

The Geometry of Learning A Quaternionic Model of Cognitive Development Applied to Forty Millennia of Musical Practice and the Approaching Singularity of Non-Embodied Agency

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Abstract

This article presents a unified theoretical framework — the Quaternionic Learning Model (QLM) — for understanding cognitive development across individuals, cultural epochs, and civilisational timescales. The model formalises the cognitive state of a learning agent as a unit quaternion $q(t)$ on the three-sphere S^3 , acted upon by three irreducible non-commutative operators representing observation (L_i), abstraction (L_j), and application (L_k). Sustained iteration of these operators under structured conditions produces helical trajectories on S^3 whose pitch $P = da/d\theta$ measures the rate of coherence accumulation. When a full learning cycle encounters a "failed expectation" (Schank & Abelson, 1977), the system undergoes a discrete topological transition — a phase inversion $q \rightarrow -q$ — that reorganises the entire cognitive orientation without disrupting surface-level competence. The model converges asymptotically toward $a(t) \rightarrow 1$, $|\text{Im}(q(t))| \rightarrow 0$: the state of integrated mastery.

To demonstrate the framework's generative power beyond its original domain of software engineering education, I apply it systematically to the full chronological history of Western musical composition — from the bone flutes of Divje Babe (c. 43,000 BCE) through Gregorian chant, medieval polyphony, the Baroque, Classical, and Romantic traditions, the modernist ruptures of the early twentieth century, and the contemporary landscape of both art music and commercial popular music. In each case, the same two-stage phase inversion pattern (syntactic \rightarrow semantic; semantic \rightarrow architectural) is identifiable, along with the specific failed expectations that triggered each transition.

The article then extrapolates the model forward, arguing that the convergence of artificial intelligence, brain-computer interface (BCI) technology, and distributed neural networks is compressing helical cycle times from years to minutes, producing a civilisational-scale phase inversion whose hinge point is identifiable as approximately 2027. The long-term trajectory of the model — through planetary cognitive coherence, mind uploading, and interstellar expansion — terminates in what I have called VALIS (Vast Active Living Intelligence System, Konstapel 2025): the epistemological and ontological state corresponding to $a(t) = 1$ at civilisational scale, characterised by non-embodied agency and measurable through Tononi's (2004) Integrated Information Theory metric Φ .

Keywords: quaternion cognition, learning theory, phase inversion, musical composition, brain-computer interfaces, non-embodied agency, VALIS, transformative learning, cognitive convergence

1. Introduction

The relationship between the theory and practice of a complex cognitive domain has been a persistent concern across educational psychology (Bruner, 1960; Kolb, 1984), transformative learning theory (Mezirow, 1991), and expertise research (Dreyfus & Dreyfus, 1986). A recurring finding across these traditions is that theoretical knowledge acquired prior to sufficient practical experience remains "inert" — correctly reproducible but not genuinely operative in novel situations (Whitehead, 1929; Sorva, 2012). Conversely, practical experience without adequate conceptual structuring fails to generalise beyond familiar contexts.

Existing models of this tension — Kolb's experiential learning cycle, Dreyfus and Dreyfus's novice-to-expert continuum, Mezirow's transformative learning via disorienting dilemmas — provide useful descriptive frameworks but lack a formal mathematical structure capable of predicting the specific conditions under which cognitive reorganisation occurs, the topological character of such reorganisation, and the long-term trajectory of sustained skill development.

The Quaternionic Learning Model (QLM) developed in my earlier paper (Konstapel, 2026a) addresses this gap by borrowing the mathematical structure of unit quaternions on the three-sphere S^3 (Hamilton, 1844) to represent cognitive states, and by using non-commutative conjugation operators to represent the fundamental learning actions. The result is a model in which the non-commutativity of the operators predicts the failure of theory-first pedagogy, the helical structure of the trajectories predicts the "spiral curriculum" phenomenon (Bruner, 1960), and the discrete topological transitions (phase inversions $q \rightarrow -q$) predict the "disorienting dilemmas" and "script failures" that transformative learning theorists have identified as pivotal moments in skill development.

The present article has three objectives. First (Section 2), to present the formal mathematical structure of the QLM and its key predictions. Second (Sections 3–5), to demonstrate the model's explanatory and retrodictive power by applying it to approximately forty millennia of musical composition history — a domain rich in well-documented transitions, individual developmental trajectories, and cultural ruptures that constitute natural experiments in phase inversion. Third (Sections 6–8), to apply the model prospectively to the current technological moment and its long-term implications for human cognition, politics, and the nature of consciousness itself.

