From Movement to Mathematics

What a search for the foundations of mathematics in the Vacuum.Net theory found, and the search it opens

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Abstract

This essay reports a search. The question was whether the Vacuum.Net theory can ground mathematics by derivation. The work used homotopy type theory in cubical Agda and went bottom-up. Where a derivation stopped, the smallest missing piece was named. The result is a ladder of twenty-two rungs, twenty-eight machine-checked files and a book of forty-nine pages [1]. The net derives, under stated hypotheses, the discrete core of mathematics and the continuum of one interval. Four assumptions remain. Two findings stand out. First, the start is not a distinction but movement: an eternal round. Second, the two ladders of the theory divide the work. The ×3 ladder gives addresses; the ×2 ladder, with the same three digits, gives computation. The search also exposed a pattern. Several fields hold pieces of the same structure, but none sees the whole. We read this as a trace of steam-engine thinking. The essay closes by announcing a new search into these blind spots, starting with decisions at thresholds.

1. The question

Mathematics can be read as a large collection of words. Each word is a unique pattern, and people gave it a name. The Vacuum.Net theory writes its patterns in three values, the trits −1, 0 and +1 [2]. The question of this search was simple. Can the theory derive these words, one from the other, instead of only translating them?

Many have tried to order mathematics. Euclid built a tree from axioms. Bourbaki built a lattice of structures. Category theory built a network of arrows. Homotopy type theory builds a space of paths [3]. An older line, from India, generated words from rules: Pāṇini for grammar, Piṅgala for metres [1]. Our ladder belongs to that older line. It is a list in the order of generation. Each rung follows from the one before by one rule.

We agreed on one goal before we started. The goal was derivation. When a derivation stops, the stopping point is a result. It shows what the theory does not yet give. It also sharpens the question.

2. Method

The language was homotopy type theory, in its cubical form [3, 4]. The tool was cubical Agda 2.8.0 with the cubical library, version 0.9 [5]. Every formal file runs under the flag --safe and uses no postulates. Twenty-eight files were checked together on 17 November 2026.

Each claim in the book carries one of four labels. “Proved in Agda” means the proof checks. “Checked by computation” means a script computes it. “Known theorem” means the literature proves it. “Reading” means an interpretation that no proof forces. The labels keep two things apart: what follows, and what we see in it.

Every deliverable was checked by a separate AI agent that had not written it. This is the author’s Respondent–Judge protocol. The judges found real errors. Their corrections are in the text. Section 7 returns to the question of who checks such work.

3. The start is movement

The written theory begins with a first distinction, as Spencer-Brown's Laws of Form does [6]. During the work the author corrected this. The first phase is not a distinction but a pulsation. In the beginning there is movement, with a direction: <- . ->. Nobody was there at the start. But without movement nothing happens. So the movement never stopped and never began. It is an eternal round.

Homotopy type theory has exactly this object. The circle S^1 is one point and one movement that returns to it [3]. Its loops are the whole numbers: every number of turns is kept [7]. A movement that goes out and back along the same way leaves nothing. Only a round leaves a trace. All of this is proved in Agda. That the round is eternal and has no start is a reading, not a theorem: S^1 has no time in it.

The correction has precursors. Martin-Löf, who founded the type theory beneath homotopy type theory, followed the order of concepts backwards. He ended at act and object [8]. Kauffman reads the imaginary unit as an oscillation in time [9]. Seen at one place, the round shows three directions: back, the point, and out. These are the three trits. We read a trit as a still picture of a movement, not as a value.

4. The ladder

From the round the book climbs twenty-two rungs. Table 1 gives the outline. Rungs 0 to 12 rest on proofs, computations and cited theorems; Rung 11 assumes the minimal node. Rung 13 adds the embedding in space. Rungs 14 to 19 each add a hypothesis from physics. Rungs 20 and 21 add infinite streams and closure as oscillation.

Rungs	What appears	Field that already had a name for it
0–2	The round, three directions, the return	Circle, integers as windings
3–4	Direction and words	$\mathbb{Z}_3$, balanced ternary; digit = winding modulo three
5–8	Windows, closures, seams	Lines of F_3^n, circulants, a second difference $D^2 = 0$
9–10	Closed without a start; the plane	Necklaces; Eisenstein integers, base $1 - \omega$
11–13	Mirror and ring; two ladders; three dimensions	PSL(2, $\mathbb{Z}$); $2^n \neq 3^k$ and the comma; the trefoil
14–19	Coupling, free path, closure on itself, registers, measure, loss	Analysis, probability, Clifford algebra (by hypothesis)
20–21	Numbers without end; closure as oscillation	Signed-digit reals; periods 3, 4, 6

Table 1. Outline of the ladder [1].

