Edges, Choice and Countable Cycles

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Abstract

A stream of three-valued digits can be read in two ways. In base three it names a cell inside a cell: an address. In base two the same stream computes. We report machine-checked results about the border between the two readings. First, two base-three names of one place differ only in a fixed edge form, and never by more than one cell. Second, every function that turns a base-two place into a base-three address proves the principle LLPO; Since LLPO is not provable constructively, no constructive computation gives this step. At the edge a decision is needed. Third, if steps are treated as paths, periodic streams carry loops that can be counted, and every edge ends in a cycle of period one. All results are checked in cubical Agda without postulates. A final section, marked as interpretation, relates the results to cycles and decisions outside mathematics.

1. The ladder in brief

The Vacuum.Net theory starts from movement. One step is a direction: forward, standing still, or back. We write such a step as a trit: R (+1), O (0) or L (−1). An infinite row of trits is a stream.

Read in base three, a stream $s = d_1d_2d_3\ldots$ has the value $\Sigma\, d_i \cdot 3^{-i}$, which lies between −1/2 and 1/2. This is balanced ternary. Each digit selects one of three sub-cells, so the stream is an address. Read in base two, the value is $\Sigma\, d_i \cdot 2^{-i}$, between −1 and 1. This is signed binary. Its cells overlap, which makes it redundant; that redundancy is what lets it compute.

We write $V_n(s)$ for the integer value of the first n digits, most significant first. Two streams are the same place when their values agree. In integer form: $|V_n(s) - V_n(t)|$ stays bounded for all n. A place with two different base-three names is an edge. The simplest example is 1/6 = 0.RLLL… = 0.ORRR….

All results below are proved in cubical Agda, a proof assistant based on homotopy type theory.[1] The work is collected in a book and an Agda package.[2]

2. What is new

- **Where edges are.** The two names of a place have exactly one possible form, and they never drift more than one cell apart.
- **Why the edge needs a decision.** Going back from computing to an address proves LLPO, a principle that constructive mathematics cannot prove.
- **Cycles can be counted.** With steps as paths, periodic streams carry loops with a definite number of turns, and edges end in such a loop.

3. Results

3.1 The edge theorem

Let s and t be two streams that agree before place k and differ at place k. Write $D(n) = V_n(s) - V_n(t)$.

> **Theorem 1.** s and t are the same place if and only if, after possibly swapping them, the digits at place k are (R, O) or (O, L), and after place k the upper stream is L L L … and the lower stream is R R R ….[3]

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[1] A. Vezzosi, A. Mörtberg and A. Abel, “Cubical Agda: a dependently typed programming language with univalence and higher inductive types”, Journal of Functional Programming 31 (2021) e8; earlier version ICFP 2019.

[2] J. Konstapel, Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0, reference edition, 19 November 2026; Agda package vacuumnet-mathematics-v7.0.

[3] Agda: twee-namen en rand-zelfde, file VacuumNetEdge.

Proof sketch. From the digit recursion, $D(n+1) = 3 \cdot D(n) + e$, where e is the difference of two digits, so $|e| \leq 2$. If $D(n) \geq 2$, then $D(n+1) \geq D(n) + 2$. The difference then grows without bound, so the streams are different places. Hence $|D|$ stays at 1 after place k. The value $D = 1$ survives a step only when $e = -2$, that is, when the upper digit is L and the lower digit is R. □

> **Theorem 2.** s and t are the same place if and only if $|V_n(s) - V_n(t)| \leq 1$ for every n. A difference of 2 or more at a single depth proves that the places differ.[4]

Two remarks, argued on paper and not in Agda. A place that is not an edge has exactly one name. The converse of the last sentence of Theorem 2, that every difference of place shows at some finite depth, is an instance of Markov's principle, which is not provable constructively. Markov's principle says: if a decidable property cannot fail everywhere, then a place where it holds exists.

3.2 The choice theorem

A way back is a function g on streams that gives, for every stream read in base two as a place x, a base-three address of x/3. With the factor 1/3, the critical input point is $x = 1/2$, a dyadic number. Its image 1/6 is an edge. This keeps the test streams finite.

> **Theorem 3.** Every way back g proves LLPO. Continuity of g is not assumed.[5]

LLPO, the lesser limited principle of omniscience, says: for a sequence of 0s and 1s with at most one 1, the 1 is not at an even place, or it is not at an odd place.

