The Foundations of Mathematics Based on the Vacuum.Net Theory

One loop, three directions, and a programme to give every named concept its trit word back

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Abstract

This essay closes a day of research with one question: can the Vacuum.Net theory be the foundation of all of mathematics? The strict reading is taken. Mathematics is not translated into the theory. It is grounded in it. The essay does two things. First, it defines the foundation in homotopy type theory and checks it in cubical Agda. The ground is one closed loop. Its return is a real distinction. Its three directions are the trinity. The words of balanced ternary are exactly the windings of the loop. The closures of the theory are paths: a meeting cancels, a ring is one winding a scale up, and a step carries. Mirroring R and L reverses the winding. Second, the essay sets out how the trits are to be re-engineered. The view taken is that mathematics is a collection of unique trit words to which people gave names. A method is given that turns a named concept into its trit word. A pilot on ten concepts gives ten different words. In it the elementary closures of the theory correspond to the simplest objects of topology. The limits are stated plainly.

Keywords: foundations of mathematics; homotopy type theory; univalence; cubical Agda; balanced ternary; Vacuum.Net

1. The question

The Vacuum.Net theory states that there is one strand and that every distinction is a loop [1]. Its numbers are the words of balanced ternary. Its selection rule is closure. The question of this essay is whether all of mathematics rests on that ground.

Two readings are possible. In the weak reading, mathematics can be written in the language of the theory. That is always true: any formal theory can be coded in any alphabet of two or more signs. In the strong reading, mathematics follows from the theory. This essay takes the strong reading.

The research sharpened the question in one step. Mathematics is read here as a collection of unique trit words, each of which received a name in history: circle, group, prime, trefoil. History handed out these names in no particular order. A study of the theory against the history of mathematics found no significant relation between the order of the theory's chain and the dates of the results it uses: over 41 results the rank correlation is 0.16 ($p = 0.31$) [2]. The task is therefore to give every named concept its trit word back, and to tie the words to the theory again. Homotopy type theory is chosen as the tool.

2. What the theory supplies

- **One strand.** There is one strand. A distinction is the strand returning to a place it has been (N1).
- **Direction.** At a return the strand leaves as it left before, goes back the way it came, or does neither. These are the three directions R, L and O.
- **Closure.** What closes persists; what does not close passes on as tension (N2). A meeting R L closes. Three equal directions close and pass one step to the next scale: the carry.
- **Address.** Every integer has exactly one balanced-ternary word (T12). Mirroring R and L gives the negative (T12.1).

The universe has no language. Physics gave the windings names, and so did mathematics. In the net a name is not chosen. It is the address the strand leaves behind.

3. Why homotopy type theory

The current foundation of mathematics is set theory, ZFC. There numbers are built from the empty set: 0 is ∅, and n + 1 is n ∪ {n}. The first distinction is static. A numeral system is only notation.

Homotopy type theory starts elsewhere [4]. An equality is a path. A type can be defined by points and paths that return. The circle is one point and one loop. Two results make it fit the theory. First, the windings of that loop are the integers: Licata and Shulman proved that the fundamental group of the circle is $\mathbb{Z}$ inside the theory itself [5]. Second, univalence says that equivalent types are equal. Two concepts with the same structure are then one concept with two names.

The history of mathematics itself ends here. Mathematics as a Chain of Proven Steps calls univalent foundations the current frontier, an independent line with no arrow back to set theory [3]. That frontier is the starting point of this essay.

The tool is cubical Agda, a proof assistant in which univalence and higher inductive types compute [7, 8]. The cubical library supplies the circle and the proof that its loops are the integers [18].

4. The foundation

The foundation consists of six statements. Each is a definition or a theorem in two Agda files, VacuumNetHoTT.agda and VacuumNetFoundation.agda [19]. Both type-check in Agda 2.8.0 with the cubical library v0.9 under the flag --safe, without postulates or holes.

F1. The strand is one closed loop

The strand is the circle: one place, base, and one return, loop. Nothing else is declared.

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Streng = S¹      terugkeer : base ≡ base
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F2. The return is a distinction

A loop that can be shrunk to standing still makes no difference. The loop of the strand cannot. If loop were equal to refl, its winding number 1 would equal 0.

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onderscheid : ¬ (loop ≡ refl)
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F3. The trinity is direction

R is the loop forward, L the loop backward, O standing still. The three values are not assumed as numbers. They are the three ways to move at a return.

Foundations III.1 says the trits are digits that add, not directions that compose. Here both readings meet. A trit is a direction of motion along the loop, and composing such paths is adding digits, with the carry (F4, F5).

```
ℓ R = loop      ℓ L = sym loop      ℓ O = refl
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F4. Addresses are windings

A word is a sequence of digits L, O, R, lowest scale first. It is a higher inductive type with one path constructor, plus set-truncation: an O on top, with nothing above it, falls away (standing closes). Each word is read as a winding, digit by digit: a digit one scale up counts three times. The theorem is that these words are exactly the loops of the circle. By univalence the two types are equal.

