64 elements of its algebra, its double-helical structure, chirality, and five-fold broken symmetry — corresponds closely to the structure of the DNA/RNA genetic code, with its four bases, 64 triplet codons, double-helical structure, chirality, and five-fold axial symmetry [5]. This is not an external analogy imposed by a third party but a recurring, self-developed theme within Rowlands' own body of work.

Rowlands and Marcer also identify an explicitly oscillatory dimension of the same dual-space structure: they list the harmonic-oscillator mechanism and *zitterbewegung* — the rapid oscillatory motion of the electron implied by the relativistic Dirac equation — together with the geometric (Berry) phase, as characteristic features of the nilpotent structure, alongside its double three-dimensionality and five-fold broken symmetry [27]. Since *zitterbewegung* and Berry phase are intrinsically periodic phenomena, the dual-space construction is not purely static; it carries a built-in oscillatory dimension, taken up again in §2.13.

2.2 Control theory and dynamical systems: state space

Modern control theory formulates any dynamical system, mechanical or economic, in terms of a state space: the set of all values its internal state variables can take. A system's entire future behavior is, in principle, determined by its position and trajectory within this abstract space rather than by any single observable output. This is structurally identical to phase space in physics, but arose independently within engineering.

2.3 Artificial intelligence: latent space and embedding space

In deep learning, a latent space is an abstract, lower-dimensional representation of data that captures its essential features and underlying structure, generated by a neural network and termed "latent" because it encodes hidden regularities not directly present in the raw input, typically produced by an encoder that maps input data into that space and a decoder that reconstructs data from it [6]. This idea extends beyond single data points to relational data: deep latent space models for graphs aim to combine the interpretability of classical Bayesian latent-variable graph models with the representational power of deep neural networks, introducing interpretable latent variables such as latent position and community membership as node representations [7].

A parallel and independently developed strand is the notion of semantic or embedding space in natural language processing. Word2vec, introduced by Mikolov and colleagues, demonstrated that words could be represented as dense vectors in a continuous space such that semantically similar words cluster together, and that relations between concepts could be captured as directions or arithmetic operations within that space [8], famously allowing analogies such as *king* – *man* + *woman* $\approx$ *queen* through vector arithmetic. This built on the older distributional hypothesis in linguistics — the idea that a word's meaning can be inferred from the statistical pattern of contexts in which it occurs [9], summarized in Firth's dictum that a word is characterized by the company it keeps.

2.4 Evolutionary biology: fitness landscapes

Sewall Wright introduced the concept of the fitness (or adaptive) landscape in 1932 as a way of visualizing evolution as movement of a population across a surface of peaks and valleys defined by reproductive success rather than as a sequence of isolated observations [10], an image that has since become, in Dennett's phrase, a standard imagination prosthesis for evolutionary theorists. The landscape is not a physical terrain; it is a relational space in which position is defined by genotype or allele frequency and elevation by fitness, with the population represented as a point driven uphill by selection but capable of becoming trapped on a local peak.

Stuart Kauffman later generalized this into the NK model, a tunably rugged fitness landscape defined over binary strings of length N , where a second parameter K controls how many other positions influence the fitness contribution of each site [11], allowing the landscape's ruggedness to be tuned continuously between a single smooth global optimum and a highly fragmented, many-peaked structure. The concept has since propagated into immunology, protein evolution, and even the social sciences [12], illustrating exactly the kind of cross-disciplinary transfer this paper's Dual Space proposal aims to make explicit rather than incidental — though this transfer has also

generated ambiguity, since the properties attributed to fitness landscapes vary considerably as the concept moves between fields.

2.5 Cognitive science: conceptual spaces

Independently of both physics and AI, Peter Gärdenfors proposed conceptual spaces as an intermediate representational level between symbolic and connectionist approaches in cognitive science [13], built from geometric structures organized around quality dimensions, intended to model concept learning and similarity in ways that neither pure symbol manipulation nor pure neural association could capture on its own. On this account, properties normally treated as abstract predicates are instead organized spatially, according to topological principles such as convexity and betweenness, so that meaning itself becomes a geometric relation rather than a symbolic assignment. This is a strikingly direct cognitive-science analogue to embedding spaces in AI, arrived at from an entirely different starting point.