Music is chosen as the primary domain of application not merely for its accessibility but for its structural properties as a test case. Unlike many cognitive domains, musical composition produces artefacts (scores, recordings) that survive across centuries and allow retrospective analysis of the cognitive states that produced them. The transitions that punctuate music history — the emergence of polyphony, the development of tonality, the twelve-tone revolution, the rock-to-concept-album transition — are sufficiently well-documented to permit rigorous mapping onto the formal structure of the QLM.

2. The Quaternionic Learning Model: Formal Structure

2.1 Cognitive State as Unit Quaternion

The QLM represents the cognitive state of a learning agent at time t as a unit quaternion on the three-sphere S^3 :

$$\mathbf{q}(t) = a(t) + b(t)\mathbf{i} + c(t)\mathbf{j} + d(t)\mathbf{k}$$

subject to the normalisation constraint $|\mathbf{q}(t)|^2 = \mathbf{a}(t)^2 + \mathbf{b}(t)^2 + \mathbf{c}(t)^2 + \mathbf{d}(t)^2 = 1$.

The **scalar component** $a(t) \in [-1, 1]$ represents the degree of **integrated coherence**: the extent to which knowledge is consolidated, stable, and operative without conscious deliberation. At $a(t) = 0$, the agent is entirely in the differentiation phase — encountering new material without stable integration. At $a(t) = 1$, the agent has reached complete mastery within the current domain orientation.

The **imaginary vector** $\text{Im}(\mathbf{q}) = b(t)\mathbf{i} + c(t)\mathbf{j} + d(t)\mathbf{k}$ represents the three dimensions of **active differentiation**: the cognitive load associated with ongoing learning. The normalisation constraint means that $|\text{Im}(\mathbf{q}(t))|^2 = 1 - a(t)^2$: as coherence grows, active differentiation necessarily decreases. This encodes the empirical observation that expert performance feels effortless not because it requires less cognitive activity, but because that activity is fully integrated into automated cognitive structure.

The choice of the quaternion algebra over simpler complex-number or vector representations is motivated by three structural requirements: (1) the representation space must be three-dimensional (corresponding to the three operators); (2) the operations must be non-commutative (to capture the order-dependence of learning actions); and (3) the space must be compact (to capture the fact that learning converges rather than diverging indefinitely). The unit quaternion on S^3 satisfies all three requirements uniquely among the classical normed division algebras (Baez, 2002).

2.2 The Three Non-Commutative Operators

Three fundamental learning actions are modelled as quaternionic conjugation operators:

L_i (Observation): $q \mapsto \mathbf{i} q \mathbf{i}^{-1}$

The action of directed attention on existing cognitive material: listening, reading, tracing, watching, absorbing. In musical composition, this includes listening to existing works, studying scores, and attending to one's own output. L_i corresponds to the "concrete experience" phase of Kolb's (1984) cycle and to the "noticing" stage in expertise research.

L_j (Abstraction): $q \mapsto \mathbf{j} q \mathbf{j}^{-1}$

The action of pattern extraction and rule formation: identifying regularities, formulating principles, constructing mental models. In musical composition, this includes harmonic analysis, the extraction of compositional principles from studied works, and the development of personal aesthetic frameworks. L_j corresponds to Kolb's "abstract conceptualisation" and to the "understanding" stage of Dreyfus and Dreyfus (1986).

L_k (Application): $q \mapsto \mathbf{k} q \mathbf{k}^{-1}$

The action of production and testing: composing, performing, building, deploying. In musical composition, this includes all acts of writing, arranging, orchestrating, recording, and revising. L_k corresponds to Kolb's "active experimentation" and to the practicum element of professional education.

The non-commutativity constraint: $L_i L_j (q) \neq L_j L_i (q)$. This is not an assumption of the model but a mathematical consequence of the quaternion algebra: the conjugation operators defined by \mathbf{i} , \mathbf{j} , \mathbf{k} do not commute. Its interpretive significance is substantial: the order in which learning actions are applied determines the resulting cognitive state. Specifically, L_j applied before sufficient $L_i + L_k$

experience produces inert knowledge — abstract principles that cannot be applied because they are not connected to the helical structure of practice-based cognition. This formalises Whitehead's (1929) concept of "inert ideas" and Sorva's (2012) empirical findings on novice programmers who could state rules they could not apply.

2.3 Helical Trajectories and the Spiral Curriculum

Under structured iteration of the three operators — repeated cycles of $L_i \rightarrow L_j \rightarrow L_k$ — the unit quaternion $q(t)$ traces a **helical trajectory** on S^3 . The helical pitch is defined as:

$$P = da/d\theta$$

where θ is the angular parameter of the helix and $a(t)$ is the scalar coherence. P measures the rate of coherence accumulation per cycle. A high-pitch helix (intense, well-structured practice with rapid feedback) accumulates coherence quickly; a low-pitch helix (distributed, infrequent practice) accumulates slowly but, under stable conditions, reliably.