```
ρ (d ▷ w) = ℓ d · (ρ w · ρ w · ρ w)

BT ≃ ΩS¹      BT ≡ ΩS¹
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Agda computes the words. Thirteen is R R R, three is O R, minus thirteen is L L L.

F5. Closures are paths

The three closures of the theory become equalities between paths.

- **Meeting.** Forward then back is no winding: ℓ R · ℓ L ≡ refl.
- **Ring.** A digit one scale up is three windings below: ρ (O ▷ w) ≡ ρ w · ρ w · ρ w.
- **Step with carry.** One step more, with carry, is one winding more: ρ (sucBT w) ≡ ρ w · loop. Where R meets R, the step writes L and carries R upward: 1 + 1 = 1T, the open pair of T7.1.

F6. Mirroring is reversal

Exchanging R and L in a word reverses its winding. This is the mirror charge T12.1 of the theory, now as a path.

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spiegeling : ρ (mirror w) ≡ sym (ρ w)
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Statement	Agda name	Status
F1 strand as loop	Streng, terugkeer	definition (library S^1)
F2 distinction	onderscheid	proved here
F3 trinity as direction	ℓ, vooruit, terug, stil	definition
F4 words are windings	BT≃ℤ, ρ≡intLoop, BT≃ΩS^1, BT≡ΩS^1	proved here; uses library ΩS^1 ≃ ℤ
F5 meeting	meeting	library law (rCancel)
F5 ring	ring	by definition of ρ and the unit law
F5 step with carry	openPair	proved here
F6 mirroring	val-mirror, intLoop-neg, spiegeling	proved here

Table 1. The foundation and its status in cubical Agda.

5. Truth, openness and depth

This section is a reading, not a formal result. It records how the foundation answers three old questions.

Truth. A statement is true when it closes in one direction, false when it closes in the other, and open when it does not close. Truth is not agreement with something outside. There is no outside.

Gödel. A sentence that speaks about itself is a loop. Gödel and Turing showed that some such loops do not close [13]. In a logic with two values that is a gap. With an open value it is tension, not contradiction. The language is complete; the net stays open.

Depth. Every reading happens at a finite depth. Cutting a balanced-ternary expansion is rounding to within half a unit of the last depth kept [1, 16]. The available depth is large beyond any need. The foundation needs no completed infinity.

6. Re-engineering the trits

6.1 The principle

Every named concept of mathematics is a unique trit word. The work is to find that word for each name, and to prove in homotopy type theory that the word belongs to the concept. Univalence then joins names that history gave to one word, and keeps different words apart.

6.2 The method

1. Write the concept as windings: its independent loops and the closures that bind them. In topology this is a presentation by cells [20].

2. Put loop i on scale i. A step forward along it is R at that scale; a step back is L.
3. Walk the boundary of each closure. The steps form a strand, and the strand is a trit word.
4. A closed strand has no starting point. Its address is taken up to rotation, as in T13 of the theory, up to reading it backwards (inversion), and over the choice of scales, in its least form.
5. Prove in cubical Agda that the type built from that word is equivalent to the concept. Univalence makes them one.

6.3 A pilot

Steps 1 to 4 were run in Python on ten concepts. The homology of each was computed alongside, as an independent check. All ten addresses differ. The point, the circle and two joined circles have no closure; they differ only in their number of loops.

Concept	Strand of its closures	What remains
point	none	nothing
circle	one loop, no closure	$\mathbb{Z}$
disc	T	0
projective plane	T T	$\mathbb{Z}/2$
$\langle a \mid a^3 \rangle$ (π_1 of the lens space L(3,1))	T T T	$\mathbb{Z}/3$
torus	T0 T 10 1 (sum 0)	$\mathbb{Z}^2$
Klein bottle	T0 T T0 1	$\mathbb{Z} \oplus \mathbb{Z}/2$
trefoil (knot group)	T0 T0 T0 1 1	$\mathbb{Z}$
figure-eight knot (knot group)	T0 T 10 T T0 1 10 T 10 1	$\mathbb{Z}$
two circles joined	two loops, no closure	$\mathbb{Z}^2$

Table 2. Trit words for ten named concepts, and their first homology.

The pilot shows a simple pattern. A closure made of n equal steps leaves $\mathbb{Z}/n$. Two equal steps, the open pair of the theory, correspond to the projective plane and $\mathbb{Z}/2$. Three equal steps, the ring, correspond to $\mathbb{Z}/3$. The trefoil binds a twofold winding to a threefold one: its knot group has the relation $a^2 = b^3$. That echoes the numbers 2 and 3 of the two ladders of the theory [1]. Whether these correspondences are more than echoes is a question for the work programme, not a result.

6.4 The dictionary

The list of names exists. Mathematics as a Chain of Proven Steps tells all 63 fields in 55 chapters, link by link [3]. Each link is a named result. The book records who found what and when. That is history, not address. Its use here is as the index of the dictionary: one entry per link, each to receive its trit word.

