How to Calculate Science

The Carry as the Rewrite Rule of a Self-Addressing Knowledge Net

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Status: the arithmetic in Parts II and III is proved and can be checked by hand. The correspondence in Part IV is read at the human winding depth. The measurements in Part VI have not yet been run.

I. The net, the mesh, and the missing rule

Picture a fishing net. One strand, knotted back into itself. The knots are not additions to the strand. They are the strand returning and holding. What closes in phase stays. What does not close comes apart. Every mesh is bounded by four crossings of one continuous thread.

The MAZE takes that picture literally and applies it to knowledge. A piece of knowledge is not an entry in a catalogue. It is a turn of the strand that keeps coming back. Its place in the net is not assigned by a librarian. It is computed from the turn itself.

The computation is short. A turn is one of three moves: over, straight, under. Write them as +1, 0, −1. A route is a sequence of turns. The address of a route is the integer

$a = \Sigma\, t_{\mathbf{k}} \cdot 3^k$

with the last turn at weight 1, the one before it at weight 3, then 9, 27, 81.

So the route (+1, −1, −1, −1, +1) has the address

$81 - 27 - 9 - 3 + 1 = 43.$

This map from routes to integers is a bijection. A route of depth *n* gives an address, and every integer between $-(3^n-1)/2$ and $+(3^n-1)/2$ gives back exactly one route. The capacity of the rings runs 1, 4, 13, 40, 121, 364. Rest is address zero.

Four properties follow without extra machinery.

Coarsening is dropping the last turn. In arithmetic that is dividing by 3 and rounding to the nearest integer. The error is at most half a unit. Coarse knowledge is a prefix of fine knowledge.

The opposite is the minus sign. Negating every turn mirrors the whole route.

Distance is subtraction. The difference between two addresses is itself a route: the way from one to the other.

Emptiness is legible. A ring has a fixed capacity. If fewer objects are encoded than the ring holds, the unoccupied addresses stay in the geometry. They are positions, not absences.

The inhabited net currently holds roughly 3.6 million pieces across about 4,800 windings, drawn from the English Wikipedia, the periodic table, and a body of essays. The periodic table is the cleanest population test. The 118 known elements were encoded into ring 5, which holds 121 positive addresses. 121 − 118 = 3. Three positions stay empty, and they stay in the structure rather than being edited out.

All of this works. And all of this is an archive.

An archive with beautiful arithmetic is still an archive. It stores, it retrieves, it navigates. It does not move on its own. The open problem in the project has carried a number for a year: **O2, the rewrite rule R.** Three conditions were set for it in advance:

1. It preserves depth.
2. It closes under coarsening.
3. $R \circ R = 0$.

The third condition is the strange one. It says: apply the rule twice and nothing is left. A rule that is its own exhaustion. It was set because the net model demands it. Loading, holding, discharge. Once a mesh has closed, closing it again must yield nothing.

For a year the rule was looked for. It did not need to be invented. It was already in the number system.

II. The carry is the rule

Take two closed addresses. Closed means every digit is a trit: −1, 0 or +1.

Loading is adding them digit by digit, without carrying yet.

$v_{\mathbf{k}} = a_{\mathbf{k}} + b_{\mathbf{k}}$

Since each digit is at most 1 in size, each sum lies between −2 and +2. A position holding −2 or +2 is a mesh under more tension than it can hold. It has become a place where two turns want the same slot.

Now the discharge. For each position:

$c_{\mathbf{k}}$ = the nearest integer to $v_{\mathbf{k}} / 3$ $d_{\mathbf{k}} = v_{\mathbf{k}} - 3c_{\mathbf{k}}$

Work it through. If $v_{\mathbf{k}} = 2$, then $c_{\mathbf{k}} = 1$ and $d_{\mathbf{k}} = -1$. If $v_{\mathbf{k}} = -2$, then $c_{\mathbf{k}} = -1$ and $d_{\mathbf{k}} = 1$. If $|v_{\mathbf{k}}| \leq 1$, then $c_{\mathbf{k}} = 0$ and $d_{\mathbf{k}} = v_{\mathbf{k}}$. Nothing else can occur.

