The Oscillating Vacuum Model: A Unified Framework Derived from Maxwell's Quaternion Electrodynamics and Rowlands' Nilpotent Constraint

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Abstract

We present the Oscillating Vacuum Model (OVM), a theoretical framework in which the quantum vacuum is an active oscillatory medium structured by the biquaternion algebra $\mathbb{C} \otimes \mathbb{H} \cong \mathrm{Cl}(3,1)$. The model unifies two historically independent structures sharing this algebra: Maxwell's original quaternion electrodynamics (1865/1873) and Rowlands' nilpotent universal computational rewrite system (NUCRS). In the BRST-extended formulation of QED, the Nakanishi–Lautrup auxiliary field B, which is BRST-invariant ($\delta_B B = 0$), is identified with the vacuum dual $\bar{\Psi} = -(ikE + \mathbf{i}p + jm)$ of Rowlands' nilpotent formalism via an explicit algebraic map $\mathcal{M}: \bar{\Psi} \to B$ preserving ghost number, cohomological class, and conserved charge. Both are BRST-closed, non-exact structures in $\mathrm{Cl}(3,1)$.

The BRST doublet decoupling theorem, which would render B cohomologically trivial, fails in asymmetric Casimir cavities: the Robin boundary condition on B at a conductor–dielectric interface is incompatible with the BRST image of that condition on the Faddeev–Popov ghost, breaking global nilpotency of the BRST charge at the boundary. The resulting incomplete quartet cancellation generates a residual B-field zero-point energy with the functional form

$$\frac{\delta F}{F_\mathrm{QED}} = \alpha_B \cdot \frac{\kappa - 1}{\kappa + 1} \cdot \left(\frac{d}{\lambda_e}\right)^{-1},$$

where κ is the dielectric constant, d the plate separation, λ_e the electron Compton wavelength, and $\alpha_B \sim \alpha$ the fine-structure constant. This is Prediction 1, falsifiable at 5×10^{-4} precision over separations 50–500 nm. A second quantum prediction concerns algebraic relationships among Higgs Yukawa coupling constants, contingent on a mass-hierarchy calculation identified as the framework's principal open problem.

An explicit dynamical operator $\Phi = \Phi_2 \circ \Phi_1$ bridges the quantum algebraic structure to macroscopic dissipative systems via Lindblad–GKSL extension and renormalization group coarse-graining. Under Born–Markov coupling via energy-diagonal jump operators, the quantum regression theorem guarantees preservation of the nilpotent resonance spectrum in the steady-state power spectral density. Irreversibility emerges structurally from the non-injectivity of the coarse-graining map. An Arnold-tongue selection principle with base frequency $\omega_\odot$, width function $K^{\max(p,q)}\omega_\odot$, and quality-factor cutoff identifies which vacuum resonances survive at macroscopic scales, generating two further falsifiable predictions: dominant paleoclimate periodicities cluster near rational multiples of the solar forcing frequency with Arnold-

tongue tolerances, and the critical renewable penetration threshold for power grid desynchronisation satisfies a parameter-free Kuramoto-derived formula.

1. Introduction

Maxwell's 1865 formulation of electrodynamics in Hamilton's quaternion algebra $\mathbb{H}$ encodes the electromagnetic four-potential as $Q = \phi + \mathbf{A}$, with scalar and vector parts on equal algebraic footing. Heaviside's 1880s vector reformulation imposed the Lorenz gauge $\partial_\mu A^\mu = 0$, eliminating the scalar combination $S \equiv \partial_t\phi + \nabla\cdot\mathbf{A}$ as a kinematic constraint. In the modern BRST-extended formulation of QED, the mechanism by which the scalar sector decouples from physical amplitudes is the Kugo–Ojima quartet: the scalar photon, longitudinal photon, ghost c, and anti-ghost $\bar{c}$ form a quartet that is BRST-exact and therefore trivial in the physical cohomology $H^0(\delta_B)$. This is not a loss of physics in flat unbounded space.

The BRST-extended Lagrangian necessarily introduces the Nakanishi–Lautrup auxiliary field B via $\mathcal{L}\mathrm{gf} = B(\partial^\mu A\mu) + \frac{\xi}{2}B^2$. The BRST transformation satisfies $\delta_B\bar{c} = B$, $\delta_B B = 0$: the pair $(\bar{c}, B)$ forms a doublet. In unbounded Minkowski space the doublet decoupling theorem (Kugo–Ojima) guarantees that B contributes trivially to cohomology — it is BRST-closed but effectively BRST-exact within the doublet. However, this theorem relies on the global nilpotency $Q_B^2 = 0$ of the BRST charge as an operator on the full Hilbert space.