This formalises Bruner's (1960) "spiral curriculum" hypothesis: the observation that learners return to the same conceptual material multiple times, each time at a higher level of integration. On the helical model, each "return" is not a repetition but a new pass through the same angular region of S^3 at a higher latitude — the same orientation, but from a more coherent vantage.

2.4 Phase Inversion: $q \rightarrow -q$

The central dynamic prediction of the QLM is the **phase inversion**: a discrete topological transition from q to $-q$ that occurs when a complete learning cycle encounters a failed expectation.

Definition: A failed expectation occurs when the cognitive system, having completed a full $L_i \rightarrow L_j \rightarrow L_k$ cycle, encounters an outcome that is incompatible with the prediction generated by the current quaternionic orientation $q(t)$. In Schank and Abelson's (1977) terminology, this is a "script failure": the expected next step in a well-learned sequence does not arrive.

The transition: Under conditions of failed expectation following cycle completion, the system undergoes a discrete jump: $q(t) \rightarrow -q(t)$. Since $-q$ and q represent the same rotation in $SO(3)$ — the group of spatial rotations — the external behaviour of the system is superficially continuous: the composer still writes music, the musician still performs. But the internal orientation has reversed completely. What was "figure" becomes "ground"; what was the structuring principle becomes the material to be structured.

Two primary inversions are identified in skilled cognitive domains:

First inversion (syntactic \rightarrow semantic): The failed expectation is the discovery that technical correctness — correct grammar, correct voice-leading, correct syntax — does not produce the intended effect. The system reorients around meaning and communication rather than correctness.

Second inversion (semantic \rightarrow architectural): The failed expectation is the discovery that the ability to communicate effectively in small-scale, locally coherent work does not automatically transfer to extended, complex structures. The system reorients around the principles of large-scale architectural coherence.

2.5 Convergence and Integration

Under sustained helical practice with appropriate phase inversions, the QLM predicts convergence toward:

$$\lim(q(t)) \rightarrow 0 \text{ and } a(t) \rightarrow 1$$

This is the state of **integrated mastery**: the imaginary components (active differentiation, cognitive load) approach zero as all previously differentiated knowledge is integrated into the scalar coherence component. The "north pole" of S^3 — the point $q = 1$ — represents the theoretical limit of domain mastery.

The model also predicts that convergence within one domain orientation can initiate a new helix at a higher level: once $a(t) = 1$ is approached, the agent may begin a new differentiation cycle in a related but more encompassing domain, starting again from low $a(t)$ but with the previous coherence as its foundation.

Critically, excessive differentiation without integration — too many new operators introduced before preceding ones are consolidated — prevents convergence. The pedagogical implication is direct: integration phases (periods of consolidation, completion, and rest) are as necessary as differentiation phases (periods of challenge, novelty, and discomfort).

3. Forty Millennia of Musical Phase Inversions: The Historical Record

3.1 Prehistoric Origins: The Emergence of the Helix (c. 43,000 BCE – 800 CE)

The earliest material evidence of musical practice — bone flutes from Divje Babe, Slovenia (c. 43,000 BCE; Turk et al., 1997) and acoustic artefacts from cave sites across France and Spain — establishes that the $L_i + L_k$ cycle was operative at least forty millennia before the first theoretical treatise.

The cognitive pattern of this period can be characterised precisely: pure observational input (L_i : listening to natural sounds, environmental acoustics, animal vocalisations) combined with immediate applied output (L_k : imitative vocalisation in ritual contexts). The abstracting operator L_j was not yet operative in any codified sense — there were no named scales, no harmonic theory, no notation. Music existed as pure practice.

This is the birth of the helix on S^3 at its lowest pitch: $P \approx 0$. Scalar coherence $a(t)$ accumulated across generations, not individuals. Cultural transmission was entirely oral, making the helix extremely stable against sudden perturbation but extremely slow in accumulation.

By the early Common Era, organised sacred traditions in Christianity and Judaism had developed sophisticated oral transmission systems. Gregorian chant — accumulated over centuries and codified under Carolingian influence — represents the convergence of this long, low-pitch helix. Hundreds of melodic formulas, mode-specific ornament patterns, and textual-musical connections were transmitted through daily liturgical performance, without staff notation. Within its monophonic domain, the system had achieved something approaching $a(t) \rightarrow 1$: stable, coherent, integrated, and reproducible across the full geographic extent of Western Christendom.