So every overloaded position resolves into two parts. A residue that is a legal trit, and a unit that moves to the next position up — one weight deeper into the net.

Define:

D(v) = the residue, digit by digit **R(v)** = the carries, shifted one position up

The identity is exact:

$\Sigma\, v_{\mathbf{k}} 3^k = \Sigma\, d_{\mathbf{k}} 3^k + \Sigma\, c_{\mathbf{k}} 3^{k+1}$

No value is lost. The sum has been split into what stays at this depth and what goes one depth up.

R is the rewrite rule. Here is why it satisfies all three conditions.

$R \circ R = 0$. Both D(v) and R(v) are closed configurations: every one of their digits is a trit. Apply R to a closed configuration and every c_k is 0, because $|v_k| \leq 1$ gives $c_k = 0$. So R of anything R produces is exactly zero. The rule exhausts itself in one application. This is not imposed. It follows from the digit range of the number system.

Depth is preserved, up to exactly one. Two closed addresses of depth *n* have magnitudes at most $(3^n-1)/2$ each. Their sum is at most $3^n - 1$. Ring $n+1$ holds $(3^{n+1}-1)/2$, which is larger. For $n = 4$: two addresses of at most 40 give at most 80, and ring 5 holds 121. So a discharge can open the next ring, and never more than one.

Coarsening commutes above the dropped position. A carry born at position 0 lands at position 1, which survives coarsening. A carry born higher is untouched. So coarsening and closing agree everywhere except for the carry generated at the position thrown away. The engineering rule is therefore: close first, coarsen after. Never the reverse.

There is one boundary, and it is worth stating precisely. The nilpotency holds when at most **four** closed addresses are loaded at once. With four, the largest digit sum is 4, giving c = 1 and d = 1, both trits. With five, a digit sum of 5 gives c = 2, which is not a trit, and R(v) is no longer closed. Four is the capacity of ring 2. Load in fours, or load pairwise.

III. Three sums, done by hand

43 + 25

43 is (+1, −1, −1, −1, +1) at weights 81, 27, 9, 3, 1. 25 is 27 − 3 + 1, so (0, +1, 0, −1, +1).

Add digit by digit, listing from weight 1 upward:

$v = (2, -2, -1, 0, 1)$

Two overloaded positions. Initial tension $\sigma = (2-1) + (2-1) = \mathbf{2}$.

Round 1. Position 0 holds 2: residue −1, carry +1 up. Position 1 holds −2: residue +1, carry −1 up.

$v^1 = (-1, 2, -2, 0, 1)$

Round 2. Position 1 holds 2: residue −1, carry +1. Position 2 holds −2: residue +1, carry −1.

$v^2 = (-1, -1, 2, -1, 1)$

Round 3. Position 2 holds 2: residue −1, carry +1 into position 3.

$v^3 = (-1, -1, -1, 0, 1)$

Read it off: 81 − 9 − 3 − 1 = **68**. And 43 + 25 = 68.

Three rounds. Watch what moved: the lowest overloaded position was 0, then 1, then 2. It rises every round. That is the termination proof. Carries only travel upward, and the ring is finite, so the process ends in at most *n*+1 rounds.

Call the number of rounds ϱ. Here $\boldsymbol{\varrho = 3}$ while $\boldsymbol{\sigma = 2}$. Small overload, long propagation.

40 + 40

40 is the full ring 4: $(+1, +1, +1, +1) = 27 + 9 + 3 + 1$.

$v = (2, 2, 2, 2)$, so $\sigma = \mathbf{4}$

Round 1. Every position holds 2. Every residue is −1. Every carry is +1, landing on the next position, where it meets a residue of −1 and cancels it to 0. The topmost carry opens a new position.

$v^1 = (-1, 0, 0, 0, 1) = 81 - 1 = \mathbf{80}$

One round. $\boldsymbol{\varrho = 1}$, $\sigma = 4$. Heavy overload, immediate discharge, and a new ring opens.

A saturated ring added to itself moves the whole configuration one depth up in a single step, leaving a single mark behind at the old depth.