Two results came out of the work itself, not from the theory. Rings of the net come to rest exactly when four divides their length. And closure on the trits is a second difference, $D^2 = 0$, so the trits carry a simple chain complex already at Rung 7.

5. Where the derivation stopped

The stopping points are the core of the report. Table 2 lists the main ones.

Stopping point	What became of it
Loops as words	Resolved. Loops are trit words; the integers are only a measuring rod.
Digit and winding	Resolved. A digit is the winding modulo three, for all loops.
The rewrite rule	Forced up to a scalar among linear rules of radius one.

Stopping point	What became of it
The base 1 − ω	Forced up to complex conjugation; the choice within the pair is not derived.
The node of Rung 11	Two hypotheses H1, H2 proved for the minimal node; that the net's node is this node remains open.
Product of streams	No finite automaton can compute it; it needs growing memory [10].
Value of an infinite word	Closed at the level of values (Section 6).
Chirality	Open. The knot group cannot see left from right; the embedding in space is needed.

Table 2. Main stopping points and their status.

Four assumptions remain. The net's node must be the minimal node of Rung 11. One base must be chosen from a conjugate pair. Space must be given for the knot at Rung 13. And Rungs 14 to 21 rest on hypotheses, physical up to Rung 19.

6. The continuum

The hardest question was the continuum. The written theory took the real numbers as an input. The search asked whether the net builds them itself. Up to Bishop's equality it does, in five steps. The identification with the Cauchy reals and the proof that the ×3→×2 conversion respects places are not formalised.

6.1 Oscillation gives the rational phases

A place within one turn can be written as a stream of trits. An oscillation repeats a finite block. In Agda, such a stream is a fixed point of "put the block in front", which is Kauffman's eigenform in the language of the net [9]. A block W of length n gives the phase $W/(3^n - 1)$. With a finite prefix, oscillations give exactly the rational phases. These lie dense on the round, but they are countable.

6.2 The net has movement that never closes

The theory has two ladders, ×2 and ×3. They never meet: $2^n = 3^k$ only when $n = k = 0$. This is proved in Agda by parity. So $\log_2 3$ is irrational. Stepping the round by $\log_2 3$ of a turn never repeats. Its nearest returns come at the closure depths 1, 2, 5, 12, 41, 53, 306 and 665. At depth 12 the miss is the Pythagorean comma. By Kronecker's theorem the steps come as close as one likes to every place [11]. We read this as follows. The comma does not drive the movement. It keeps the movement from repeating, and so it lets it reach everything.

6.3 The limit step is a path

A place is a descent through the scales. At each scale one trit is added, and what is left goes on to the next scale. This is the carry of the theory. Two descents are the same place when their difference, carried scale by scale, stays in a window for ever. In Agda this is an equivalence relation. The places form a set quotient. A quotient adds a path between two descents that meet. We read this as: the continuum of one turn is made of descents and of the movement between them. The descents 0.1TTT… and 0.0111… differ in their first trit, yet they are one place.

6.4 The value of a place

Agda proves that the carried rest after n steps is the difference of the two truncated values. So the same place means a bounded difference at every scale. That is Bishop's equality of real numbers [12]. The value of a place is a real number in Bishop's sense, and the integer bounds behind this are machine-checked. The identification with the Cauchy reals of the HoTT book is not formalised [3].

6.5 The two ladders divide the work

Arithmetic revealed a division of labour. On the ×3 ladder three digits are not redundant. The first digit of a sum can depend on a digit arbitrarily far down. On the ×2 ladder the same three digits are

redundant [13]. There the average works one digit at a time; in Agda it acts on places themselves. The conversion from ×3 to ×2 is only computed. Constructively the difference goes further. An argument on paper shows: if every real number in [−1/2, 1/2] had a ×3 place, one could decide $x \leq 0$ or $x \geq 0$ for every real x. That is the principle LLPO, which constructive mathematics does not prove [14]. The ×2 places need no omniscience principle, though countable choice is used to reach the Cauchy reals [15]. We read this as a division of labour: the ×3 ladder gives addresses, the ×2 ladder gives computation and the full continuum.

7. Who checks such work

A common answer is: a human referee. We doubt that this is still the right answer. Voevodsky, the founder of univalent foundations, changed his research because he feared that proofs had become too complex to check by hand [16]. He had found an error in his own work seven years after publication. His answer was to let computers check proofs.

