

Round Trips: Periods, Complex Multiplication and the Vacuum.Net

An open letter to Marco Streng (Mathematical Institute, Leiden) on ReSCue, closure and the ternary ground

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Why this is addressed to you

Your new NWO project asks whether a curve can be recovered from its period matrix — from the numbers recorded by going once around each of its closed loops. That is the question the Vacuum.Net asks of everything, and this essay shows where the two meet.

A *streng* is a strand. For a framework that reads the world as closed strands, the name is a happy coincidence; the overlap in substance is not. Your work computes the invariants of closed curves with extra rotational symmetry, the step at which points on them return, and the primes at which their loops pinch. Those are exactly the operations a theory of closure needs.

The essay does three things. It reads your seventeen listed publications as one map of closure. It adds one computed result that moves the Vacuum.Net a step forward: its ternary unit, the trit, turns out to be the natural residue in the Eisenstein integers, the lattice behind your Picard curves. And it proposes ReSCue as a bridge, in both directions, with three concrete questions at the end.

The Vacuum.Net in three sentences

The Vacuum.Net describes everything that persists as a closed curve: a path that returns to where it started. Points are places in a position space; curves run through them; where curves meet, there is a connection point. A thing is a turn that keeps coming back.

Two consequences matter for what follows. First, the address of a closed path has no starting point: it is a rotation class, so the number of distinct addresses of length n is the necklace count $g(n) = (1/n) \cdot \sum \phi(n/d) \cdot 3^d$, not 3^n . Second, the unit of the address is the trit, a digit from $\{-1, 0, +1\}$, read off balanced ternary rather than assigned by hand.

The longer statement is in "Curves in the Universe" (constable.blog, 24 October 2026); the rotation-class address is worked out in "The Positron and the Cogwheel" (26 October 2026). Nothing below depends on more than these two consequences.

Your work as a map of closure

Read in the language of the net, your publications cover five operations on closed paths, and together they form one coherent programme.

Operation on a closed path	Your papers	Reading in the net
Giving a closed path a basis-free address	Computing Igusa class polynomials (2014); Generalized class polynomials (2022, with Houben); Schertz style class invariants for higher degree CM fields (2026, with Enge)	The j-invariant and its higher-genus analogues name a torus independently of the chosen basis — an address without a starting point
Counting paths that share one rotational symmetry	Polarised class groups of quartic CM orders (2017, with Bisson); The CM class number one problem for genus 2 (2023, with Kiliçer)	Complex multiplication is a turn that maps the lattice into itself; the class number counts how many paths share that turn; class number one is a unique closure
Finding the step at which a point returns	Divisibility sequences for CM elliptic curves (2008); Algebraic divisibility sequences over function fields (2012); Primitive divisors with constant j-invariant (2020, with Naskręcki); Modular units for $\Gamma_1(N)$ (2023)	A primitive divisor of the n-th term is a prime modulo which P first closes at step n; $\Gamma_1(N)$ is the structure of points that close after N steps
Finding where a loop pinches	Primes dividing invariants of CM Picard curves (2019); A bound on bad-reduction primes for genus-3 CM curves (2020); Plane quartics over $\mathbb{Q}$ with CM (2018)	Bad reduction is a connection point: modulo that prime two strands of the path fall together
Using closure as a working tool	CM construction with p-rank 1 (2011); Prescribed embedding degree (2008); Curves in characteristic 2 with 2-torsion (2014); (l,l) -endomorphisms (2013); CM curves over the reflex field (2015)	Closure arithmetic already runs as practical machinery; the 2008, 2011 and 2014 papers serve curve-based cryptography

The fourth row contains the family that matters most here: the Picard curves $y^3 = f(x)$, with $\deg f = 4$. They carry a built-in threefold turn, multiplication by a cube root of unity ω . That is where the net's ternary ground and your work share a lattice.

The step forward: the trit lives in $\mathbb{Z}[\omega]$

In the Eisenstein integers the trit is not a convention but the residue modulo the prime above 3, and its digit dynamics split every number into two kinds: those that come to rest and those that end in a perpetual 60° turn.

The residue. Let ω be a primitive cube root of unity and $\beta = 1 - \omega$, of norm 3. Then $\mathbb{Z}[\omega]/(\beta)$ is the field with three elements, and since $\omega \equiv 1 \pmod{\beta}$, the class of $a + b\omega$ is $a + b \pmod{3}$. Its natural representatives are $-1, 0, +1$: exactly the balanced-ternary digits.

The digit map. Write $d(z)$ for that balanced residue and set $T(z) = (z - d(z))/\beta$. Repeating T gives an expansion

$$z = d_0 + d_1\beta + d_2\beta^2 + d_3\beta^3 + \dots, \quad d_k \in \{-1, 0, +1\}$$

What the computation shows. I ran T on every $a + b\omega$ with $|a|, |b| \leq 100$. Every orbit ends in one of exactly three places: at 0, or at one of two fixed points $\pm(1 + \omega)$. The share that ends at a fixed point rather than at 0 grows with the box: 27% for $|a|, |b| \leq 40$ (1,788 of 6,561) and 41% for $|a|, |b| \leq 100$ (16,546 of 40,401). Whether it settles at a limit is open.

Why this holds for all of $Z[\omega]$. The norm drops at every step: $N(T(z)) = N(z - d)/3 \leq (\sqrt{N(z)} + 1)^2/3$, which is smaller than $N(z)$ whenever $N(z) \geq 2$. So every orbit falls into the elements of norm 0 or 1, the zero and the six units. Checking those seven by hand: 0, ± 1 and $\pm\omega$ run to 0, while $\pm(1 + \omega)$ map to themselves.

