

E8, the Bronze Mean and the Vacuum.Net Theory

J. Konstapel, Leiden, 4 October 2026.

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Abstract

Three structures are laid side by side. The first is E8, the largest exceptional Lie algebra, as used in the physics model of Frank (Tony) Smith. The second is the Bronze Mean, the number $(3 + \sqrt{13})/2$, with its sequence 1, 1, 4, 13, 43, 142. The third is the Vacuum.Net theory, which writes every place in the net as a string of three-valued signs.

They meet at one place: the fifth ring of the net. A string of five signs has $3^5 = 243$ forms. Three of them are uniform. The other 240 equal in number the root vectors of E8. The algebra E8 has 248 dimensions and sits inside a Clifford algebra of $2^8 = 256$. The gap between the two ladders is $256 - 243 = 13$, the number under the root of the Bronze Mean. The bronze sequence follows the uniform rim of the rings for three steps. At its fourth step, 43, it leaves the rim and enters the fifth ring.

Every count in this essay is written out and can be checked by hand. What is counted, what is read, and what is still open are kept apart in the last section. This is the reading of 4 October 2026.

1. The net and its rings

The image of the Vacuum.Net is a fishnet in water. There is one strand. Where it crosses itself it passes over, passes under, or does not cross. Those are three states: +1, -1 and 0. One such sign is called a trit.

A place in the net is written as a string of trits. The string is read as a number in balanced ternary. The leftmost sign counts for the highest power of three. An example:

• $- - - + = 81 - 27 - 9 - 3 + 1 = 43.$

Every whole number has exactly one such string. Zero is the string of rests. The mirror of a string, with every sign reversed, is the negative of its number.

Strings of n signs form ring n. Ring n has 3^n strings. Its largest number is the string of n plus signs. That number is the ceiling of the ring.

Ring n	Strings 3^n	Ceiling $(3^n - 1)/$ 2	Ceiling as string
1	3	1	+
2	9	4	++
3	27	13	+++
4	81	40	++++

5	243	121	+++++
6	729	364	++++++

The ceilings obey $c(n) = 3 \cdot c(n-1) + 1$. Each ring holds three uniform strings: all plus, all rest, all minus. They are the ceiling, zero and the mirror of the ceiling.

The net has one rule for neighbours. Three neighbouring trits form a window. The window is closed when the three add to zero, counted modulo 3. With two of the three fixed, exactly one value closes the window.

2. The E8 model in brief

Smith builds his model on the Clifford algebra $Cl(8)$. It has $2^8 = 256$ dimensions, graded as

$$1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256.$$

Two copies multiply to $Cl(16) = Cl(8) \otimes Cl(8)$. Inside it lies E8, with 248 dimensions:

$$248 = 120 + 128.$$

The 120 is the algebra $Spin(16)$. The 128 is one of its half-spinors. E8 has rank 8 and 240 root vectors. They come in 120 pairs, each root with its opposite.

Smith reads the 240 roots as physics.

- 24 + 24 roots of two copies of D4. One copy gives gravity, the other the forces of the Standard Model.
- 64 roots for eight dimensions of spacetime times eight Dirac gammas.
- 64 + 64 roots for eight kinds of fermion particles and eight kinds of antiparticles, each times eight.

Together: $24 + 24 + 64 + 64 + 64 = 240$. Three features of the model matter here.

Triality. The three blocks of 64, spacetime, particles and antiparticles, are related by the triality of $Spin(8)$. Three equal things are bound so that two fix the third.

Three generations. A fermion of the first generation is one octonion. One of the second is a pair. One of the third is a triple.

The number 256. The book opens with $16 \times 16 = 256$ patterns, and with the wheel of Ramon Llull: 16 points joined by 120 lines. The 120 lines are the 120 generators of $Spin(16)$.

3. Where two meets three

Smith counts in twos. Eight binary choices give $2^8 = 256$. The net counts in threes. Five trits give $3^5 = 243$.

The two ladders never coincide, because $\log_2 3 = 1.58496\dots$ is not a fraction. They come close where $3^q \approx 2^p$. Those places follow from the continued fraction of $\log_2 3$. Its convergents are $2/1$, $3/2$, $8/5$, $19/12$, $65/41$ and further. The third one says $3^5 \approx 2^8$.