2.6 Mathematics and statistics: information geometry

Shun-ichi Amari's information geometry treats the set of all probability distributions of a given family as a differentiable (Riemannian) manifold — a statistical manifold — equipped with a metric derived from Fisher information [14], providing the geometric foundation for studying information transfer, data analysis, and statistical inference. Here the "points" of the space are not particles or states but entire probability distributions, and distances between them quantify statistical distinguishability rather than physical separation — again the same underlying move: relations replace objects as the primary subject matter.

2.7 Neuroscience: attractor manifolds and neural state spaces

Grand-scale recordings of neural population activity show that, despite thousands of measured dimensions, activity typically traces out a low-dimensional manifold whose dimension and topology match the encoded variable — a manifold perspective that allows the encoded variable to be decoded without prior knowledge of the input, output, behavior, or topography [15]. This was demonstrated directly for the mammalian head-direction circuit, where the possible states form a topologically non-trivial one-dimensional ring, stable across waking and REM sleep [15]. Recurrent neural networks exhibit attractor dynamics, in which network activity evolves toward specific, stable patterns within the state space of possible neural activation patterns [16], with point attractors, line or ring attractors, and limit cycles representing different geometric forms of the same underlying low-dimensional, stable manifold in population activity. This is structurally identical to what is called phase space in physics and latent space in AI — independently rediscovered within neuroscience.

2.8 Ecology: the niche as hypervolume

Independently of physics, AI, and neuroscience, Hutchinson defined the ecological niche as a hypervolume in an n -dimensional, abstract environmental space that allows a population to persist indefinitely [17], distinguishing between the fundamental niche (before interspecific competition) and the realized niche (after). Each point in this space corresponds to a possible state of the environment under which the species can survive [18]; the axis dimensions are not physical coordinates but biotic and abiotic factors that influence birth and death rates. Here too: not the object (the organism) but a relational space of possible conditions is central — and the concept has since spread, like the fitness landscape, into functional morphology, community ecology, and evolutionary biology at large.

2.9 Economics and social systems: state space and agent space

Modern economic theory relies heavily on state-space models, in which each decision-making unit — household, firm, government body — is modeled as a collection of state-conditioned transition processes [19]. Agent-based computational economics rejects the externally imposed rationality and equilibrium conditions of classical models and instead generates a phase portrait of the system: which regions of the state space are credibly reachable, and which basins of attraction lead to which equilibria [20]. The same concepts as in the previous sections — state space, basin of attraction, phase portrait — are thus independently redeployed, now for socioeconomic rather than physical or biological systems.

2.10 Linguistics: language as a system of relations

Ferdinand de Saussure's structural linguistics rejected the idea that words directly represent physical entities, arguing instead that a language is a system of interdependent terms in which the value of each term results solely from the simultaneous presence of the others [21]. Meaning is relational and differential: a sign has no positive content on its own, only differences and oppositions relative to other signs in the system [22]. This is, formally, the same claim later made independently by embedding-space semantics in AI (§2.3): a linguistic unit is defined by its position relative to other units in a structured space, not by an intrinsic property of its own — arrived at more than half a century before the vector-space formalization.

2.11 Psychology: the topological life space

Kurt Lewin, drawing explicitly on the physical concept of the field, proposed a psychological field or "life space" as the locus of a person's needs, goals, and behavior, and adapted topology — a branch of mathematics dealing with spatial relationships — to map the regions, boundaries, and pathways within that life space [23]. Behavior, in this account, is a function of the entire life space at a given moment, not of isolated stimuli or traits [24]; vectors represent the forces moving a person toward or away from goals within it. Lewin's attempt at full mathematical

formalization did not survive intact, but the underlying move — treating behavior as movement through a relational, geometric space of possibilities rather than as a property of the person in isolation — anticipated, independently, the same architecture found in physics, AI, and ecology.

2.12 Cosmology: superspace and minisuperspace

In quantum cosmology, the wave function of the universe is defined as a functional on the configuration space of all possible three-dimensional spatial metrics and matter-field configurations — termed "superspace" by John Wheeler — on which the Einstein equations describe the classical mechanics of a configuration point moving through that space [25]. Because this space is infinite-dimensional, one works in practice with minisuperspace: a finite-dimensional truncation in which only a limited number of degrees of freedom, such as the scale factor of the universe, are retained [26], on which the Wheeler–DeWitt equation is then solved. This is perhaps the most extreme example in this paper of the generic pattern: even the universe as a whole is not described as an object, but as a point moving through an abstract relational configuration space.

