The Missing Weight and the Medium

What the Vacuum Model Says About Dark Matter — A Solution Direction

J. Konstapel · Leiden — companion piece to The Vacuum as Weather *(3rd ed., 29-7-2026)*

I. The question, turned around

The dark matter problem, stripped of its name, is this: on galactic scales and above, there is roughly five times more gravity than there are identifiable sources. The mainstream response has been to postulate a new substance and hunt for its particle. Forty years of increasingly sensitive detectors have returned null results throughout.

The vacuum model suggests turning the question around. If gravity is not a fundamental force but the elastic response of the vacuum medium to deformation — Sakharov's induced gravity, the founding assumption of our framework — then "missing gravity" is not evidence of missing *stuff*. It is evidence that **the medium's response is not everywhere the same.** The question shifts from *what invisible substance has been added to space?* to *under what conditions does the medium respond more strongly than the laboratory calibration predicts?*

That is not a rhetorical move. It relocates the problem into exactly the machinery the vacuum model has already built: a nonlinear condensate with an internally generated length, a response that depends on context, and a single readable knob at the bottom. This essay works out what that machinery says, what already exists in the literature that says it quantitatively, what our framework adds that the literature lacks, and — in the discipline the third edition established — what would settle it.

II. Three anchors any solution must hit

A solution direction is only worth writing down if it is aimed at the sharpest available data. Three empirical facts constrain everything.

Anchor 1: One acceleration, not one mass. The tightest regularity in galactic dynamics is the radial acceleration relation: the observed acceleration in disk galaxies is a function of the acceleration the visible matter alone would produce — a single curve, with remarkably little scatter, across hundreds of galaxies of every size and type (McGaugh, Lelli & Schombert, 2016). The extra gravity switches on below one critical acceleration:

$\mathbf{a_0 \approx 1.2 \times 10^{-10}\ m/s^2}$

And this number is not anonymous. Take the Hubble rate $H_0 \approx 70$ km/s/Mpc $= 2.27 \times 10^{-18}\ s^{-1}$, multiply by c and divide by 2π:

$c \cdot H_0/2\pi = (3.0 \times 10^8 \times 2.27 \times 10^{-18})/6.28 \approx \mathbf{1.1 \times 10^{-10}\ m/s^2}$

Before reading anything into that number, state the physics that requires it — the order matters, because without the physics the comparison is numerology. A medium whose largest standing wave has wavelength $\lambda_H = c/H$ is not in a strain-free ground state: the mode itself is a deformation of the medium. Follow what that deformation does. A wave of wavelength λ_H propagating at speed c imposes velocity gradients of order $c/\lambda_H = H$ on everything embedded in it — the Hubble rate, read as the shear of the largest mode. Anything riding the medium therefore accumulates velocity changes of order c over one mode period λ_H/c, which sets the characteristic acceleration scale of the background: $a \sim c^2/\lambda_H = cH$, with an order-unity geometric factor (2π for a full wavelength). This is not one acceleration among many; it is the strain rate of the medium itself, and no embedded dynamics can be defined independently of it. Any local dynamics *slower* than this background strain cannot be treated as motion against a quiet medium; the medium's response to it is necessarily conditioned by the largest mode. A medium picture therefore *demands* a transition acceleration of order $cH/2\pi$ before a single galaxy has been measured. Only then read the number: the threshold at which galaxies stop obeying laboratory gravity coincides, to within measurement, with exactly that scale. In a particle picture this is a coincidence requiring elaborate tuning. In the medium picture it is the expected signature of top-down conditioning — Tomes's orientation appearing in the data, the largest scale shaping the behaviour of every scale beneath it.

Note also the structural form: a *threshold* separating two response regimes. That is the same architecture as the Zebiak–Cane coupling μ_c — the signature form of the tension–release machine that runs at every layer of the hierarchy.

Anchor 2: Cores with a fixed length. Dwarf galaxies do not show the sharp central density cusps that particle-based simulations predict. They show cores — flat central regions of a definite size, of order a kiloparsec. The Gross–Pitaevskii–Poisson fit to DDO 168 (2026) gives a solitonic core radius of 2.4 kpc. A core of fixed size is the fingerprint of a medium with an internal length — the astrophysical analogue of the healing length, as §VII of the third edition argued.