Number	What it is
243	3^5 , the strings of ring 5
248	the dimension of E8
256	2^8 , the dimension of Cl(8)

E8 lies between the two ladders. It is 5 above the ladder of three and 8 below the ladder of two. Those are the two exponents themselves. The whole gap is

$$256 - 243 = 13.$$

4. The count

Take ring n and remove its three uniform strings. What remains is $3^n - 3$.

Ring n	$3^n - 3$	In the E8 model
3	24	the 24 roots of D4
5	240	the 240 roots of E8

Both numbers are central in Smith's structure. D4 appears twice inside E8, and its 24 roots form the figure called the 24-cell.

The grouping agrees as well. Rotate a string: move its first sign to the end. A string of three signs has three rotations. The 24 strings of ring 3 fall into 8 classes of 3. The 24-cell, in its standard description, consists of three figures of 8 points each. Triality carries one into the next. Both times the count is 3×8 .

On ring 5 the 240 strings fall into 48 classes of 5. And 48 is the number of roots of F4, the symmetry group of the exceptional Jordan algebra of 27 dimensions. That algebra is $3^3 = 27$, the full count of ring 3. Smith also cites the ternary Golay code, with $3^6 = 27 \times 27 = 729$ words. That is the full count of ring 6.

The mirror agrees too. The 240 roots form 120 pairs of opposites. Ring 5 holds the numbers 1 to 120 and their mirrors -1 to -120 . Zero and ± 121 are the three uniform strings. So the positive roots match the numbers 1 to 120 in count, and the mirror of a string matches the opposite of a root. In Smith's reading the opposite of a particle root is its antiparticle.

One more count gives the inner shape of ring 5. Sort the 240 strings by their number of rests.

Rests in the string	Strings
0	30
1	80
2	80
3	40

4	10
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The sum is 240. The first row is $2^5 - 2 = 30$: the strings without a rest, less the two uniform ones.

5. Triality is the closure rule

Triality binds three things of equal size so that any two fix the third. The closure rule of the net says the same about three neighbouring trits. With two fixed, one value closes the window.

Read as a trit, Smith's three blocks are: particle +1, spacetime 0, antiparticle -1. Their sum is zero. A window with all three values is one of the six closed windows that are not uniform.

The three generations fit the same ladder. One octonion, a pair, a triple: that is depth 1, 2 and 3 on a base of eight instead of three.

Smith also finds a system with three states for the heaviest quark and the Higgs: a low, a middle and a high state, joined by one line. That again has the form of one trit.

6. The Bronze Mean

The Bronze Mean is the positive root of $x^2 - 3x - 1 = 0$:

$$\sigma = (3 + \sqrt{13})/2 = 3.30278\dots$$

Its continued fraction consists of threes only: $3 + 1/(3 + 1/(3 + \dots))$. Its sequence obeys $b(n) = 3 \cdot b(n-1) + b(n-2)$:

1, 1, 4, 13, 43, 142, 469.

The step is written out: $4 = 3 \cdot 1 + 1$, $13 = 3 \cdot 4 + 1$, $43 = 3 \cdot 13 + 4$, $142 = 3 \cdot 43 + 13$. Each term is three times the one before, plus the one before that. The second part is the memory term: the sequence carries what lies two steps back.

Three things connect it to ring 5.

6.1 The gap is thirteen

The gap between the ladders is $2^8 - 3^5 = 13$. Thirteen is a term of the bronze sequence. It is also the number under the root in σ .

6.2 The sequence leaves the rim at ring 5

Write each bronze term as a string.

Bronze term	String	Ring	Ceiling of that ring
1	+	1	1
4	++	2	4
13	+++	3	13

43	+ - - - +	5	121
142	+ - - + - +	6	364

The check for 142: $243 - 81 - 27 + 9 - 3 + 1 = 142$.

The first three terms are the ceilings themselves. They are uniform strings. While the memory term is 1, the bronze rule and the ceiling rule are the same rule: three times the last, plus one.

