# The Height of the Tower

**Deriving Scale Separation, Tower Height, and the Human Level from the Vacuum.Net Base Model**

J. Konstapel & Claude (Anthropic), Leiden, 16-8-2026. Second edition.

## Revision note (edition 2)

The first edition was reviewed and four repairs were required. All four are made, and one open point is closed by calculation rather than left standing.

(i) The human level was stated as an equality, $R = 9q^m$. That does not follow. From "the last level that still fits" follows only an interval, and §7 now carries the interval. The tables give minimum values of R, not predicted ones.

(ii) The rest fraction $f = 1/3$ was used as though derived. It is empirical input. A second, unstated step sat underneath it: rest fraction was silently identified with sleep fraction. An equalisation phase is defined by $J \approx 0$, and whether sleep is that phase is a separate claim. Both are now listed as inputs.

(iii) "The human is not a chosen depth" was too strong. The finite depth is conditional on q, on the growth law, on Choice 2d, and on the inputs of (ii). §7 says so.

(iv) The growth law $C_{min,k} = 3q^k$ was an open point. It is now derived from the same contact counting that gives $\phi$, in §5. The result is still exponential, with the same base q and a shifted prefactor. The correction costs at most one level. Section 6 gains an exact form.

Two further items came out of the review and are taken up in §8: the internalisation fraction of a pair is made time-dependent, and the pair is given its own window condition. One recommendation was not taken — replacing the growth law by a free function g(C, q) — because a free growth function removes the height and the scaling law and leaves a framework that forbids nothing. The counting in §5 is the alternative.

The paper is self-contained. Every step and every number is in the text.

## 1. What is carried over

Only the minimum is repeated. The vacuum is one strand. Where the strand touches itself there is a contact. A winding is a cycle of segments and contacts that keeps coming back. The net is the graph of all of this; the meshes are its cycles. One scalar field $\chi$, the tension, lives on the strand, and its total is conserved. The only dynamics is redistribution of tension toward equalisation, along the strand and across contacts.

Four results of *The Human as Throughflow Closure* are used, and nothing else.

**(R1) Retention time.** A winding W with capacity $C_W$ and boundary conductances $\eta_b$ has one characteristic time

$$\tau_W = C_W / \Sigma_b \eta_b.$$

Capacity over boundary conductance. This holds for windings that have an inside — a large internal spectral gap.

**(R2) The closure lemma.** Closing constituents $W_i$ into one winding S internalises part of their boundary conductance. With $G \equiv \Sigma_i \Sigma_b \eta_{b,i} = \Sigma_i C_i/\tau_i$, and $\eta_{int}$ the internalised part,

$$\tau_S = \Sigma_i C_i / ( G - \eta_{int} ).$$

**(R3) The maintained pattern.** A closure re-lays its own contacts while current runs through it. Contacts decay at rate $\lambda$. Re-lay runs at rate $\mu J$ during load, drift out of the identity pattern at rate $\nu J$, repair at rate r during equalisation phases. Stationary:

$$f = \nu J / (\nu J + r), \quad C_W^*(J) = \mu r J / [ \lambda ( \nu J + r ) ].$$

**(R4) The window.** Closure requires $C_W^* \geq C_{min}(N)$, the smallest sub-realisation that still closes every defining cycle of the identity pattern. The constitutional condition is

$$\mu r > \lambda \nu C_{min}(N).$$

# 2. Scale separation is a count of boundary contacts

Let q constituents be closed into one winding. Each constituent carries b boundary contacts of comparable conductance $\eta_b$. Then $\tau_i = C_i/(b \eta_b)$, and $G = q b \eta_b$.

A closure lays c new contacts running over the $W_i$ as wholes. Each new contact consumes one boundary contact on each of two constituents and turns both into interior. So $\eta_{int} = 2c \eta_b$.

Define the closure density $\zeta \equiv c/q$, new contacts per constituent. The minimum that closes a cycle over q constituents is a ring, $c = q$, so $\zeta \geq 1$. The maximum is $c = qb/2$, where no boundary is left.

Define the internalisation fraction

$$\boldsymbol{\phi \equiv \eta_{int} / G = 2\zeta / b.}$$

Substituting into R2, with identical constituents so that $\Sigma_i C_i / G = \tau_i$:

$$\boldsymbol{\tau_S = \tau_i / (1 - \phi), \text{ hence } \sigma \equiv \tau_S/\tau_i = b / (b - 2\zeta).}$$

Three readings follow.