1 + 1, and 1 + 1 + 1

At depth 1: $v = (2)$. Residue −1, carry +1. The result is $(+1, -1) = 3 - 1 = 2$.

Two identical windings produce one deeper winding **plus their own opposite** at the original depth. The pair generates its own antithesis. Nothing else could happen; the arithmetic leaves no other option.

Three identical: $v = (3)$. Residue 0, carry +1. The result is $(+1, 0) = 3$.

Three identical windings promote cleanly. Residue zero. Nothing left behind at the old depth.

Two is a controversy. Three is a fact. That is a theorem about a number system, and we will meet it again outside the machine.

IV. Where the same arithmetic already runs

The net model does not treat the human scale as a different kind of thing. It treats it as the same strand at another winding depth. So if the carry is the rewrite rule of the net, discovery should show the same shape. It does. Not as a metaphor for how minds work, and not as a cause. As the same arithmetic, read at the depth where people live.

The residue that does not fit at its own depth

By 1845 the observed motion of Uranus had drifted from the computed motion by about two arcminutes. That is a residue. Small, stubborn, and impossible to absorb at the position where it appeared.

Le Verrier did not adjust the position. He placed the excess one weight further out and computed what would have to sit there. Johann Galle pointed a telescope at the predicted place on 23 September 1846 and found Neptune within about a degree.

That is the carry, exactly. An overloaded position resolves into a residue plus a unit at the next weight up. The unit is not a correction to the old position. It is a new occupied address.

The empty positions kept in the ring

Mendeleev's table of 1869 had gaps, and he refused to close them. He computed what the empty positions required.

For the space below aluminium he gave an atomic weight near 68 and a density near 5.9 grams per cubic centimetre. Gallium was isolated in 1875: atomic weight 69.7, density 5.91. For the space below silicon he gave a weight near 72 and a density near 5.5. Germanium arrived in 1886: weight 72.6, density 5.35.

He was not searching. He was reading unoccupied addresses in a ring whose capacity he trusted more than his inventory. The MAZE does the same thing structurally, which is why 118 elements in a ring of 121 leave three positions rather than a rounding error.

The mirror position

Dirac's equation of 1928 produced solutions with the sign reversed. In 1931 he stopped treating the reversal as an artefact and read it as an address: the same winding, mirrored. Carl Anderson photographed the positron in 1932.

Negation is one of the four native operations of the net. The mirror of an occupied address is a legitimate position. If it is empty, that emptiness is information.

Two, and three

Wegener published continental displacement in 1912. One line of evidence, well closed in itself. It held tension for fifty years and generated its own opposite: an equally organised body of argument that continents cannot move. That is 1 + 1 at the human depth — the pair produced a deeper winding and left its antithesis standing at the old one.

What ended it was the third. Palaeomagnetic reversals, the magnetic striping of the seafloor read by Vine and Matthews in 1963, and the seismicity of the trenches. Three independent routes landing on one address. Residue zero. Within a decade there was no controversy left to have, and very few minds were changed by argument.

Three is not a sociological threshold. It is where the digit sum reaches 3 and the residue is 0.

The saturated ring

By 1900 chemistry had a nearly full ring. Positions occupied, weights measured, valences tabulated. Then the measurements doubled: spectra, ionisation, radioactivity, X-ray frequencies. Ring 4 added to itself does not extend ring 4. It opens ring 5.

The occupied positions did not go away. They acquired a deeper address — the electron configuration — and the old table survived as a coarsening of the new one. Which is what coarsening means: drop the last turn and the previous ring is still exactly right.

The human layer

None of this makes the loading painless.

Poincaré described the pattern without arithmetic in 1908. Weeks of work with no result. Then the interruption of the ordinary. Then the moment on the step of the omnibus at Coutances, when the Fuchsian functions closed, complete, before he had sat down. He noted that the sudden part is never the work. The work is the loading.

That is what a person actually does. They hold a configuration that does not fit. They hold it while it costs them sleep, income, standing, and often their place in a department. The tension is real and it is carried in a body. Mendeleev held an incomplete table. Wegener died on the ice with his question open.