Anchor 3: Clusters behave differently. In galaxy clusters — the Bullet Cluster above all — the lensing mass is displaced from the visible gas, and the simple one-acceleration law of Anchor 1 under-predicts the discrepancy. Whatever produces the extra response in galaxies behaves, in clusters, more like a component that can separate from the baryons. A pure modification of the force law fails here; a pure particle fails at Anchors 1 and 2. The solution must be capable of *both behaviours*, in different contexts.

Three anchors; and note that the third demands context-dependence explicitly. The data themselves are saying: the medium does not respond the same way everywhere.

III. The solution direction: the medium's own collective mode

Here is the proposal in one paragraph. **Dark matter is not a new particle added to the vacuum. It is the vacuum's own long-wavelength collective mode — the medium doing, at galactic scale, what every condensate does: carrying a coherent excitation whose effective mass and healing length are set not by a new constant of nature but by the local context of the medium.**

Unpack the three claims inside that.

It is a collective mode, not a constituent. In the vacuum model, the constituent scale of the medium terminates at the proton (the smallest stable knot, $\bar{\lambda}_p \approx 0.21$ fm). But every medium supports excitations far below its constituent energy scale — a phonon in a crystal has an effective mass tens of orders of magnitude below the atoms that carry it, and it is no less physical for that.

Compute the scale gap here. A galactic core of ~1 kpc, read as a collective healing length $\xi = \hbar/(\sqrt{2}\cdot m\_eff\cdot c)$, corresponds to an effective mass:

$$m\_eff = \hbar/(\sqrt{2} \cdot \xi \cdot c) \approx 1.055\times10^{-34} / (1.41 \times 3.1\times10^{19} \times 3\times10^{8}) \approx 8\times10^{-63}\ \text{kg} \approx \mathbf{5\times10^{-27}\ eV/c^2}$$

(If the relevant length is de Broglie-type rather than Compton-type, with galactic velocities $v \sim 10^{-3}c$, the mass scale rises to the $\sim10^{-22}$–10^{-23} eV of the "fuzzy dark matter" literature.) Either way: the mode sits **30 to 35 orders of magnitude below the proton.** The ratio m_p/m_eff is comparable to the ratio between an atom and the softest phonon in a macroscopic crystal. Dark matter, on this reading, is to the vacuum what sound is to a solid.

Its scale is contextual, not constant. Here the framework departs from every existing condensate model — and the departure is the testable content. Fuzzy dark matter and axion models take the boson mass m as a new universal constant, one more number added to the list. Fractal Context Dynamics (axiom A4) holds that the medium's parameters are state functions of the local vacuum: $L(s) = \Phi(T|s)$. Applied here: the effective mass of the collective mode — and hence the core radius it produces — is set by the local tension of the medium, not fixed once for the whole universe. The Scale-Invariant Vacuum programme of Maeder and Gueorguiev arrives at the same doorstep from the opposite side: its own equations make scale effects grow as density falls — which is exactly the MOND phenomenology of Anchor 1 — but its conformal factor is forbidden by construction to vary in space, and its authors themselves name that spatial dependence as the open direction. **The context principle is precisely the missing piece: SIV supplies the density-dependence of the response; FCD supplies the licence for it to vary from place to place.** Two programmes, each holding half of the mechanism.

Both behaviours of Anchor 3 come out of one medium. This is not conjecture; the machinery exists. Berezhiani and Khoury (2015) constructed a model in which the dark component is a superfluid inside galaxies — where its phonons couple to baryons and mediate exactly the one-acceleration force law of Anchor 1 — while in the hotter, more disturbed environment of clusters the superfluid phase is destroyed and the same medium behaves as ordinary separable matter, which is what the Bullet Cluster shows. Read through our framework, their model is a worked demonstration of the central claim: *one medium, two response regimes, with a threshold between them set by context*. The tension–release architecture again — and the phase boundary of the superfluid plays the role of μ_c.