$\phi = 1$ means fully sealed: no boundary, no exchange, no throughflow. Retention becomes infinite and the closure is dead. So $b > 2\zeta$ is not a technical restriction. It is the statement that a living closure keeps part of its boundary open.

$\sigma > 1$ holds for every $\phi > 0$. Scale separation is not a condition to be imposed. It follows from closing anything at all.

$\sigma$ is a ratio of two small integers. Minimal ring closures:

| b | 3 | 4 | 5 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| $\sigma = b/(b-2)$, $\zeta = 1$ | 3 | 2 | 5/3 | 3/2 | 4/3 | 5/4 |

Denser closures give larger values: $b = 5$, $\zeta = 2$ gives $\sigma = 5$; $b = 6$, $\zeta = 2$ gives $\sigma = 3$.

# 3. Drift, and what falls out of the constitutional condition

**Choice 2d (drift scaling).** A re-lay is pattern-preserving when the replacement contact lands in the same chord interval, bounded by the neighbouring contacts of the pattern. A pattern with more contacts has shorter intervals, so the hit probability falls with pattern size. Write the hit probability as $w = C_{min}^{-a}$:

$$\nu = \mu \left( 1 - C_{min}^{-a} \right), \quad a = 1 \text{ or } 2.$$

$a = 1$ corresponds to one effective degree of freedom in the landing, $a = 2$ to two independent endpoints.

Substitute into R4:

$$\mu r > \lambda \mu \left( 1 - C_{min}^{-a} \right) C_{min}.$$

$\mu$ cancels. Define the repair-to-decay ratio $R \equiv r/\lambda$. Then

$$R > C_{min} - C_{min}^{1-a}.$$

For $a = 1$: $C_{min} < 1 + R$. For $a = 2$: $C_{min} < R + 1/C_{min}$. To leading order, in both cases,

$$\mathbf{C_{min,max} = R.}$$

The largest identity pattern a closure can hold equals its repair-to-decay ratio. The re-lay coefficient $\mu$, which looked like a material parameter of life, does not appear in the existence condition at all. This is a consequence of Choice 2d, not of R1–R4 alone.

The exponent $a$ is visible only at the smallest closures. At $C_{min} = 3$ the two versions require $R > 2$ and $R > 2{,}67$. So the smallest closure needs repair running roughly three times faster than decay.

# 4. The rest fraction is not free

From R3, write $x \equiv \nu J/r$, so $f = x/(1+x)$. Then

$$C_W^* = \mu J / [ \lambda (1 + x) ] = (\mu/\nu) \cdot (\nu J/r) \cdot (r/\lambda) / (1+x) = (\mu/\nu) \cdot R \cdot f.$$

With $\mu/\nu = 1/(1 - C_{min}^{-a})$, exactly:

$$C_W^* = R f / \left( 1 - C_{min}^{-a} \right).$$

For patterns of any size the correction factor is close to one, giving the clean form $C_W^* = R \cdot f$. Closure requires $C_W^* \geq C_{min}$, hence

$$\mathbf{f \geq C_{min} / R.}$$

Rest is not recovery from exertion and it is not a schedule. It is the fraction of the pattern that repair must reach per cycle. A larger pattern on the same repair capacity must rest more.

# 5. The growth law, derived

Each level closes q constituents with ζq new contacts. The identity pattern of the new level contains the q constituent patterns, plus the contacts that tie them:

**$C_{min,k+1} = q \cdot C_{min,k} + \zeta q$.**

This is the same counting that produced φ in §2. It is not a separate hypothesis.

Solve it. The fixed point is $C^* = -\zeta q/(q-1)$. Write $B \equiv \zeta q/(q-1)$ and $A \equiv C_{min,0} + B$. With the smallest stable winding $C_{min,0} = 3$ — the trefoil, three crossings:

**$C_{min,k} = A\, q^k - B$, $A = 3 + B$, $B = \zeta q/(q-1)$.**

For $\zeta = 1$:

| q | A | B | C_min,0..4 |
|---|---|---|---|
| 2 | 5,000 | 2,000 | 3, 8, 18, 38, 78 |
| 3 | 4,500 | 1,500 | 3, 12, 39, 120, 363 |
| 10 | 4,111 | 1,111 | 3, 40, 410, 4110, 41110 |

Growth is still exponential with base q. Only the prefactor shifts, from 3 to A. In levels the shift is $\log(A/3)/\log q$: 0,74 at $q = 2$; 0,37 at $q = 3$; 0,22 at $q = 5$; 0,14 at $q = 10$. The open point of edition 1 therefore costs at most one level.