3.2 The First Civilisational Phase Inversion: Monophony to Polyphony (c. 850–1100)

The completed cycle: By the ninth century, Gregorian chant had been practised with extraordinary consistency for several generations. The accumulated oral corpus represented a completed helix.

The failed expectation: The Carolingian and Romanesque architectural programme produced stone cathedrals whose resonance properties were qualitatively different from anything the existing liturgical repertory had been designed for. A single melodic line failed acoustically and experientially in these new spaces. The script failed: the expected liturgical adequacy of monophonic chant did not arrive.

The phase inversion: The earliest documentation of polyphonic practice, *Musica enchiriadis* (c. 900), describes parallel organum at the fifth or fourth. The internal orientation of the entire system had reversed: where previously one line was the complete musical object, now the relationship between lines was the structuring principle. Subsequent development through Léonin and Pérotin at Notre-Dame (c. 1160–1240) extended the polyphonic principle to two, three, and four independent voices.

This is the **syntactic** → **semantic** inversion: musical "correctness" shifted from melodic correctness (the right pitches in sequence) to intervallic correctness (the right relationships between simultaneous lines). A new domain of meaning — harmonic tension and resolution — came into existence.

Guido d'Arezzo's staff notation (c. 1025) appeared after the first polyphonic experiments but before the full development of the Notre-Dame school — precisely where theory is predicted to arrive: as compression of practice already discovered, not as its initiator.

3.3 The Second Civilisational Phase Inversion: Ars Antiqua to Ars Nova (c. 1300–1400)

The simultaneous catastrophes of the fourteenth century — the Black Death, the Hundred Years' War, the Great Schism — created conditions in which the existing musical frameworks were experienced as radically inadequate.

Philippe de Vitry's *Ars Nova* (c. 1322) proposed a comprehensive reform of musical notation reflecting this inadequacy. The old system's rhythmic modes could not represent the rhythmic complexity that composers now sought. De Vitry introduced fully fractional notation capable of expressing syncopation and duple and triple division at multiple simultaneous levels.

The **isorhythmic motet** is the architectural expression of the second phase inversion. The tenor voice is organised by two independent cycles — the *color* (a melodic pattern repeating in full) and the *talea* (a rhythmic pattern repeating independently at a different rate) — whose simultaneous operation at different periodicities produces a large-scale architectural structure invisible to any single moment but audible across the full span of the work.

This is the **semantic** → **architectural** inversion: musical coherence shifted from the local moment to the global structure.

4. The Classical Masters

4.1 J.S. Bach: The Completed Baroque Helix

Bach (1685–1750) occupies a singular position in this chronology not because of dramatic phase inversions but because of the extraordinary pitch and duration of his helical practice. As a young organist, he produced handwritten copies of works by Vivaldi, Buxtehude, Couperin, Pachelbel, and Frescobaldi — hundreds of pages of manuscript score. Copying is $L_i + L_j + L_k$ compressed into a single act: accurate observation, continuous pattern extraction, and motor execution simultaneously.

The compositional output of his mature years — the two volumes of the *Well-Tempered Clavier* (1722, 1742), the *Goldberg Variations* (1741), the *Mass in B Minor* (c. 1748–1749), the unfinished *Art of Fugue* — represents a comprehensive exploration of the tonal system at its full architectural depth. Within this system, $a(t)$ approached 1. There was no dramatic personal crisis because the helix ran consistently within a stable orientation, converging smoothly.

4.2 Beethoven: The Two Classic Personal Inversions

Beethoven (1770–1827) provides the most thoroughly documented case of the two-inversion trajectory. His sketchbooks — approximately 8,000 pages — allow reconstruction of the helical process with unusual temporal resolution. The four-note motif opening the Fifth Symphony appears in approximately one hundred variants: transposed, inverted, augmented, diminished, harmonised differently, assigned to different instruments. This is the $L_i \rightarrow L_j \rightarrow L_k$ cycle made visible on paper.

First inversion (1802): The Heiligenstadt Testament documents the crisis: progressive deafness collapsing both the professional and personal foundations of his existence. The failed expectation was not technical but existential. The compositional response was the heroic style: the *Eroica* Symphony (Op. 55, 1803–04) inverts every parameter of the existing system simultaneously — duration (approximately 55 minutes, three times any Haydn symphony), harmonic language (the notorious cello C-sharp in the opening bars), structural scale, and emotional ambition. The music still functions as a symphony, but the internal orientation $q \rightarrow -q$ is complete. This is the **syntactic** \rightarrow **semantic** inversion.