The machine does not take that over, and should not be sold as though it does. What it takes over is the bookkeeping — which position is overloaded, how far the correction has to travel, which addresses are empty and how often something has landed near them. Those are the parts that currently take a career and a large institution, and they are the parts that are pure arithmetic.

V. The engine

The rule turns the archive into a machine because the output re-enters as input. Seven steps.

1. Encode. The utterance becomes a route, the route becomes an address a at depth d.

2. Load. Add a digit by digit to the running state s. Up to four items at a time.

3. Close. Apply R until R returns zero. Guaranteed to terminate in at most $d+1$ rounds.

4. Measure. σ is the initial overload, summed as $\Sigma(|v_k| - 1)$ over overloaded positions. ϱ is the number of rounds. Both are integers. σ says how much did not fit. ϱ says how far the correction had to travel. They are different quantities and both matter.

5. Land. Look at the resulting address. Three outcomes, and only three. *Inhabited*: the occupants are the answer. *Empty*: report the vacancy and record the load against it. *Beyond the ring capacity*: coarsen by one turn and land again — a coarser answer, which is the net's honest form of "roughly this".

6. Read out. The trace is $s' - a$. A subtraction, which is itself a route, which is a sequence of over, straight and under. The justification of the answer is not narrated. It is arithmetic that a reader can redo.

7. Write back. Increase the occupancy of the landed address. If the address was empty, the load stays on it as a counter.

The engine stops when the address stops moving. A fixed point. In the vocabulary of the model: what comes back in phase stays. The number of iterations to that fixed point is the effort the question cost.

Three consequences follow directly from the arithmetic, without any additional design.

It can return nothing. A machine that predicts the next token always has a next token. This engine can land on an empty address, and an empty address is a legitimate terminal state. Not a failure mode bolted on afterwards — a position in the geometry.

Its reasoning is a subtraction. There is no interpretability problem to solve, because there is nothing hidden to interpret. The route from question to answer is a difference of two integers.

Its vacancies accumulate evidence. An empty address that keeps being approached carries a counter. That counter is a prediction with a number attached, generated by the structure rather than proposed by a person.

VI. How science gets calculated

Now the forward view. Not what the machine could become in principle, but what changes in practice once the rule runs.

The prediction ledger writes itself. Today a research programme's open questions are chosen. Someone decides what counts as a gap worth funding. In a net with computed addresses, the gaps are the unoccupied positions, and their load counters are computed from what has actually landed near them. The list of open questions in a field becomes an output rather than an opinion. Mendeleev did this once, by hand, for one ring.

Archives merge without an authority. Two laboratories, two continents, two vocabularies. If both compute addresses from the material rather than assigning them, their nets add. There is no committee to harmonise ontologies and no controlled vocabulary to negotiate. This is the practical difference between a computed address and an assigned one, and it is the reason the whole scheme was built this way.

Disagreement becomes a number. Two results that will not close leave a permanent residue at a known position, with a known σ and ϱ. "These two findings conflict" stops being a matter of tone and becomes a coordinate. And the arithmetic already tells you what to expect: two will generate their own opposite, and only a third independent landing gives residue zero. A field can see, before it starts arguing, whether it has a two-problem or a three-answer.

Coarse answers replace confident wrong ones. When the closure lands beyond the ring, the engine coarsens and answers at lower resolution. That is a working machine saying "at this level, this much". It is worth more than a fluent paragraph at full resolution built on nothing.

Priority and provenance are trivial. The route is the record. Who landed where, at what depth, when.

Who can do it changes. This matters more than the rest. The bookkeeping of a discipline is currently held by the discipline, and access to it runs through appointments, journals, and review. Arithmetic on public material has none of those doors. A person outside every institution can compute an address, land it, and read the vacancy. Whether they are believed is another matter, and always was. But the calculation itself stops being a privilege.

What does not change: someone still has to hold the tension. The engine can tell you that position 7 is overloaded by 4 and that the correction travels three rings. It cannot want to know. It cannot spend eleven years on a table that will not close. That part stays with people made of flesh and blood, and it is the part that has always cost them.