# 6. The height of the tower

The tower stops at the last level whose pattern still fits under the bound of §4:

$A\, q^m - B \le R\, f$.

**$m = \lfloor \log( (R f + B) / A ) / \log q \rfloor$.**

With $f = 1/3$ and $\zeta = 1$. The value in brackets is the edition-1 result with the uncorrected growth law:

| $R = r/\lambda$ | q = 2 | q = 3 | q = 5 | q = 10 |
|---|---|---|---|---|
| $10^2$ | 2 (3) | 1 (2) | 1 (1) | 0 (1) |
| $10^3$ | 6 (6) | 3 (4) | 2 (2) | 1 (2) |
| $10^4$ | 9 (10) | 6 (6) | 4 (4) | 2 (3) |
| $10^6$ | 16 (16) | 10 (10) | 7 (7) | 4 (5) |

| | | | | |
|---|---|---|---|---|
| $10^9$ | 25 (26) | 16 (16) | 11 (11) | 7 (8) |
| $10^{12}$ | 35 (36) | 22 (23) | 15 (15) | 10 (11) |

The tower is finite in every case, and short. A repair-to-decay ratio of a million supports four to sixteen levels, depending on how many constituents each closure gathers.

Two properties of the shape. The height depends on R only through its logarithm, so it is insensitive to the exact value: a factor of a thousand in repair capacity buys three to ten extra levels. And the height falls as q rises. A closure that gathers many constituents at once climbs faster and stops sooner.

# 7. The consistency check

Two series were built from different countings. Pattern size grows as $C_{min,k} = A q^k - B$. Retention time grows as $\tau_k = \tau_0 \sigma^k$. Eliminate k. Because $q^k = (C_{min,k} + B)/A$, the relation is exact in the shifted variable:

$$\tau = \tau_0 \cdot \left[ (C_{min} + B) / A \right]^{( \log \sigma / \log q )}.$$

The exponent:

| b, ζ, q | σ | exponent |
|---|---|---|
| 3, 1, 3 | 3 | 1,000 |
| 4, 1, 2 | 2 | 1,000 |
| 4, 1, 4 | 2 | 0,500 |
| 3, 1, 10 | 3 | 0,477 |
| 5, 1, 5 | 5/3 | 0,317 |

One case gives exactly one. When $b = q = 3$ and $\zeta = 1$, retention time is proportional to pattern size plus a constant of 1,5. That is the case in which three is the only number in the model: three crossings in the smallest winding, three constituents per closure, three boundary contacts per constituent, one ring to close them. There

$$\tau_m / \tau_0 = ( C_{min,m} + 1{,}5 ) / 4{,}5.$$

With $m = 10$ this gives $\sigma^m \approx 5{,}9 \times 10^4$ and $C_{min} \approx 2{,}66 \times 10^5$.

# 8. The human level, as an interval

The human is the last level that still clears its own bound. That statement is conditional, and the conditions are named here rather than in the ledger alone. It holds given q, given the growth law of §5, given Choice 2d, and given the two inputs below.

**Input 1.** The rest fraction f is empirical. This paper uses f = 1/3.

**Input 2.** The identification of the equalisation phase with sleep. An equalisation phase is defined by J ≈ 0. Whether sleep is that phase is a separate claim, and it is what allows f to be read off a daily cycle at all. If rest is distributed differently — many short phases rather than one long one — f changes and every number below scales with it.

From C_min,m ≤ R f < C_min,m+1, with f = 1/3:

**3 C_min,m ≤ R < 3 C_min,m+1.**

So R is bracketed within a factor of roughly q, not fixed. The table gives the lower bound, which is what the earlier equality actually was:

| q | m | C_min, human | R ≥ | R < | τ_m/τ_0 at σ = 3 |
|---|---|---|---|---|---|
| 3 | 10 | $2{,}66 \times 10^5$ | $7{,}97 \times 10^5$ | $2{,}39 \times 10^6$ | $5{,}9 \times 10^4$ |
| 10 | 5 | $4{,}11 \times 10^5$ | $1{,}23 \times 10^6$ | $1{,}23 \times 10^7$ | $2{,}4 \times 10^2$ |
| 3 | 16 | $1{,}94 \times 10^8$ | $5{,}81 \times 10^8$ | $1{,}74 \times 10^9$ | $4{,}3 \times 10^7$ |

What the construction does fix: the pattern size and the retention time are locked to one another by §7, and the rest fraction and the pattern size are locked by §4. What remains free is the count — b, ζ, q, a — and the ratio R within its bracket.