Proof sketch. For such a sequence α, build the stream that starts with R, then O everywhere, except an L or an R at the place of the 1. If the 1 is at an odd place, the place lies just above the edge and the address must start with R. If it is at an even place, the address may not start with R. Reading the first digit of g on this stream therefore decides which half of α contains no 1. □

Known theorem: LLPO is classically true but not provable in Bishop's constructive mathematics.[6] Weak countable choice is a mild form of the axiom of choice for countable families. With it, every Cauchy real has an expansion in a base exactly when LLPO holds.[7] Theorem 3 is a formal, machine-checked instance of this phenomenon on our ladder. Its consequence is a meta-statement: a way back cannot be constructed without an extra principle.

3.3 Countable cycles

The book glues the two ends of the interval into a ring called Kring. Kring is a set in the sense of homotopy type theory: any two paths between the same two points are equal. A set cannot carry a turn:

> **Theorem 4.** Every map from the circle S^1 to Kring is constant.[8]

To count turns, we let a step be a path instead of a function. The type Wind has a point for every stream s and a path from s to its tail. This is the homotopy coequalizer of the identity and the tail map. A map tel from Wind to S^1 sends every point to the base and every step to one turn; a step back counts -1.[9]

> **Theorem 5.** If a stream returns to itself after k steps, the k steps close into a loop. Under tel this loop winds exactly k times. For $k \geq 1$ the loop is not trivial. Going round m times winds $m \cdot k$ times.[10]

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[4] Agda: zelfde-scherp, hoogstens1-zelfde and verschil-zichtbaar, file VacuumNetTwo.

[5] Agda: kiezen-LLPO, file VacuumNetChoice. The earlier result that no continuous way back exists is geen-terugweg, file VacuumNetBack.

[6] E. Bishop and D. Bridges, Constructive Analysis, Springer, 1985; D. Bridges and F. Richman, Varieties of Constructive Mathematics, Cambridge University Press, 1987.

[7] nLab, "lesser limited principle of omniscience", consulted 19 November 2026.

[8] Agda: kring-vergeet, file VacuumNetWinding.

[9] J. Konstapel, Upward in the Windings, plan and first result, version 0.2, 20 November 2026; Agda file VacuumNetWinding.agda, file version 1.0.

[10] Agda: periode-winding, lus-niet-triviaal and herhaal-winding, file VacuumNetWinding.

Theorem 6. Each name of an edge reaches, in finitely many steps, a constant stream L L L … or R R R …. A constant stream is its own tail, so it carries a loop that winds exactly once under tel.[11]

Not yet proved: that all loops at a periodic point of Wind are exactly the integers. Argued on paper and checked by an independent reviewer (an AI agent that did not write the file): every connected part of Wind contains at most one cycle, because every stream has exactly one tail. A part without an eventually periodic stream is an infinite tree. Every stream in it has three predecessors. Such a tree is contractible and holds no loop. A part with a periodic stream is homotopy equivalent to the circle S^1.

4. Place in the literature

Brouwer asked in 1921 whether every real number has a decimal expansion.[12] Turing met the same problem and in 1937 proposed another way of writing computable numbers.[13] Computable analysis explains why redundant digit systems are needed for computing.[14] Constructive reverse mathematics classifies such statements by omniscience principles such as LLPO.[15] Our contribution is a machine-checked account on one ladder: the address and the computation share one alphabet of three directions, and the border between them is located exactly.

For the circle and higher windings we build on homotopy type theory. Licata and Shulman proved by machine that the loops around the circle are the integers.[16] Ljungström and Mörtberg computed the Brunerie number in cubical Agda and so proved $\pi_4(S^3) \cong \mathbb{Z}/2$.[17] The next rungs of this programme lead towards those results.

5. Beyond mathematics (interpretation)

This section is a reading. It is not proved.

The results suggest a simple picture. Computing reaches an edge but cannot cross it. At the edge the two descriptions of one situation stay within one step of each other, and further detail repeats the same split. Crossing needs a decision from outside the computation. Turns arise exactly where a pattern returns, and the simplest edges end in a cycle.

This picture underlies a practical project, the Guide of Humanity, which helps people and organisations find their cycles, the edges in those cycles, and the decision at each edge. It connects with the author's earlier work on windings in financial markets, which never fully close,[18] and on Paths of Change, in which every successful change is a closed path.[19]

One tension is open. Theorem 3 says that crossing needs a decision. Elsewhere the author argues that civilisations pass through phase transitions rather than choices.[20] Both can hold if the decision comes from outside the computation, either from a person or from a collective that tips. Which of the two acts at a given edge is an empirical question.

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[11] Agda: rand-naar-lus and const-lus-winding, file VacuumNetWinding.

[12] L. E. J. Brouwer, "Besitzt jede reelle Zahl eine Dezimalbruchentwicklung?", Mathematische Annalen 83 (1921), 201–210.