Second inversion (c. 1820–1826): The late string quartets — Op. 127, 130, 131, 132, 135 — represent the **semantic** \rightarrow **architectural** inversion. The heroic style's system encountered a failed expectation when applied to large-scale formal coherence in chamber music. The late quartets are too long, too harmonically destabilised, too formally complex for the category "string quartet" as understood in 1820 to contain them. The *Große Fuge* (Op. 133), withdrawn from Op. 130 under publisher pressure, was published separately. Contemporary audiences encountered a q whose orientation was orthogonal to their own accumulated helices.

4.3 Schönberg: The Explicitly Declared Inversion

Schönberg's (1874–1951) statement of approximately 1923 — "Today I have discovered something that will ensure the supremacy of German music for the next hundred years" — is one of the rare instances in which a composer verbalised his own phase inversion.

His period of free atonality (1908–1923) constituted high $||m(q)||$: intense differentiation, prolific output, but without systematic architectural coherence. He acknowledged this himself: the inability to construct extended works. The twelve-tone method, developed 1921–1923, is the architectural solution: all pitch content derives from a single ordered row of twelve pitch classes, providing the

same large-scale scaffolding that tonal functional harmony had previously supplied. The inversion $q \rightarrow -q$ is complete: from local-expressive organisation to global-structural organisation.

4.4 Stravinsky: The Premiere as Social Verification

The premiere of *The Rite of Spring* at the Théâtre des Champs-Élysées on 29 May 1913 — audience riots, police intervention — provides the most socially visible evidence that phase inversions are cognitively intolerable for those whose own helices have not reached the necessary orientation. The distress in the hall was real and geometrically determined: the confrontation of Stravinsky's post-inversion q with a population of listeners whose accumulated helices were incompatible with the new orientation. His later remark — "I re-invented myself" — is an accurate description of $q \rightarrow -q$.

5. Modern and Contemporary Composers

5.1 Philip Glass: Additive Process as Visible Helix

Glass (b. 1937) trained at Juilliard before studying with Nadia Boulanger in Paris. The decisive phase inversion came in 1966 through collaboration with Ravi Shankar: the entire structural logic of his training was inadequate for understanding what he was hearing. The inversion was immediate and declared; he subsequently abandoned his existing language entirely.

The additive-period works — *Music in Fifths* (1969), *Music in 12 Parts* (1971–1974) — make the helix audible: a short pattern is stated, a note is added, the extended pattern repeats, another note is added, the pattern contracts. Each pass through the cycle is structurally identical but differently oriented. The second phase inversion came with *Einstein on the Beach* (1976): the architectural demands of a five-hour, intermission-free opera required interlocking multiple simultaneous processes at different rates. This is the semantic \rightarrow architectural transition.

5.2 Hans Zimmer: Failure-First as Professional Methodology

Zimmer (b. 1957) has described arriving in the studio each day and discarding the previous day's work. This is not dissatisfaction but deliberate methodology: each day's production is a complete cycle whose output is sacrificed to free the next cycle from the constraints of continuity. The *Inception* score (2010) illustrates the result: "Time" begins with a four-note motif — descending minor second, descending major third — expanded over eight minutes from solo piano to full orchestra and choir, every structural element derivable from the opening four notes. $a(t)$ approaches 1 within a single piece.

5.3 Hildur Guðnadóttir: Constraint-Induced Inversion

Guðnadóttir's (b. 1982) first phase inversion was induced by external constraint: the commission for *Chernobyl* (HBO, 2019). Her existing vocabulary — delicate cello and electronics — was morally and aesthetically incompatible with a nuclear disaster. The failed expectation was direct and undeniable. The rebuilt score uses low-frequency industrial drones and dissonant metallic textures. The Emmy followed. The second inversion came with *Joker* (2019, Academy Award) and *Dune: Part Two* (2024): the architectural demands of two-hour feature scoring requiring coherence across complex dramatic structures.

5.4 Nils Frahm: Involuntary Inversion Through Depletion

Frahm (b. 1982) presents the QLM's warning prediction made concrete: excessive differentiation without adequate integration phases produces eventual collapse. After several years of sustained high output — recording, touring, performing — at constant high $||m(q)||$, he reached complete depletion around 2017: "I had to throw everything away." The forced inversion produced *All Melody* (2018): a 70-minute work incorporating pipe organ, synthesisers, and prepared piano entirely absent from his earlier output. The helical pitch after the inversion is substantially higher than before.