Those are countable quantities, not fitted constants. Three model choices have been replaced by five things one could go and count, plus two named inputs.

# 9. Two closures

A second closure at the same depth is not a second object. Two closures can be constituents of one closure, by the same operator. Three things follow.

**The internalisation fraction is a state, not a constant.** For a pair the counting of §2 is not fixed by construction, because the number of shared contacts varies in time. Write it directly:

**ϕ_ij(t) = η_int,ij(t) / ( G_i + G_j ), τ_ij(t) = ( C_i + C_j ) / ( G_i + G_j − η_int,ij(t) ).**

Interaction is then the time course of η_int,ij. Part of what was boundary for each becomes interior for the pair. The measurable quantity is not one number but a curve.

**The pair has its own window.** Nothing guarantees that two closures form one. The condition of §4 applies unchanged at the higher level:

**R_ij f_ij ≥ C_min,ij.**

By the growth law of §5 with q = 2, C_min,ij = 2 C_min + 2ζ. So the pair must maintain twice a person's pattern plus its tie contacts. Since a person satisfies C_min = R f with f = 1/3, and f_ij ≤ 1:

$R_{ij} \geq 2\, C_{min} + 2\zeta \approx (2/3)\, R$.

A pair closes only if its joint repair-to-decay ratio reaches about two thirds of an individual's. Most pairings do not meet that. This predicts that closure between people is the exception, not the rule, and that where it holds it is costly — $f_{ij}$ must be large.

**Ordering.** With $\phi_{pair}$ small, $\tau_{pair}$ exceeds the members' retention time while $\varrho_{pair}$ stays below theirs. The pair is more slowly held and more weakly held at once. That is the content of the earlier claim that group closures are looser than their members, now with both quantities named.

What is not supplied here: which concrete interaction corresponds to which value of $\eta_{int,ij}$. The model gives the quantity and the condition. The mapping from attention, speech, joint action or entrainment onto $\eta_{int,ij}$ is measurement work, not derivation.

# 10. What this does to the tests

**T4 becomes sharp.** Adjacent tempi in one individual stand in the ratio $\sigma = b/(b - 2\zeta)$, with b and $\zeta$ small positive integers and $2\zeta < b$. So 3, 2, 5/3, 3/2, 5, 4/3 — not an arbitrary factor. A robust measured ratio of, say, 2,7 falsifies either the counting of §2 or the equal-conductance assumption behind it.

**T5 becomes a curve.** Measure $\phi_{ij}(t) = 1 - (C_i + C_j)/[(G_i + G_j)\, \tau_{ij}(t)]$ through an interaction, rather than one value at its end. The predicted signature is a rise during interaction and a decay after it, with the closure condition of §9 separating pairs that hold from pairs that do not.

**T6 (height).** The number of separable tempo levels in one individual is finite and equals m of §6. Counting levels, and independently measuring R from repair and decay rates, must give the same m within the bracket of §8. Levels found beyond that count falsify the constitutional bound of §3.

**T7 (rest scales with pattern).** From §4, $f \geq C_{min}/R$. Within one individual, deeper closures require larger rest fractions in proportion to pattern size. Measured rest fractions independent of pattern size falsify Choice 2d.

**T8 (growth law).** From §5, pattern size at adjacent levels satisfies $C_{min,k+1} = q\, C_{min,k} + \zeta q$, with the same q and $\zeta$ that set $\sigma$ in T4. So T4 and T8 must return consistent integers. Two levels whose tempo ratio implies one q and whose pattern ratio implies another falsify the counting of §2 and §5 together.

Failure map, extended: T6, T7 → Choice 2d. T8, T4 → the contact counting.