[13] A. M. Turing, "On computable numbers, with an application to the Entscheidungsproblem. A correction", Proceedings of the London Mathematical Society (2) 43 (1937), 544–546.

[14] K. Weihrauch, Computable Analysis, Springer, 2000.

[15] H. Diener, Constructive Reverse Mathematics, habilitation thesis, arXiv:1804.05495 (2018).

[16] D. R. Licata and M. Shulman, "Calculating the Fundamental Group of the Circle in Homotopy Type Theory", LICS 2013, 223–232.

[17] A. Ljungström and A. Mörtberg, "Formalizing $\pi_4(S^3) \cong \mathbb{Z}/2\mathbb{Z}$ and Computing a Brunerie Number in Cubical Agda", LICS 2023 (arXiv:2302.00151).

[18] J. Konstapel, "Understanding Financial Markets through Vacuum.Net Theory", constable.blog, 5 September 2026.

[19] J. Konstapel, "About Paths of Change (PoC)", constable.blog, 10 September 2020.

[20] J. Konstapel, "Waarom 2027–2032 cruciaal is voor onze toekomst", constable.blog, 15 May 2026.

6. Open questions

- Are all loops at a periodic point of Wind exactly the integers?
- Does LLPO, conversely, give a way back on streams?
- Can “one name off the edges” be proved for places rather than streams?
- Does a ladder of pairs of trits close into the sphere S^2, and what are its higher turns?
- How reliably can the edge form be recognised in human conversation?

7. How to check

All results are in the Agda files named in the footnotes. They check with Agda 2.8.0 and the cubical library v0.9, under the option --safe and without postulates. Each new file was reviewed by an independent reviewer, an AI agent that did not write it; its corrections are recorded in the package.

Annotated references

J. Konstapel, Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0, reference edition, 19 November 2026; Agda package vacuumnet-mathematics-v7.0. *The closed reference edition. Theorems 1–3 and the ladder.*

J. Konstapel, Upward in the Windings, plan and first result, version 0.2, 20 November 2026; Agda file VacuumNetWinding.agda, file version 1.0. *Theorems 4–6 and the programme for higher windings.*

A. Vezzosi, A. Mörtberg and A. Abel, “Cubical Agda: a dependently typed programming language with univalence and higher inductive types”, Journal of Functional Programming 31 (2021) e8; earlier version ICFP 2019. *The proof assistant used.*

L. E. J. Brouwer, “Besitzt jede reelle Zahl eine Dezimalbruchentwicklung?”, Mathematische Annalen 83 (1921), 201–210. *The question whether every real has a decimal expansion.*

A. M. Turing, “On computable numbers, with an application to the Entscheidungsproblem. A correction”, Proceedings of the London Mathematical Society (2) 43 (1937), 544–546. *The correction that changed how computable numbers are written.*

E. Bishop and D. Bridges, Constructive Analysis, Springer, 1985; D. Bridges and F. Richman, Varieties of Constructive Mathematics, Cambridge University Press, 1987. *Standard references for constructive analysis and the status of LLPO.*

nLab, “lesser limited principle of omniscience”, consulted 19 November 2026. *States the equivalence of expansions and LLPO under weak countable choice. A wiki page, not peer reviewed.*

H. Diener, Constructive Reverse Mathematics, habilitation thesis, arXiv:1804.05495 (2018). *Survey of constructive reverse mathematics, including LLPO and its equivalents.*

K. Weihrauch, Computable Analysis, Springer, 2000. *Standard text on computable analysis and digit representations.*

D. R. Licata and M. Shulman, “Calculating the Fundamental Group of the Circle in Homotopy Type Theory”, LICS 2013, 223–232. *First machine proof that the loops around the circle are the integers.*

A. Ljungström and A. Mörtberg, “Formalizing $\pi_4(S^3) \cong \mathbb{Z}/2\mathbb{Z}$ and Computing a Brunerie Number in Cubical Agda”, LICS 2023 (arXiv:2302.00151). *Machine proof of $\pi_4(S^3) \cong \mathbb{Z}/2$ by computation.*

J. Konstapel, “Understanding Financial Markets through Vacuum.Net Theory”, constable.blog, 5 September 2026. *The author’s windings in finance: address, tension, loading, discharge.*

J. Konstapel, “About Paths of Change (PoC)”, constable.blog, 10 September 2020. *Paths of Change as closed paths of change.*

J. Konstapel, “Waarom 2027–2032 cruciaal is voor onze toekomst”, constable.blog, 15 May 2026. *The author’s argument for phase transitions rather than choices.*