VII. Status ledger

Proved, checkable by hand. The bijection between routes and integers. The carry decomposition $v = D(v) + R(v)$. The nilpotency $R \circ R = 0$ for loads of up to four closed addresses. Termination in at most $n+1$ rounds. Depth growth of at most one ring. Commutation with coarsening above the dropped position.

Built. The addressing kernel, with tests that recompute the published numbers exactly, using arbitrary-precision integers so the bijection holds at any depth. The inhabited net. The live prototype.

To be measured next. Exhaustive verification of $R \circ R = 0$ over all pairs of closed addresses to depth 5, which is 59,049 pairs — small enough to run completely rather than sample. Verification that closing preserves the integer sum. The landing rate: when addresses that belong together are loaded, how often does the closure land on an inhabited address rather than an empty one? That single percentage is the first real measurement of the engine, and it is the number this essay is written to make worth taking.

Open. Whether the encoder is stable enough that two independent encodings of the same material land on the same address. That is where the arithmetic meets the world, and no theorem covers it.

Annotated references

Mendeleev, D. (1869/1871). On the relation of the properties to the atomic weights of the elements. *Why read?* The original act of treating an empty position as information. The 1871 paper contains the numerical predictions for eka-aluminium and eka-silicon. *Reading tip:* Read only the predictions and then the 1875 gallium and 1886 germanium measurements. The agreement is the argument.

Le Verrier, U. (1846). Recherches sur les mouvements d'Uranus. *Why read?* A residue of two arcminutes resolved by placing a new occupied position one weight further out. The clearest historical instance of a carry.

Dirac, P. A. M. (1931). Quantised singularities in the electromagnetic field. ***Proc. R. Soc. A*** **133.** *Why read?* The paper where a sign reversal stops being an artefact and becomes an address. Negation as a structural operation, three years before anyone saw a positron.

Vine, F. & Matthews, D. (1963). Magnetic anomalies over oceanic ridges. ***Nature*** **199.** *Why read?* The third independent landing that ended a fifty-year controversy. Read it against Wegener to see the difference between two and three.

Poincaré, H. (1908). ***Science et Méthode*****, book I, chapter 3.** *Why read?* The most exact first-hand account of loading and discharge ever written by a working mathematician. He insists the sudden part is not the work.

Hadamard, J. (1945). ***The Psychology of Invention in the Mathematical Field.*** *Why read?* Poincaré's account tested against many others. The four-phase pattern recurs across people who had no contact with each other.

Knuth, D. (1997). ***The Art of Computer Programming*****, vol. 2, §4.1.** *Why read?* The technical treatment of balanced ternary, including the carry behaviour used here. Knuth calls it perhaps the prettiest number system, and the reason is the symmetry that makes negation free.

Hayes, B. (2001). Third Base. *American Scientist* 89(6). *Why read?* The most readable short account of balanced ternary and of the Setun machine. *Reading tip:* Start here if the arithmetic in Part II is unfamiliar.

Brusentsov, N. (1958). The Setun computer. *Why read?* Balanced ternary has been built in hardware and it worked. Fifty machines, in service for years. The number system is not a curiosity.

Leibniz, G. W. (1686). *Discours de métaphysique*, §§5–6. *Why read?* The requirement that maximum richness follow from minimum assumption. This is the criterion the whole construction is held to.

Chaitin, G. (2005). *Meta Math!*. *Why read?* Leibniz's criterion made quantitative as program length. Useful for judging whether a rule is genuinely simple or merely short to state.

Gödel, K. (1931). On formally undecidable propositions. *Why read?* The ancestor of computed addressing: statements receive numbers derived from their own structure. Everything here is a descendant of that move.

Rowlands, P. (2007). *Zero to Infinity*. *Why read?* A physics built on a rewrite system whose operator squares to zero. Independent arrival at the nilpotency condition, from a different direction. *Reading tip:* The rewrite chapters. The rest is a longer commitment.

Kuhn, T. (1962). *The Structure of Scientific Revolutions*. *Why read?* The description of a field filling its ring and then opening the next one. Read the account of saturation and ignore the vocabulary of paradigms, which has since been worn smooth by use.