What the fixed point is. $1 + \omega = -\omega^2 = e^{i\pi/3}$, a sixth root of unity: a 60° turn. Its expansion is the digit -1 repeated forever, because $-\omega^2 = -1 + \beta \cdot (-\omega^2)$. The ordinary number 2 already shows the split: $2 \rightarrow 2 + \omega \rightarrow 1 + \omega$, and from there the turn never stops. The number 1, by contrast, comes to rest in one step.

The reading in the net. The ternary ground, placed in your lattice, reproduces the net's basic dichotomy without being told to. A finite trit word comes to rest at 0. An eventually periodic word does not rest: it ends in a unit that keeps turning, a turn that keeps coming back. Taking $-\beta = \omega - 1$ as base instead merges the two fixed points into one cycle of length 2, the same turn seen from the other side.

This is the step forward for the Vacuum.Net: its unit of address and its two states of rest and return follow from the arithmetic of $Z[\omega]$ itself.

ReSCue as the bridge

ReSCue asks how to recover a superelliptic curve $y^n = f(x)$ from its period matrix ([Leiden University, 26 October 2026](#)). Read in the net, that is the reverse journey: from the record of round trips back to the path that made them.

From the net to ReSCue. For $n = 3$ the curve is a Picard curve, and $(x, y) \mapsto (x, \omega y)$ is an automorphism of order 3. It acts on the loops and on the differentials, so the period lattice is a module over $Z[\omega]$. Because $Z[\omega]$ is a principal ideal domain, that lattice has a basis over $Z[\omega]$, and the digit map of the previous section applies to the coordinates of every period in that basis: each coordinate has a balanced-trit expansion in base $1 - \omega$, finite or ending in the 60° turn. Whether that address helps reconstruction is the first question below.

From ReSCue to the net. The Vacuum.Net needs exactly the procedure ReSCue is building: given only measured closures, rebuild the curve. A text, a dataset or a physical measurement gives numbers, not a path. If periods determine the curve, then closures determine the path, and the net's position space can be reached from data rather than postulated.

The physics side. The three papers below were checked against their arXiv records on 27 October 2026. In Seiberg–Witten theory (1994) a four-dimensional field theory is encoded by a curve, and its periods are the physical quantities: the masses of stable particles and the couplings (the abstract states the exact mass formulas; their derivation from the periods is in the body of the paper). Gregory Moore ("Arithmetic and Attractors", 1998) showed that for $N=4$ and $N=8$ string compactifications the attractor varieties of supersymmetric black holes are built from elliptic curves with complex multiplication, and that for $N=4$ the number of inequivalent backgrounds with the same horizon area is the class number of an imaginary quadratic field — the quantity your class-number papers compute. Gukov and Vafa (2002) found that the rational conformal field theories with a two-torus target are those whose torus has complex multiplication. In each case the physics sits on closed curves through their periods or their complex multiplication — the two things ReSCue and your earlier papers work with.

So ReSCue already stands on a bridge to physics; it has not been named as such. The net offers a third bank: a ternary address for the same periods, and a reading in which rest and return are the only two states.

Three questions to Marco Streng

Each question can be checked in an afternoon with Sage or PARI, and each answer, either way, tells the net something.

1. **A trit address for periods.** For Picard curves, does the balanced expansion in base $1 - \omega$ give a useful coordinate on the period lattice — for instance in lattice reduction, or in deciding to what precision periods must be known before reconstruction succeeds? And which period vectors end in the 60° turn rather than at rest?
2. **First closures and necklace counts.** In the divisibility sequences of a CM curve, the index at which a prime first appears is the step at which a point first closes. Do these indices, gathered over many primes, follow a distribution that the necklace count $g(n) = (1/n) \cdot \sum \phi(n/d) \cdot 3^d$ can be compared with — especially for curves with CM by $\mathbb{Z}[\omega]$?
3. **Reconstruction as a general procedure.** If ReSCue yields an algorithm from period matrix to curve, could the same algorithm take any set of measured closures — not only periods of an algebraic curve — and return the simplest closed path that produces them?

The Vacuum.Net offers in return a reading in which your results are not only arithmetic but structural statements about rest, return and connection. I would welcome a conversation in Leiden.

References

- [T.C. Streng, publication list, Universiteit Leiden](#) — all seventeen papers cited in the table above.
- [Leidse onderzoeksprojecten ontvangen Open Competitie financiering, Universiteit Leiden, 26 October 2026](#) — ReSCue.
- J. Konstapel, "[Curves in the Universe](#)" (PDF title "Curves in the Vacuum.Net", v1.0), constable.blog, 24 October 2026, updated 26 October 2026 — points in position space, curves, knots and seams; trits and balanced ternary.
- J. Konstapel, "[The Positron and the Cogwheel](#)", constable.blog, 26 October 2026 — the address as a rotation class (necklace), worked example $g(3) = 11$.
- N. Seiberg and E. Witten, "Electric-magnetic duality, monopole condensation, and confinement in $N=2$ supersymmetric Yang–Mills theory", Nuclear Physics B 426 (1994) 19–52, [arXiv:hep-th/9407087](#).
- G. Moore, "Arithmetic and Attractors", [arXiv:hep-th/9807087](#) (1998).
- S. Gukov and C. Vafa, "Rational Conformal Field Theories and Complex Multiplication", [arXiv:hep-th/0203213](#) (2002).
- Computation of section 4: Python script over all $a + bw$ with $|a|, |b| \leq 100$, run 27 October 2026; the norm argument makes the two end states general.