6.5 The work programme

1. Extend the pilot to the knot table and the closed surfaces, where presentations are standard.
2. Fix the least presentation for each concept, as knot tables fix the least diagram.
3. For each entry, build the type from its trit word in cubical Agda and prove the equivalence.
4. Let univalence join synonyms. Keep a record of which names fell together.
5. Work through the dictionary field by field, starting with the fields the theory already touches: number theory and knot theory.

7. Limits

- The word type uses a set-truncation constructor. It is a set by construction, not by proof. Altenkirch and Scoccola show how integers can be a higher inductive type without it [6].
- F4 uses the library theorem that the loops of the circle are the integers, and the library lemma that winding respects addition [5, 18]. The integers also serve as the measuring type for the value of a word. The new content is that balanced-ternary words, read digit by digit, are those loops. The integers are not yet derived from the loop without that theorem.
- The ring and the meeting hold by definition or by a general law. They are recorded, not discovered.
- The address of a concept depends on its presentation. A least presentation must be fixed, and that is not yet done.
- The pilot covers ten concepts and runs in Python, not in Agda. Its readings in Section 6.3 are correspondences, not theorems.
- Section 5 is a reading. It is not formalised.
- The Agda files and the pilot were written and checked by the same AI system. An independent check by a person is still needed.

8. Conclusion

The foundation is small. One closed loop. A return that is a real distinction. Three directions at the return. Words that are exactly the windings. Closures that are paths. Mirroring that is reversal. Each of these is a definition or a theorem in cubical Agda, checked by the machine.

On that foundation, mathematics is a dictionary. Each name points to a trit word. The method to find the word exists, and a first pilot works. In it the elementary closures of the theory correspond to the simplest objects of topology. What remains is the long work of going through the names, one by one, and letting univalence join what was always one.

Tool disclosure. The Agda files, the pilot and this essay were made with the AI system Claude (Anthropic) under the direction of the author. The author set the question and decided what counts as a result. Every Agda statement was accepted by the type checker.

Annotated references

[1] Konstapel, J. (2026). The Vacuum.Net Theory, Ninth Edition: Foundations 9.2 and Physics 9.0. Constable Research, Leiden, 27 October 2026.

The theory this essay grounds. Read N1–N3, the trit rule in III.1, T7 and T7.1 for the closures, T12 for the address and T13 for rotation classes.

[2] Konstapel, J. (2026). The Vacuum.Net Theory Read Against the History of Mathematics. Constable Research, Leiden, 29 October 2026.

Places the 45 historical results the theory rests on. Shows that knot theory carries the most and that the theory reads history rather than retelling it.

[3] Konstapel, J. (2026). Mathematics as a Chain of Proven Steps: The 63 Fields of Mathematics, from the First Proofs to 2026. Constable Research, Leiden.

The list of names. Use it as the index of the dictionary in Section 6.4. Its chapter on numbers and foundations ends with univalent foundations.

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The reference for paths as equality, higher inductive types and univalence. Start with chapters 2, 6 and 8.

[5] Licata, D. R., & Shulman, M. (2013). Calculating the fundamental group of the circle in homotopy type theory. Proceedings of LICS 2013, 223–232. arXiv:1301.3443.

Proves inside the theory that the loops of the circle are the integers. The result F4 builds on.

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Integers as a higher inductive type without set-truncation. The route to remove the first limit of Section 7.

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[11] Spencer-Brown, G. (1969). Laws of Form. Allen & Unwin, London.
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[12] Kauffman, L. H. (2005). EigenForm. Kybernetes 34(1/2), 129–150.
A form that reproduces itself. The reading of closure as a fixed point.

[13] Gödel, K. (1931). Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatshefte für Mathematik und Physik 38, 173–198. Turing, A. M. (1936). On computable numbers, with an application to the Entscheidungsproblem. Proceedings of the London Mathematical Society 42, 230–265.
The two classic limits. Read with Section 5: both are loops that do not close.

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The strong three-valued logic, with an open value. The logic in which an unclosed loop is not a contradiction.

[15] Fox, R. H. (1962). A quick trip through knot theory. In M. K. Fort (ed.), Topology of 3-Manifolds and Related Topics, 120–167. Prentice-Hall. See also Crowell, R. H., & Fox, R. H. (1963). Introduction to Knot Theory. Ginn, Boston.
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[17] Awodey, S. (2014). Structuralism, invariance, and univalence. Philosophia Mathematica 22(1), 1–11.
Why univalence lets isomorphic objects be identified. The principle behind joining synonyms in Section 6.

[18] The agda/cubical library, version 0.9. Modules Cubical.HITs.S1.Base and Cubical.Data.BinNat.BinNat. https://github.com/agda/cubical
Supplies the circle, the proof that its loops are the integers, and Mörtberg's binary numbers, the model for the ternary words here.

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The formal side of Section 4. Check with Agda 2.8.0 and cubical v0.9.

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