At the next step they part. The ceiling of ring 4 is $3 \cdot 13 + 1 = 40$. The bronze term is $3 \cdot 13 + 4 = 43$. The number 43 is larger than 40. It does not fit on ring 4. It moves to ring 5, where it is the string + - - - +.

So the memory term does one thing. It lifts the sequence off the uniform rim and into the interior of ring 5. That interior is the set of 240 strings counted in Section 4. The number 43 is the first bronze term that is not a ceiling. In the count it is one of the 240.

The difference is $43 - 40 = 3$. The 240 itself is $243 - 3$.

This gives a place to an old pair of numbers. Built labyrinths count to forty: the classical one counts $7 \cdot 5 + 5 = 40$, the one in Chartres $11 \cdot 3 + 7 = 40$. The Sri Yantra has 43 triangles. Forty is the rim of ring 4. Forty-three is the first step inside ring 5.

6.3 The fourth power is 119

The powers of σ , taken with their mirror, give whole numbers: $\sigma^n + (-1/\sigma)^n$. They obey the same rule, starting from 2 and 3:

2, 3, 11, 36, 119, 393.

So $\sigma^4 + \sigma^{-4} = 119$, exactly. The roots of E8 form 120 pairs. The two numbers differ by one.

The same 119 occurs elsewhere. From the Planck length to the width of the observable universe is a factor of about 5×10^{61} . Since $\log_{10} \sigma = 0.5189$, that is 119 bronze steps: $119 \times 0.5189 = 61.7$.

7. Gold and bronze

In Smith's book the mean is gold, not bronze. He builds E8 from two copies of a figure with 120 points, related by the Golden Ratio. That gives $120 + 120 = 240$.

Gold and bronze are family. Both are metallic means, the roots of $x^2 - m \cdot x - 1 = 0$.

Mean	m	Value	Continued fraction	Symmetry of its tiling
Gold	1	1.618 ...	all ones	fivefold
Silver	2	2.414 ...	all twos	eightfold
Bronze	3	3.303 ...	all threes	sixfold

Gold belongs to the division in five. Bronze belongs to the division in three. E8 carries both, and the count of Section 4 shows how. The 240 roots are the strings of five signs on a base of three. The five is the side that Smith describes. The three is the side of the net.

8. What this says about the MAZE

The MAZE is the address system built on the net. Every item has a number, and the number is a string of trits.

Ring 5 is the ring of E8. The MAZE has 243 addresses on ring 5. Take away rest and the two ceilings, and 240 remain.

The ceilings are the uniform strings. The series 1, 4, 13, 40, 121 is the rim. The roots are everything between.

The mirror is the antiparticle. The mirror column of the MAZE, n against $-n$, is the same operation as root against opposite root.

The 256 and the 243 are neighbours. A byte holds 256 values. An address of five trits holds 243. Ring 5 is the place where the two-valued world of present computers and the three-valued MAZE can be exchanged, with a remainder of 13.

Smith supplies inhabitants. The MAZE has the addresses of ring 5. Smith has names for 240 roots: forces, spacetime, particles, antiparticles. Joined, that is a body of physics with an address by construction.

9. Status

Counted. These are exact and checked.

- $3^5 - 3 = 240$ and $3^3 - 3 = 24$.
- 8 classes of 3 on ring 3; 48 classes of 5 on ring 5.
- 120 mirror pairs on ring 5.
- $2^8 - 3^5 = 13$, and $243 < 248 < 256$.
- The bronze terms 1, 4, 13 are ceilings; $43 = + - - - +$ lies on ring 5; $142 = + - - + - +$ lies on ring 6.
- $\sigma^4 + \sigma^{-4} = 119$.

Read. These are readings, not derivations.

- Triality as the closure rule, with particle, spacetime and antiparticle as +1, 0 and -1.
- The three generations as depths 1, 2 and 3.
- The memory term as the step from the rim into ring 5.

Open. Two things are not yet there.