2.13 Oscillation and synchronization as an independently recurring pattern

The preceding sections describe Dual Space primarily as a *static* relational configuration space: a set of possible coherent states. A complementary reading is available in at least three of the disciplines already discussed, in which not the configurations themselves but the **oscillatory processes that couple configurations together** carry the explanatory weight.

In physics and applied mathematics, Yoshiki Kuramoto described a population of coupled phase oscillators with distinct natural frequencies; as coupling strength increases, an increasing fraction of the oscillators lock into a shared rhythm, until, above a critical coupling, the entire population oscillates as a single coherent unit [28]. The Kuramoto model has since been applied to systems ranging from cardiac pacemaker cells to chemical oscillations [29], providing a generic mathematical language for synchronization that is independent of the underlying physical substrate.

In neuroscience, Pascal Fries proposed that synchronization of neuronal oscillations, particularly in the gamma-frequency band, is the mechanism by which groups of neurons communicate effectively: synchronized excitation arrives quickly enough to escape subsequent inhibition and drive postsynaptic neurons effectively, a hypothesis he termed "communication through coherence" [30]. Two decades of subsequent research into theta, gamma, and cross-frequency coupling have substantiated this proposal [31]. This is a direct, independently developed parallel to the attractor manifolds of §2.7: here it is the synchronization between oscillatory states, rather than the static state itself, that carries functional information.

In ecology, Lotka and Volterra independently developed, in the 1920s, a model in which predator and prey populations do not converge to a static equilibrium but continue to oscillate: as the prey population grows, the predator population grows with it; that growth then reduces the prey population, which in turn causes the predator population to decline, restarting the cycle [32]. This is a concrete, quantitative instance of oscillation within the niche hypervolume of §2.8 — not merely a metaphor, but the standard model for population dynamics in ecology.

Taken together, these three independently developed literatures suggest that Dual Space has, alongside its static configuration-space reading, a legitimate dynamic reading in which coherence is synchronization between oscillators rather than mere co-location in configuration space. This dynamic reading is best supported, at the time of writing, for physics/mathematics (Kuramoto synchronization), neuroscience (communication through coherence), and ecology (Lotka–Volterra population cycles); extending it to artificial intelligence or linguistics would require oscillatory mechanisms that have not, to date, been established in the literature on latent spaces or distributional semantics, and any such extension should be flagged explicitly as speculative rather than presented as an established parallel.

3. The Common Principle

The central observation is surprisingly simple.

Every scientific space describes possible configurations, possible relationships, possible transformations, possible trajectories.

None of these spaces primarily describe matter itself. Instead they describe constraints upon matter. They are therefore relational rather than material.

What section 2 makes explicit is that this pattern is not merely conceptual — in several cases the mathematical machinery converges as well. Fitness landscapes, state spaces, latent spaces, and niche hypervolumes are all, formally, manifolds or graphs equipped with a scalar or vector field (fitness, cost, activation, survival probability) whose gradient structure determines dynamics. Information geometry supplies a metric structure that can, in principle, be imposed on any of them. As §2.13 shows, a subset of these spaces additionally converges on the same *dynamic* mechanism — synchronization between coupled oscillators — as the means by which coherent states are reached and

maintained. This commonality suggests the existence of a deeper abstraction, rather than a family of unrelated metaphors.

4. Beyond Individual Disciplines

Current science treats these spaces as discipline-specific constructions.

Physics possesses Hilbert space and Rowlands' dual vector space. Artificial intelligence possesses latent space and embedding space. Biology possesses fitness landscapes. Ecology possesses the niche hypervolume. Neuroscience possesses the attractor manifold. Control theory and economics possess state space. Cognitive science possesses conceptual space. Linguistics possesses the system of differential signs. Psychology possesses the topological life space. Statistics possesses the statistical manifold. Cosmology possesses superspace. Synchronization theory possesses the Kuramoto model of coupled oscillators.