That answer has now arrived. In 2026 AI systems prove research-level theorems, and formal proof assistants check them [17]. In such work the reader is the kernel of the proof checker. For the proved parts of this search that holds too. Twenty-eight files check under --safe.

One limit remains. A checker tests whether a proof fits its definition. It does not test whether the definition means what we intend. That S^1 is the eternal round is such a step. In the book these steps carry the label “Reading”. There the author is the reader. The files, scripts and build script are published so that anyone, person or machine, can check them again.

8. The new search: what steam-engine thinking left out

The author has long argued that energy is a poor concept from the age of the steam engine [26]. That way of thinking treats the world as stocks that are used up. Our results contradict it at several points. Where they clash, we expect blind spots. Table 3 sets them out.

Steam-engine assumption	What the search found	Expected blind spot
Stuff first, then motion	Movement first; a place is a descent	Measuring phase and rhythm instead of amounts
Two values: on or off	Three directions; the middle is a direction	The resting middle as an active state
Loss is waste	What does not close goes on to the next scale	Rest and noise as carriers
Closing is perfect	What never closes reaches every place	Near-resonance as an ordering principle
Everything can be measured and decided	At a boundary a ×3 place cannot be decided	Decisions at thresholds and norms
One base for all	Two ladders divide the work	Computers and AI built on trits

Table 3. Blind spots predicted by the search.

A first survey checked each blind spot against the literature. None is wholly unexplored. Each exists as a niche.

- Phase. Timing treatment to the body clock can change the effect and toxicity of cancer treatment, yet its use in health care is still scarce [18].
- The middle. Sensory neurons split into ON and OFF types that signal increase and decrease [19]. As capacity grows, optimal coding goes from binary to ternary and beyond [20].
- Noise. In stochastic resonance, added noise improves detection instead of degrading it [21]. The passing of a rest to the next scale we did not find.

- Near-resonance. Systems with two competing frequencies lock at rational ratios, with quasi-periodic behaviour in between [22]. A ladder of closure depths as an ordering principle we did not find.
- Thresholds. Measurement practice knows that a true value near a limit may lie on either side, and uses guard bands and decision rules [23]. Recent work shows that at the threshold itself, acceptance tends to a coin toss [24].
- Trits in computing. One model family now uses weights −1, 0 and +1 throughout the network [25]. They use trits for compression, not the division of labour between the two ladders.

The result of the survey is a pattern. What has never been studied is not one of these fields. It is the fact that they are one structure. Movement first, three directions, a rest that goes on, the difference between closing and not quite closing, and a boundary that cannot be decided but can be computed: each field solves one piece. We read this as a trace of steam-engine thinking: it divided these pieces over separate disciplines, so nobody saw the whole.

9. The first target: decisions at thresholds

We start where the search offers most. Norms, limits and diagnostic thresholds govern health, environment and finance. Practice treats the trouble at a threshold as a statistical nuisance. The search gives a principled reason. On a non-redundant scale, a place exactly at a boundary cannot be decided digit by digit. It can be computed only with redundancy, where neighbouring codes overlap. Our hypothesis is that the guard bands of measurement practice are such redundancy. They work, but they are seen as a patch.

The new search will test whether this reading improves real decisions. Its questions are concrete. Where do current norms decide on a non-redundant scale? What would a redundant decision rule look like, built on the ×2 ladder? And what changes for the people who must live with the decision? The nitrogen deadlock in the Netherlands, diagnostic cut-offs in medicine and limits in finance are the first cases.

10. Conclusion

The search asked whether the Vacuum.Net theory can ground mathematics by derivation. Within stated hypotheses, it derives the discrete core and the continuum of one interval. Where it stops, the missing piece is named. That was the goal.

The search also changed its own starting point. It begins with movement, not with a distinction. And it found that the theory's two ladders divide the work of mathematics: one names, the other computes.

Finally, the search pointed outward. Fields that look unrelated hold pieces of one structure. The next search follows those pieces, beginning where decisions meet their limits.

Data and files. The book (v3.0), the twenty-eight Agda files, the scripts and the build script are kept in the author's repository [1]. All Agda files check with Agda 2.8.0 and cubical v0.9 under --safe.

Tool disclosure. The Agda files, computations and this text were made with the AI system Claude (Anthropic) under the direction of the author. The author chose the method and decides what counts as a result.

Annotated references

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