What our framework adds to Berezhiani–Khoury is the same thing it adds to fuzzy dark matter: their superfluid is a new substance with new constants; ours proposes that the superfluidity is a property of the vacuum itself, with its parameters anchored — like everything else in the medium — to the single knob. Which yields the strongest and strangest prediction of the set.

Adjacent programmes, and where this one differs. Three other lines of work treat space itself as the responding object, and the family resemblance should be stated rather than discovered by a referee. Verlinde's emergent gravity (2016) derives an extra gravitational response of order cH_0 from the elastic back-reaction of the dark-energy medium to the presence of matter — the closest neighbour to the present proposal, and the same top-down logic; it differs in deriving the response thermodynamically from entanglement entropy rather than from a condensate's collective mode, and in carrying no context-dependence of the medium's parameters. Einstein-Æther theories equip spacetime with a dynamical preferred frame — a medium with structure — but promote its couplings to new universal constants, which is precisely the move the single-knob argument forbids. Blanchet's dipolar dark matter models the medium as gravitationally polarizable, recovering MOND phenomenology as a polarization effect; again the mechanism is medium-response, and again the medium's susceptibility is a fixed constant. The present direction sits in this family and is distinguished from every member of it by the same two commitments: no parameter beyond α, and contextual rather than universal medium response. Those two commitments are what predictions 2 and 3 below test.

IV. The single-knob corollary

Section IV of the third edition established that everything an internal observer can read off the vacuum compresses to one dimensionless number, α, and that any local structure or tension in the medium can only manifest as a local variation of α. There is no second channel.

Apply that here, and the proposal becomes falsifiable in a way no particle model is. **If the galactic response of the medium is contextual, then the context is real physical structure in the vacuum — and real structure in the vacuum must express itself, however faintly, as spatial structure in α.** The prediction is not that α varies wildly; it is that the *pattern* of any measured $\Delta\alpha/\alpha$ across the sky should correlate with the acceleration environment — with where systems sit relative to the a_0 threshold — and not simply with gravitational potential. (The distinction matters: FCD already forbids a gravitationally-explained dipole; a context-correlated pattern is a different object, keyed to the medium's regime boundary rather than to potential wells.)

This also explains, rather than excuses, the terrestrial null results. An Earth laboratory sits at accelerations ten orders of magnitude above a_0 — deep in the medium's stiff, calibrated regime. On this model, as on SIV's own account, **the laboratory is structurally the worst place in the universe to look.** The null results of optical-clock networks ($|\delta\alpha/\alpha| < 5\times10^{-17}$) are predicted, not embarrassing. The signal, if it exists, lives where the medium is soft: in quasar absorption systems sorted by their host environment, and in the outskirts of galaxies.

V. The constitutive equation: g(χ)

Everything in this essay funnels through one function, and it deserves its own section rather than a passing mention: the dependence of the medium's interaction strength on its local state, **$g(\chi)$**. This is the vacuum's constitutive equation — its analogue of Hooke's law for a solid or the equation of state for a gas — and once it is known, everything else in the proposal follows mechanically: the healing length $\xi(\chi)$, the sound speed of the collective mode, its effective mass m_eff(χ), the transition at a_0, the α-pattern of §IV, and the environment-dependence of cores. One function, six phenomena.

Define the variable first. χ is the dimensionless state of the medium at a point: local dynamics measured against the medium's own background strain. For the galactic problem the operational definition is

$$\boldsymbol{\chi \equiv a / (cH)}$$

— the local gravitational acceleration in units of the strain rate of the largest mode (§II). Dimensional analysis motivates the choice before any data do: a point in the medium exposes exactly one local dynamical quantity, the acceleration a, and the background mode exposes exactly one scale, cH; the only independent dimensionless measure of local dynamics relative to the background is their ratio. The data then confirm what dimensions suggest. The radial acceleration relation keys, empirically, to acceleration and not to gravitational potential: systems at equal potential but different internal acceleration behave differently. Sharper still is the external-field effect: at fixed internal properties, isolated galaxies deviate more strongly from Newtonian behaviour than galaxies embedded in a strong external field — a signature claimed detected in the SPARC rotation-curve sample (Chae et al., 2020). For particle dark matter, a galaxy's internal dynamics depending on the acceleration of its *surroundings* is close to inexplicable; for a contextual medium it is the defining behaviour. (The FCD benchmark for strong-field α-tests used a potential-type variable, $|\Phi|/c^2$; whether that variable and $\chi = a/(cH)$ are two limits of one state function is an open point the framework must eventually close, and it is flagged here rather than papered over.)