There is almost no attempt to ask a more fundamental question: why do all mature sciences eventually invent an abstract configuration space?

This paper proposes that the answer is simple. Every complex system requires a relational description. The space appears whenever object-based descriptions become insufficient — when what matters about a system is not what it *is* at a given instant, but the structure of what it *could become*.

5. Dual Space as the General Concept

Peter Rowlands introduced Dual Space within the mathematical framework of nilpotent quantum mechanics. His Dual Space possesses precise algebraic meaning: a second Clifford algebra, structurally identical to the first but commutative with it, dual to it in the sense of carrying no additional physical information, with the duality fixed by a zero-totality condition [2], a construction the author argues supplies the mathematics needed for both quantum mechanics and other areas of physics simultaneously.

The proposal developed here is different. Dual Space is generalized into a universal concept.

Definition. Dual Space is the abstract relational configuration space in which the possible coherent states of a system are represented independently of their physical realization.

This definition deliberately avoids restricting Dual Space to quantum mechanics, consciousness, biology, or artificial intelligence. Instead it becomes a universal scientific abstraction — a meta-space of which Hilbert space, latent space, fitness landscape, niche hypervolume, attractor manifold, state space, conceptual space, the linguistic sign system, the psychological life space, the statistical manifold, and cosmological superspace are all specific coordinate realizations, each carrying its own metric, dynamics, and interpretation. Where §2.13 applies, the same coordinate space additionally supports a dynamic reading, in which coherence corresponds to synchronization between oscillatory components rather than static co-location.

6. Why Science Needs Such a Concept

Scientific revolutions repeatedly occur when relations become more fundamental than objects.

Newton replaced isolated observations by phase trajectories. Einstein replaced forces by geometry. Quantum mechanics replaced particles by state vectors — and, in Rowlands' extension, by a duality between two vector spaces. Artificial intelligence replaced symbols by embeddings. Modern biology increasingly studies fitness landscapes and niche hypervolumes rather than isolated organisms. Neuroscience increasingly studies population manifolds rather than single neurons, and, following Fries, synchronization between oscillating populations rather than isolated firing rates. Economics increasingly studies phase portraits of agent populations rather than representative individuals. Ecology, since Lotka and Volterra, has treated population levels as oscillating rather than static quantities. Linguistics, a century ago, already replaced the isolated word by the system of differences. Psychology, in Lewin's hands, already replaced the isolated trait by the topological field.

The historical trend is unmistakable. Science progressively shifts from objects towards relations. Dual Space provides a common language for this transition.

7. Consciousness as One Application

Consciousness is not the motivation for introducing Dual Space. It is merely one application.

If consciousness represents a dynamical organization of coherent relationships, then it naturally occupies a region within Dual Space. The same conceptual framework can equally describe ecological systems, neural systems, economic systems, social systems, artificial intelligence.

Dual Space therefore precedes consciousness. It is not invented to explain consciousness.

8. Implications

Viewing scientific spaces as manifestations of a single abstraction has several consequences.

First, results from different disciplines become mathematically comparable. A ruggedness measure defined for NK fitness landscapes, for instance, may translate directly into a statement about the geometry of a loss landscape in machine learning, since both are scalar fields over a high-dimensional combinatorial or continuous space.

Second, theories developed in one field may become transferable to another, as already happened when fitness-landscape thinking migrated from population genetics into immunology and the social sciences, and when field-theoretic thinking migrated from physics into psychology via Lewin.

Third, apparently unrelated concepts such as latent spaces, attractor manifolds, niche hypervolumes, and semantic spaces become

different coordinate systems defined upon a common relational architecture.

Fourth, where a dynamic, oscillatory reading applies (§2.13), coherence within Dual Space can be given an operational definition — synchronization between coupled oscillators — that is independently testable in physical, neural, and ecological systems, rather than remaining a purely structural analogy.

Finally, Dual Space becomes a candidate for a scientific meta-language rather than another discipline-specific theory.

9. Conclusion

The history of science reveals a remarkable convergence. Independent disciplines repeatedly invent abstract relational spaces whenever direct object-based descriptions become inadequate — a pattern that, as this paper has shown, extends from subatomic physics to the topology of a single human life space, and from the genetic code to the geometry of the universe as a whole. In at least three of these disciplines, moreover, the relevant space is not merely a static container of possible states but a medium for synchronization between oscillatory components, giving Dual Space both a structural and a dynamic register.