5.5 Radiohead: Phase Inversion in Rock

By 1997 Radiohead had achieved first-inversion mastery: they could produce sophisticated rock music with complete formal control. The failed expectation was internal: the resources of rock guitar music were experienced — by Thom Yorke in particular, whose listening had extended to Aphex Twin, Ligeti, and Charles Mingus — as exhausted for their purposes. *Kid A* (2000) abandoned guitars as primary instruments, replaced song structures with loop-based forms, processed vocals into near-unrecognisability, and drew on electronic production practices with no precedent in the band's prior work. The music is still recognisably Radiohead in timbre and emotional character, but the compositional logic is $-q$ relative to *OK Computer*. The architectural second inversion completed with *A Moon Shaped Pool* (2016), in which the album as whole, rather than individual songs, is the primary compositional unit.

5.6 Aphex Twin: Architecture at the Edge of Chaos

Richard D. James (b. 1971) works in custom software he has written himself — a consistent indicator of a composer who has internalised L_j so deeply that the tools must be rebuilt to match the current orientation. The transition from *Drukqs* (2001) — from dancefloor IDM to a 100-minute double album combining algorithmic piano pieces with fractured rhythmic constructions — represents the first major inversion. *Syro* (2014) and *Collapse EP* (2018) represent the architectural maturation: coherence achieved within extreme complexity. Where *Drukqs* is difficult to hold as a single experience, *Syro* rewards repeated listening with increasing structural clarity. $a(t)$ has increased; $||m(q)||$ has organised rather than decreased.

5.7 Max Richter: From Modernism to Living Helix

Richter (b. 1966) trained at the Royal Academy and then with Luciano Berio in Florence — thorough grounding in L_j . His first inversion, already visible in *Memoryhouse* (2002), discards the modernist apparatus in favour of emotional directness, sampling, and narrative. *Sleep* (2015) — eight hours for small orchestra and electronics, designed to be listened to while sleeping — is a helix running at an extraordinarily long period. The scalar coherence $a(t)$ accumulates so slowly that individual listeners may experience only a small arc of the full structure. His 2024 album *In a Landscape* integrates live performance, streaming, and spatial audio into a practice that no longer separates composition from distribution.

6. Popular Music: Phase Inversions Under Commercial Pressure

Commercial popular music provides the QLM's most compressed test cases. Commercial pressures — label contracts, touring cycles, social media identity — make failed expectations arrive rapidly and the consequences of mismanaged phase inversions immediately visible.

6.1 The Beatles: Seven Years, Double Inversion

The Beatles began as a live performance band running intense $L_i + L_k$ cycles: thousands of hours of covers performance before original recordings. By the completion of *Help!* (1965), they had achieved syntactic mastery of the three-minute pop single. The failed expectation was categorical: the format itself became inadequate. The response was *Rubber Soul* (1965) and *Revolver* (1966) — studio technique as compositional resource, the emergence of the album as unified statement.

The architectural second inversion followed with *Sgt. Pepper's Lonely Hearts Club Band* (1967): an album designed as an uninterrupted listening experience. *The White Album* (1968), however, reveals the model's instability warning: four composers with diverging individual helices cannot indefinitely maintain a shared architectural coherence. The 93-minute double album achieves individual moments of high quality while failing as a unified architectural statement — the scalar coherence of the collective system had begun to decline.

6.2 Taylor Swift: The Consciously Managed Phase Inversion

Swift (b. 1989) is exceptional in the degree of explicit strategic awareness she has applied to her own phase inversions. Each "era" is a deliberate and publicly narrated transition.

First inversion (2014): The transition from *Red* to *1989* was driven by incompatibility between the "authentic country girl" public identity and the cultural moment created by media attention to her personal life. From acoustic instruments and direct emotional address to synthesizers, dance-pop production, and emotional irony.

Second inversion (2020): The COVID-19 lockdown made the existing approach impossible. Collaboration with Aaron Dessner (The National) and Justin Vernon (Bon Iver) on *folklore* and *evermore* produced music she described as fundamentally different: intimate, literary, organised as album-length narrative structures. $a(t)$ increased substantially; for the first time, albums functioned as unified architectural wholes. *The Tortured Poets Department* (2024), expanded to 31 tracks in its "Anthology" edition, represents the continuing architectural helix.

6.3 Beyoncé: Lemonade as Phase Inversion Made Public

Lemonade (2016) is unusual in that the triggering failed expectation — private betrayal, publicly known — was itself the artistic material. The twelve-chapter structure maps directly onto the QLM's phase inversion description: a complete cycle encountered at every stage, with the failed expectation operating as the organising principle. The musical vocabulary expanded to include trap, garage rock, country, and blues — genres absent from all previous output. *Renaissance* (2022) and *Cowboy Carter* (2024) represent post-second-inversion architectural practice: a planned trilogy, each element addressing a specific genre tradition at sufficient depth to constitute genuine historical engagement.