The function is already pinned in three places.

- *$\chi \gg 1$ — the stiff regime*. Laboratory and solar system sit ten orders of magnitude above the threshold. Here g must be indistinguishable from a constant: the collective mode is heavy and short-ranged, and the bounds are quantitative — optical clocks ($|\delta\alpha/\alpha| < 5\times10^{-17}$) and solar-system ephemerides. This is the calibrated vacuum of the third edition.
- *$\chi \sim 1$ — the crossover.* Here is the point that turns the constitutive equation from a postulate into a measured object: the radial acceleration relation, traced by hundreds of galaxies through the transition region, **is the image of g(χ) through its crossover.** The curve exists, with error bars, in published data. It has been photographed without being recognized as a constitutive law of the medium.
- *$\chi \ll 1$ — the soft regime*. In the outskirts of dwarfs the healing length reaches kiloparsecs; core radii (DDO 168's 2.4 kpc among them) measure g at low tension.

Closure (step 5 of §VII) is then a sharp question: does a *single monotonic* g(χ) pass through all three regimes? If yes, the model stands and its parameters are over-determined. If no interpolation works, the direction fails — which is what a constitutive equation is for.

The single-knob consistency condition. One constraint elevates g(χ) above curve-fitting. Section IV established that the medium carries exactly one readable degree of freedom. The constitutive equation may therefore not be a second knob: it must factor *through* the knob. The ultimate form is not g(χ) but

$$\mathbf{g = g(\alpha(\chi))}$$

— context sets the local value of the one readable parameter, and that parameter sets the medium's interaction. Nothing couples to context directly; everything couples through α. On this form the FCD benchmark relation $\Delta\alpha/\alpha = \beta\cdot\Delta\chi$ is not a separate hypothesis but the inner function of the composition, read in the weak-field limit — while the rotation curves trace the outer function through its crossover. One composed function, read at one end by atomic clocks and at the other by galaxy kinematics. Deriving that function, rather than fitting it, is the programme's central task — and the reason it is *the* task is that every quantity in this essay is downstream of it.

VI. What this direction predicts

Following the discipline of the third edition: a framework earns its keep by forbidding things.

1. No particle will ever be found. Not "the particle is hard to detect" — *there is no particle*. A collective mode of the medium does not scatter in an underground xenon tank, for the same reason one cannot catch a sound wave in a butterfly net. A confirmed direct detection of a dark matter particle falsifies this entire direction cleanly. Four decades of null results are consistent with this interpretation.

2. Core radii are contextual, not universal. Fuzzy dark matter, with its universal boson mass, predicts a tight core-radius–halo-mass relation with no environmental residue. The context model predicts a measurable *residual dependence on environment* at fixed halo mass: isolated dwarfs and dwarfs in dense surroundings should show systematically different cores. Rotation-curve samples (SPARC and its successors) can test this now. DDO 168 is one point; the claim lives in the population.

3. The threshold tracks the cosmos. If a_0 is anchored to the medium's largest standing wave — $a_0 \approx cH/2\pi$ — then it is not a constant: it should follow H(z), running higher in the early universe. High-redshift rotation curves should show the anomalous response switching on at systematically higher accelerations than local galaxies do. Particle dark matter predicts no such drift; a universal-constant MOND predicts none either. This is the cleanest discriminator on the list, and JWST-era kinematics of $z \approx 2$ disks is beginning to reach it.

4. Any α-structure correlates with acceleration environment. As §IV: the pattern of $\Delta\alpha/\alpha$, if any survives the systematics debate around the Webb data, must key to the a_0 regime boundary. A confirmed α-variation correlating with *potential* rather than with *acceleration environment* would falsify the specific mechanism proposed here (and is independently forbidden in FCD).