1. *The map.* The counts agree, but no rule yet says which root belongs to which string. There is a real obstacle. All 240 roots of E8 are alike: the symmetry of E8 carries any root to any other. The 240 strings are not alike: they differ in their number of rests, as the table in Section 4 shows. A map must say what those five kinds become on the side of E8.
2. *The sum.* In E8 two roots add to a third root, or they do not. Each root has exactly 56 partners with which it adds to a root. In the MAZE two addresses add with a carry. If the map exists, the two sums can be compared. Claude Code can run that comparison: 240

roots, 240 strings, one sum on each side. If they agree, the open rewrite rule of the MAZE is the structure of E8. If they do not, the agreement is a count and no more.

The Bronze Mean itself keeps its earlier status. It is not derived from closure here. What this essay adds is a place: the point where the bronze sequence and the rim of the rings part is the ring where E8 lives.

Annotated references

Smith, F. D. (Tony), Jr. (2008). *Physics of E8 and $Cl(16) = Cl(8) \otimes Cl(8)$* . Georgia, June–July 2008. *Why read?* The E8 model used here: the 256 of $Cl(8)$, the split $248 = 120 + 128$, the reading of the 240 roots, triality among the three blocks of 64, the three generations, and E8 as two figures related by the Golden Ratio. *Reading advice:* the Technical Introduction and "Structure of the E8 Physics Model"; then " $H_4 + H_4 = E_8$ ".

Konstapel, J. (2026). *The Climate on the Ladder of Three*. Constable Research, Leiden, 3 October 2026. *Why read?* The derivation of the layers from the trit, the closure rule on three neighbours, and the closure depths where $3^q \approx 2^p$. *Reading advice:* Section 3.

Konstapel, J. (2026). *Everything Has Its Number in the Maze*. Constable Research, Leiden, August 2026. *Why read?* The unique address in balanced ternary, with $43 = + - - - +$ as the worked example, the mirror, and the series of ceilings.

Konstapel, J. (2026). *Counting the Universe*. Constable Research, Leiden, August 2026. *Why read?* The bronze sequence, the memory term, the test of 43 against 40, and the 119 bronze steps from the Planck length to the universe.

Konstapel, J. (2026). *The Labyrinth Counts to Forty*. Constable Research, Leiden, September 2026. *Why read?* Why built labyrinths count 40 and not 43: they record the rim, not the rule of growth.

Baez, J. C. (2002). "The Octonions." *Bulletin of the American Mathematical Society* 39, 145–205. *Why read?* The clearest account of triality, the exceptional Jordan algebra of 27 dimensions, F_4 and E8, by someone who writes for readers. *Reading advice:* the parts on triality and on the exceptional Jordan algebra.

Conway, J. H., and Sloane, N. J. A. (1999). *Sphere Packings, Lattices and Groups*, third edition. New York: Springer. *Why read?* The 240 root vectors of E8, their inner products, and the binary and ternary Golay codes. *Reading advice:* Chapter 4.

Coxeter, H. S. M. (1973). *Regular Polytopes*, third edition. New York: Dover. *Why read?* The 24-cell and its three inscribed figures of eight points, and the figure of 120 points behind the golden construction of E8.

Moody, R. V., and Patera, J. (1993). "Quasicrystals and icosians." *Journal of Physics A* 26, 2829–2853. *Why read?* How E8 falls into two golden copies of a figure with 120 points. It is the mathematics behind Smith's " $H_4 + H_4 = E_8$ ".

de Spinadel, V. W. (1999). "The family of metallic means." *Visual Mathematics* 1(3). *Why read?* Gold, silver and bronze as one family, with their continued fractions.

Dotera, T., Bekku, S., and Zihler, P. (2017). "Bronze-mean hexagonal quasicrystal." *Nature Materials* **16**, 987–992. *Why read?* The Bronze Mean realised as a sixfold tiling of triangles and rectangles. It shows that bronze is the mean of the division in three.

Hardy, G. H., and Wright, E. M. (1938). *An Introduction to the Theory of Numbers*. Oxford: Clarendon Press. *Why read?* Continued fractions and their convergents: the ground under $3^5 \approx 2^8$ and under σ as a fraction of threes. *Reading advice:* Chapter X.

Knuth, D. E. (1997). *The Art of Computer Programming*, Vol. 2, §4.1. Reading: Addison-Wesley. *Why read?* Balanced ternary: every whole number has one string, and the mirror is the negative.