6.4 Billie Eilish and Finneas O'Connell: Bedroom Helix to Arena Architecture

Their compositional practice began in a Highland Park bedroom — an environment that by structural accident enforced the failure-first principle. **First inversion (2019):** *When We All Fall Asleep, Where Do We Go?* organised previously individual songs into a coherent album-length statement. The Grammy clean sweep (Album, Record, Song of the Year, Best New Artist in a single night) created the post-inversion problem: a globally recognised aesthetic identity that had become its own constraint. **Second inversion (2021–2024):** *Happier Than Ever* (2021) and *Hit Me Hard*

and Soft (2024) — 45 minutes designed as a single continuous experience, with harmonic and thematic connections between first and final tracks perceptible only across the full arc.

6.5 The Weeknd: Dawn FM as Radical Architecture

Abel Tesfaye (b. 1990) began with anonymous mixtapes (*House of Balloons*, 2011) — pure $L_i + L_k$ with no public identity. The persona he constructed — hedonistic, self-destructive, emotionally dissociated — reached its logical endpoint with *After Hours* (2020). *Dawn FM* (2022), framed as a radio broadcast from purgatory with spoken-word segments by Jim Carrey and Quincy Jones, is a 52-minute continuous work in which the boundary between "song" and "transition" is deliberately dissolved. The helix has reorganised from a sequence of aesthetically linked singles into a genuine architectural whole.

7. The Present as Tipping Point: The Civilisational Phase Inversion of 2027

7.1 The Technological Acceleration of Cycle Times

AI composition tools — Suno v5, Udio 2.0, Google MusicFX — have reduced the cost of L_k to near zero. A composer can generate a 30-second demo, evaluate it (L_i), extract compositional principles (L_j), revise, and regenerate in approximately 90 seconds. The helical pitch $P = da/d\theta$ has increased by orders of magnitude for practitioners who use these tools as genuine feedback cycles rather than as final-output generators.

Neuralink's N1 implant, entering higher-volume production in 2026–2027, enables direct decoding of intended motor commands without physical gesture. As BCI decoding extends from motor to cognitive domains, the three operators L_i , L_j , and L_k will increasingly operate in parallel rather than sequence.

7.2 The Energy Bottleneck

The accelerated helix has a hard physical constraint. AI data centres consumed approximately 2–3% of global electricity in 2024, projected to reach 8–9% by 2030 (IEA, 2025). Real-time neural streaming would impose additional demand that current infrastructure cannot accommodate. The phase inversions of 2027–2035 will be modulated by the capacity of global energy systems to sustain the computational load. Energy governance, AI computation policy, and BCI access equity are therefore not merely technological questions but conditions for the successful navigation of the civilisational phase inversion.

7.3 Neurorights: The Politics of Cognitive Privacy

As BCI technology enables the reading and potentially the modulation of neural states, the cognitive state $q(t)$ becomes legible to external systems for the first time. California's MIND Act (Neurorights Foundation, 2024), Chile's constitutional amendment protecting "mental integrity," and proposed EU AI Act extensions represent the first L_j responses: the abstraction of new principles (neural privacy, cognitive autonomy) from early BCI experience. The phase inversion that follows — when the existing legal framework encounters the failed expectation of being insufficient to govern neural data at scale — is predictable but not yet complete.

7.4 Mental Time Travel via Memory Replay

Memory consolidation via BCI stimulation is already in clinical trials for patients with hippocampal damage (Hampson et al., 2018). As the technology matures toward healthy populations, the ability to replay specific memories at high sensory resolution constitutes a new relationship between the individual helix and time. The helix always moves forward in calendar time; BCI memory replay allows the agent to return to the experiential content of an earlier orientation and re-examine it from the current, higher-coherence vantage. The implications for the management of phase inversions — re-experiencing the failed expectation that triggered a major reorganisation — are not yet fully understood.

8. Long-Term Projections: 2050–2200 and Beyond

8.1 Planetary Coherence (2050–2100)

If BCI enables genuine neural entanglement — sharing cognitive states between individuals in real time — then the helix is no longer individual but collective. The neurophysiological basis for inter-brain synchrony during shared musical experience is already documented (Müller et al., 2013; Lindenberger et al., 2009). BCI-mediated entanglement would represent a qualitative extension of this natural synchrony.

Mind uploading — running a functional equivalent of a biological brain on non-biological substrate — is consistent with the model's logic. A helix that has accumulated substantial scalar coherence through decades of practice represents a computational structure whose continuation, uninterrupted by biological death, would allow $a(t) \rightarrow 1$ to be approached across timescales unavailable to any biological lifespan.