VII. What is honestly open

Three problems, stated without decoration.

The CMB. The acoustic peaks of the cosmic microwave background are the particle model's strongest evidence: their pattern fits a universe in which a non-interacting component was already gravitating, as a fluid, at recombination. Any medium-response model must reproduce that, and "the medium responded differently then" is not a calculation. Berezhiani–Khoury inherit standard cosmology at early times precisely by *being* a substance at high temperature; the pure-vacuum reading has not yet earned that. This is the largest open front.

The effective mass has no derivation — but the derivation has a definite shape. The 10^{-27}–10^{-22} eV scale of the collective mode is, at present, read off the observed core sizes — fitted, not derived. The framework's own standard applies: this number must come out of the medium's dynamics or the proposal remains descriptive. What can be stated now is the exact chain the derivation must run, because it is standard condensate machinery at every step but one:

1. *Vacuum equation*. Write the medium as an order parameter Ψ obeying a Gross–Pitaevskii-type nonlinear equation, with one non-standard ingredient: the interaction strength is a state function of the local vacuum tension, $g = g(\chi)$, with $\chi = a/(cH)$ as defined in §V — not a universal constant.
2. *Linearization*. Expand around the local ground state. This is the Bogoliubov step, textbook material (Pitaevskii & Stringari, ch. 4).
3. *Dispersion relation*. The linearized modes obey the standard Bogoliubov spectrum, $\omega^2 = c^2k^2(1 + \xi^2k^2)$, with $\xi^2 = \hbar^2/(2\, m^2\, c^2)$ set by $g(\chi)$. Long wavelengths propagate as sound in the medium; short wavelengths as free constituents — the two-regime signature yet again.
4. *Effective mass*. The small-k sector of that spectrum defines the collective mode's effective mass, $m_{eff} \propto \hbar/(\xi\, c)$, which inherits the context-dependence of $g(\chi)$: a stiff medium (laboratory, high tension) carries a heavy, short-ranged mode; a soft medium (galactic outskirts, low tension) a light, kiloparsec-ranged one.
5. *Closure*. One function, $g(\chi)$, then carries the whole model, and it is already constrained at two points — the laboratory stiffness (where the mode must be invisible) and the galactic core scale (where it must produce kiloparsec healing lengths). Predictions 2 and 3 test whether a single monotonic $g(\chi)$ can pass through both.

Steps 2–4 are mechanical. The physics is entirely in step 1: what fixes $g(\chi)$? That is not a new question — it is the question the whole programme already rests on, *what fixes α and what makes the medium nonlinear*, wearing its galactic face. The structure of the missing calculation is familiar from the other end of the ladder too: the knot literature fixes topology but not scale, the condensate fixes scale but not topology, and the answer must come from the combination. The same missing step as "why four," one layer up. It is probably the *same* calculation.

The stripping test. The third edition's inverse renormalization says the vacuum is what remains when organization is removed. The collective-mode reading adds a subtlety: the galactic dark response would itself be *organization in the medium* — a layer, not the zero layer. That is consistent (the zero layer is the medium's ground state; the mode is its excitation) but it means the ladder of §VIII acquires an entry between "atom" and "star" that was not there before: the medium's own galactic-scale coherence. Whether that entry earns its place is exactly what predictions 2 and 3 decide.

VIII. Where this leaves the reader — and the programme

For the readers of the blog series: the "missing weight" stops being a hunt for invisible stuff and becomes a materials question about space itself. The same medium whose stiffness sets the size of atoms and the limits of ASML's machines responds, in the thin outer regions of galaxies where accelerations fall below one ten-billionth of a metre per second squared, more strongly than its laboratory calibration — and that changed response *is* the phenomenon the press calls dark matter.

For the programme: the direction is attractive precisely because it purchases nothing new. No new particle, no new constant, no new force — only the two properties the framework already rests on: that the vacuum is a nonlinear medium, and that its parameters are contextual. The price of that economy is steep and specific: predictions 1–4 above, each of which can fail. The particle hunters have spent forty years unable to lose their bet. This one can be lost within a decade — which is, by the standard this programme has set for itself, its strongest recommendation.

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