Rather than treating these constructions as unrelated inventions, this paper proposes that they are instances of a single higher-order concept. Dual Space is therefore introduced not as an additional physical dimension nor as a speculative metaphysical realm. It is proposed as the universal configuration space of coherent relations from which discipline-specific mathematical spaces emerge.

Whether this proposal ultimately proves useful will depend on its ability to unify existing theories without replacing their successful mathematics. If successful, Dual Space may provide the common conceptual language that modern science has independently approached from many different directions without explicitly recognizing their shared foundation.

Annotated References

- [1] Gibbs, J. W. (1902). *Elementary Principles in Statistical Mechanics*. New Haven: Yale University Press. The founding text of statistical-mechanical phase space and the ensemble concept; source for the opening move of §2.1.
- [2] Rowlands, P. "A Dual Space as the Basis of Quantum Mechanics and Other Aspects of Physics," in *Scientific Essays in Honor of H. Pierre Noyes*. Rowlands' most direct formulation of why a single vector space is insufficient and why a second, dual Clifford algebra is required. Core source for §2.1 and §5.
- [3] Rowlands, P. (2010). "Physical Interpretations of Nilpotent Quantum Mechanics." arXiv:1004.1523. Accessible introduction to the "two vector spaces" construction; a good entry point for readers who want to follow the mathematics further.
- [4] Rowlands, P. (2007). *Zero to Infinity: The Foundations of Physics*. Singapore: World Scientific. Chapter 19, "Application of Geometrical Structures to DNA and Genetic Coding." Rowlands' own book-length treatment of the dual-space/DNA correspondence.
- [5] Hill, V. & Rowlands, P. "Nature's Code" (I–III, 2007–2009+), presented at successive ANPA/CASYS conferences. The recurring, self-developed series in which Rowlands and Hill work out the correspondence between the nilpotent dual-space algebra and the structure of the genetic code in geometric detail.
- [6] GeeksforGeeks. "Latent Space in Deep Learning." Accessible, non-technical definition of latent space; useful for a general-audience footnote.
- [7] Yang, H., Kong, Q. & Mao, W. "A Deep Latent Space Model for Graph Representation Learning." arXiv:2106.11721. Example of latent space applied to relational (graph) data rather than isolated data points — closely aligned with this paper's "relations rather than objects" thesis.

[8] Mikolov, T., Chen, K., Corrado, G. & Dean, J. (2013). "Efficient Estimation of Word Representations in Vector Space." arXiv:1301.3781; and Mikolov, T., Sutskever, I., Chen, K., Corrado, G. & Dean, J. (2013). "Distributed Representations of Words and Phrases and their Compositionality." The founding word2vec papers — the canonical example of "meaning as geometry."

[9] Harris, Z. (1954). "Distributional Structure." *Word*, 10(2–3). The original distributional hypothesis in linguistics, later summarized in Firth's dictum that a word is characterized by the company it keeps.

[10] Wright, S. (1932). "The Roles of Mutation, Inbreeding, Crossbreeding and Selection in Evolution." The original paper introducing the fitness landscape; cited as the founding document by virtually every secondary source on the topic.

[11] Kauffman, S. A. & Weinberger, E. D. (1989). "The NK Model of Rugged Fitness Landscapes and its Application to Maturation of the Immune Response." *Journal of Theoretical Biology*, 141. Generalizes Wright's landscape to tunable ruggedness — important if Dual Space is to encompass combinatorial (discrete) as well as continuous spaces.

[12] Diaz Ochoa, J. G. et al. "The Evolution of Wright's (1932) Adaptive Field to Contemporary Interpretations and Uses of Fitness Landscapes in the Social Sciences." *Biology & Philosophy*. Documents exactly the kind of cross-disciplinary transfer described as an implication in §8.

[13] Gärdenfors, P. (2000). *Conceptual Spaces: The Geometry of Thought*. Cambridge, MA: MIT Press. Primary source for §2.5; presents a geometric intermediate level between symbolic and connectionist modeling of cognition.