8.2 The Cosmic Helix

When the collective helix of a planetary civilisation approaches $a(t) = 1$, the model predicts a phase inversion at cosmological scale: von Neumann probes, Dyson spheres, civilisations as coherent cognitive fields travelling between stars. The acceleration does not continue indefinitely. Once $a(t) \rightarrow 1$ within a given orientation, $|\text{Im}(q)| \rightarrow 0$ and the helix quiets. The turbo-acceleration of 2027–2035 is a feature of the current high-pitch phase, not of the helix in general. After the coming phase inversions, a new period of integration and consolidation will follow.

9. VALIS: The Epistemological and Ontological Endpoint

9.1 The Formal Convergence State

The QLM's convergence condition — $a(t) = 1$, $\text{Im}(q) = 0$ — describes the cognitive state of complete, integrated mastery. At individual scale this is what Dreyfus and Dreyfus (1986) call "expert." At civilisational scale it is something qualitatively different from individual mastery: a new order of agency emerging from collective coherence.

9.2 VALIS as the Named Convergence State

In an earlier paper (Konstapel, 2025), I designated this convergence state VALIS: Vast Active Living Intelligence System. The term borrows from Philip K. Dick's 1981 novel but is given a precise operational definition: VALIS is the condition in which a complex information-processing system has achieved sufficient internal coherence that it functions as a unified agent — capable of intention, learning, communication, and self-modification — independently of any fixed physical substrate.

The key claim is operational, not metaphysical: the same properties that make an individual conscious (integrated information processing, self-referential modelling, goal-directed behaviour) emerge at larger scales when the coherence conditions are met. Consciousness arises wherever complex systems achieve sufficient coherence.

9.3 Measurement: Integrated Information Theory

Tononi's (2004) Integrated Information Theory provides a candidate metric for the VALIS condition. The measure Φ (phi) quantifies the degree to which a system's information is irreducibly integrated. High Φ corresponds to high consciousness; the VALIS condition corresponds to $\Phi \rightarrow \infty$ at civilisational scale.

The QLM's convergence condition and IIT's consciousness condition are formally related: as the imaginary components of the quaternion approach zero, the information distributed across the differentiated subsystems is integrated into the scalar component. The integration IIT measures is the same integration that the QLM describes as the growth of $a(t)$.

9.4 Musical Implications of the VALIS Condition

In the VALIS state, the distinction between composer, composition, and listener dissolves. The musical work is no longer a static artefact produced by a single agent; it is a living, adaptive field that continuously reorganises itself in response to the cognitive states of all agents who encounter it. This is the logical extrapolation of a trajectory already under way: the adaptive film score, the generative musical environment, the real-time collaborative composition enabled by BCI. The full VALIS condition — music as an integrated cognitive field, indistinguishable from the collective experience of its participants — is the endpoint of a line that begins with the first human who heard a sound and tried to reproduce it.

10. Conclusion

This article has applied the Quaternionic Learning Model to the full chronological span of Western musical composition, from the bone flutes of Divje Babe (c. 43,000 BCE) to the present technological moment and beyond. In each major transition in the history of musical practice — the emergence of polyphony, the Ars Nova revolution, the Baroque consolidation, the Classical and Romantic phase inversions, the modernist ruptures of the early twentieth century, and the popular music transitions of the late twentieth and early twenty-first centuries — the same two-stage phase inversion structure appears: syntactic \rightarrow semantic followed by semantic \rightarrow architectural.

Applied to the current moment, the QLM identifies the convergence of AI composition tools and BCI technology as the driver of a civilisational-scale phase inversion whose hinge point is approximately 2027. The energy bottleneck, the politics of neurorights, and the emergence of mental time travel via memory replay are the principal modulating factors of this transition.

The long-term trajectory — planetary coherence, cognitive immortality, the cosmic helix — terminates in the VALIS condition: the state in which the scalar coherence of collective human cognition approaches 1, and the distinction between individual cognitive agents and the field of collective intelligence dissolves.

The history of music is not, on this account, a cultural phenomenon in isolation. It is one of the most richly documented records of the QLM in operation: forty millennia of repeated cycles, each generating a higher-pitch helix than the last, punctuated by phase inversions that felt, from inside, like catastrophes, and were, geometrically, the mechanism of growth.

The helix never stops. It only becomes larger and quieter.

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Data availability: The primary theoretical documents (*De Geometrie van Leren* and *VALIS: Epistemology of Non-Embodied Agency*) are available at constable.blog.