# 11. Status ledger

**Carried over.** R1–R4 from *The Human as Throughflow Closure*, edition 8.

**Derived.** $\phi = 2\zeta/b$ and $\sigma = b/(b - 2\zeta)$. The cancellation of $\mu$, and $C_{min,max} = R$, given Choice 2d. $C_W^* = Rf$ and $f \geq C_{min}/R$. The growth law $C_{min,k+1} = q\, C_{min,k} + \zeta q$ and its closed form. The height m. The exact scaling relation in $(C_{min} + B)$. The pair window $R_{ij}\, f_{ij} \geq C_{min,ij}$.

**Chosen.** Choice 2d, the drift scaling $\nu = \mu(1 - C_{min}^{-a})$. That constituents at one level carry comparable boundary conductance, which is what allows $G = qb\eta_b$. That every level uses the same q and $\zeta$.

**Empirical input.** $f = 1/3$. The identification of the equalisation phase with sleep. Both are §8, Inputs 1 and 2. Neither is derived here, and the human numbers scale directly with them.

**Removed.** Choice 3b, scale separation as an admissibility condition. It follows from $\phi > 0$.

**Open.** The values of b, $\zeta$, q for an actual closure. The exponent a, decidable only at the smallest closures. The form of N — the model does not yet distinguish two patterns of equal depth. The mapping from concrete interaction onto $\eta_{int,ij}$. And the operationalisation of T1–T8 precise enough that two independent researchers reach the same criteria.

# Annotated References


**Konstapel, J. & Claude (2026). *The Human as Throughflow Closure*, edition 8. constable.blog.** The paper this one continues. R1–R4 above are its Sections 2–4 in condensed form; its Choice 3b is what §2 removes. *Read first if the base model is unfamiliar; this paper assumes only its four results, not its full derivation.*

**Konstapel, J. (2026). *The Vacuum.Net Theory: Foundational Paper*. constable.blog.** The base model in full: one strand, contacts, windings, tension, and the single diffusion postulate. The source of $C_{min,0} = 3$, the trefoil as smallest closed pattern.

**Thomson, W. (Lord Kelvin) (1867). On Vortex Atoms. *Proc. Roy. Soc. Edinburgh* 6, 94–105.** Things as persistent windings in a medium rather than as particles in a container. The oldest statement of the picture this paper counts in.

**Faddeev, L. & Niemi, A. (1997). Stable knot-like structures in classical field theory. *Nature* 387, 58–61.** Knotted field configurations carrying a conserved topological charge. Why the pattern of §2 is discrete and therefore countable at all. *Read for the discreteness, not for the field equations.*

**Williamson, J.G. & van der Mark, M.B. (1997). Is the electron a photon with toroidal topology? *Ann. Fond. L. de Broglie* 22, 133.** The smallest thing as a closed circulation rather than an object. The physical anchor for the level $W^0$ from which the tower is counted.

**Onsager, L. (1931). Reciprocal relations in irreversible processes. *Phys. Rev.* 37, 405; de Groot, S.R. & Mazur, P. (1962). *Non-Equilibrium Thermodynamics*. North-Holland.** Redistribution toward equalisation in its standing form. The background against which R1 is an ordinary result rather than a novelty.

**Prigogine, I. & Nicolis, G. (1977). *Self-Organization in Nonequilibrium Systems*. Wiley.** Order that exists only while current runs through it. The nearest existing statement of §4, without the identity pattern and therefore without a bound on pattern size.

**Kuramoto, Y. (1984). *Chemical Oscillations, Waves, and Turbulence*. Springer. Read with Dumas et al. (2010), Inter-brain synchronization during social interaction, *PLoS ONE* 5, e12166.** Coupled closures and their joint slow modes, and the measurement side of §9. *Read Kuramoto for the mathematics of $\phi_{ij}$ and Dumas for what a two-person recording actually delivers; the mapping between them is the open point named at the end of §9.*

**Jacobson, T. (1995). Thermodynamics of spacetime. *Phys. Rev. Lett.* 75, 1260.** The demonstration that a fixed background can be replaced by the thermodynamics of a medium. The same move, one level up from where this paper works.

**Bak, P., Tang, C. & Wiesenfeld, K. (1987). Self-organized criticality. *Phys. Rev. Lett.* 59, 381.** Loading and discharge with thresholds as a general dynamic. Useful for reading the four states of the base model as a state-plane trajectory rather than as separate regimes.

**Konstapel, J. (2026). *How to Visualize the Vacuum*. constable.blog.** The seven pictures of the net, including the mesh, the tension, and the mending. The imagery this paper counts in: b is how many strands leave a mesh, ζ is how many new knots the closure ties.