[14] Amari, S. (1985). *Differential-Geometrical Methods in Statistics*. Springer Lecture Notes in Statistics 28; and Amari, S. & Nagaoka, H. (2000). *Methods of Information Geometry*. AMS/Oxford University Press. The founding texts of information geometry, usually cited together.

[15] Chaudhuri, R. et al. (2019). "The Intrinsic Attractor Manifold and Population Dynamics of a Canonical Cognitive Circuit Across Waking and Sleep." *Nature Neuroscience*. Direct experimental demonstration of a continuous attractor manifold (the head-direction ring) — the strongest empirical source for §2.7.

[16] "Models of Attractor Dynamics in the Brain" (2026), arXiv. Accessible overview of point, line, ring, and limit-cycle attractors in neural systems.

[17] Hutchinson, G. E. (1957). "Concluding Remarks." *Cold Spring Harbor Symposia on Quantitative Biology*, 22. The founding paper of the n-dimensional hypervolume niche concept — core source for §2.8.

[18] Blonder, B. et al. (2014). "The n-Dimensional Hypervolume." *Global Ecology and Biogeography*. Modern, methodological elaboration of Hutchinson's concept.

[19] Tesfatsion, L. "Agent-Based Computational Economics." Overview, Iowa State University. Explicit source for the contrast between classical, equilibrium-imposing state-space models and agent-based phase portraits in economics — core source for §2.9.

[20] Naumann-Woleske, K., Knicker, M. S., Benzaquen, M. & Bouchaud, J.-P. "Exploration of the Parameter Space in Macroeconomic Agent-Based Models." Demonstrates the phase-portrait approach to economic state space in practice.

[21] Saussure, F. de (1916). *Cours de linguistique générale*. Posthumously published lecture course; source of the definition of language as a system of interdependent terms — core source for §2.10.

[22] Secondary literature on Saussurean structuralism (e.g., Falk, J. S., "Ferdinand de Saussure," EBSCO Research Starters), summarizing the differential theory of the sign for a general readership.

[23] Lewin, K. (1936). *Principles of Topological Psychology*. New York: McGraw-Hill. The founding text of field theory in psychology and the topological life space — core source for §2.11.

[24] Lewin, K. (1951). *Field Theory in Social Science*. New York: Harper & Row. Later elaboration and the $B = f(P, E)$ formulation of behavior as a function of the life space.

[25] Wheeler, J. A. Original formulation of "superspace" as the configuration space of three-dimensional geometries; discussed in Kiefer, C., "On the Interpretation of Quantum Cosmology," arXiv:gr-qc/9605019. Core source for §2.12.

[26] Misner, C. W. et al.; overview in recent minisuperspace literature (e.g., "Bohmian Quantum Cosmology from the Wheeler–DeWitt Equation," arXiv:2512.18818). Source for the minisuperspace truncation used to make superspace tractable.

[27] Marcer, P. & Rowlands, P. (2017). "Nilpotent Quantum Mechanics: Analogs and Applications." *Frontiers in Physics*, 5. Explicit listing of the harmonic-oscillator mechanism, zitterbewegung, and Berry/geometric phase as characteristics of the nilpotent structure — core source for the oscillatory dimension of §2.1 and §2.13.

[28] Kuramoto, Y. (1975). Original formulation of the model for synchronization of coupled phase oscillators; see also Acebrón, J. A. et al. (2005), "The Kuramoto Model: A Simple Paradigm for Synchronization Phenomena," *Reviews of Modern Physics*, 77, for the standard modern overview. Core source for §2.13.

[29] Overview literature on applications of the Kuramoto model across physical, biological, chemical, and social systems (see [28]).

[30] Fries, P. (2005). "A Mechanism for Cognitive Dynamics: Neuronal Communication through Neuronal Coherence." *Trends in Cognitive Sciences*, 9. Founding article of "communication through coherence" — core source for §2.13.

[31] Fries, P. (2015). "Rhythms for Cognition: Communication through Coherence." *Neuron*, 88. Extended, later formulation with two decades of additional experimental evidence.

[32] Lotka, A. J. (1925). *Elements of Physical Biology*; Volterra, V. (1926). Independently developed, classical predator–prey equations; see Scholarpedia, "Predator-Prey Model," for an accessible overview. Core source for §2.13.