Independently, Rowlands' nilpotent quantum mechanics (1994–2007) defines a vacuum dual $\bar{\Psi} = -(ikE + \mathbf{i}p + jm)$ for every fermion state $\hat{\Psi} = (ikE + \mathbf{i}p + jm)$ within $\mathrm{Cl}(3,1)$. This dual satisfies $\hat{\Psi} + \bar{\Psi} = 0$ and encodes non-locality, Berry phase, and the thermodynamic arrow of time algebraically. Both B and $\bar{\Psi}$ are BRST-closed, ghost-number-zero structures in $\mathrm{Cl}(3,1)$.

This paper's central claim is that B and $\bar{\Psi}$ are the same algebraic object, and that the geometry where this identification becomes physically observable — an asymmetric Casimir cavity with one conductor and one dielectric plate — is precisely the geometry where the doublet theorem fails. Section 2 establishes the algebra and the identification. Section 3 proves the boundary-induced BRST incompleteness and derives the B-field Casimir correction. Section 4 states the quantum predictions. Section 5 constructs the macro dynamical operator Φ. Section 6 states the macroscopic predictions. Appendix A provides the full BRST analysis; Appendix B the algebraic map $\mathcal{M}$.

2. Mathematical Framework

2.1 The Shared Algebra $\mathrm{Cl}(3,1)$

Hamilton's quaternion algebra $\mathbb{H}$ has basis ${1, \mathbf{i}, \mathbf{j}, \mathbf{k}}$ with $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$. The complexified algebra $\mathbb{C} \otimes \mathbb{H}$ is isomorphic to the Clifford algebra $\mathrm{Cl}(3,1)$ of Minkowski spacetime, with generators satisfying $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$. Both Maxwell's four-potential $Q = \phi + \mathbf{A}$ and Rowlands' creation operator $\hat{\Psi} = (ikE + \mathbf{i}p + jm)$ are elements of $\mathrm{Cl}(3,1)$. This is the necessary algebraic condition for comparison; it does not by itself imply physical unification.

The quaternion differential operator $\nabla_\mathbb{H} = \partial_t + \mathbf{i}\partial_x + \mathbf{j}\partial_y + \mathbf{k}\partial_z$ acts on Q to yield

$$\nabla_\mathbb{H} Q = \underbrace{(\partial_t\phi - \nabla\cdot\mathbf{A})}{S-} + \underbrace{(-\nabla\phi - \partial_t\mathbf{A})}{\mathbf{E}} + \underbrace{(\nabla\times\mathbf{A})}{\mathbf{B}_\mathrm{mag}},$$

where S_- is a Lorentz scalar. The Lorenz gauge sets $S \equiv \partial_t\phi + \nabla\cdot\mathbf{A} = 0$.

2.2 BRST Structure and the Nakanishi–Lautrup Field

The BRST-extended QED Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + B(\partial^\mu A_\mu) + \frac{\xi}{2}B^2 + \bar{c}\Box c,$$

with transformations $\delta_B A_\mu = \partial_\mu c$, $\delta_B c = 0$, $\delta_B\bar{c} = B$, $\delta_B B = 0$. The BRST operator is nilpotent: $\delta_B^2 = 0$, the field-theoretic analogue of $\hat{\Psi}^2 = 0$.

Physical observables are elements of the cohomology $H^0(\delta_B)$. The scalar combination $\partial^\mu A_\mu = \delta_B\bar{c}$ is BRST-exact and therefore trivial. The field B satisfies $\delta_B B = 0$ and forms a doublet $(\bar{c}, B)$ with $\delta_B\bar{c} = B$. The doublet decoupling theorem states that in **unbounded** spacetime, any BRST-closed quantity built from a doublet pair is cohomologically trivial. In bounded geometries this theorem fails, as established in Section 3.

2.3 Rowlands' Nilpotent Operator

The fermionic state is $\hat{\Psi} = (ikE + \mathbf{i}p + jm)$ with $\hat{\Psi}^2 = -E^2 + p^2 + m^2 = 0$, yielding the mass-shell relation. The vacuum dual $\bar{\Psi} = -(ikE + \mathbf{i}p + jm)$ satisfies $\hat{\Psi} + \bar{\Psi} = 0$. It is BRST-closed in the sense $(\hat{\Psi} + \bar{\Psi})^2 = 0$ with $\bar{\Psi} \neq 0$ individually. The NUCRS formalises an irreversible process conforming to all three laws of thermodynamics, encoding the arrow of time as an algebraic consequence of nilpotency (Rowlands & Marcer, 2010).

2.4 The Core Identification and the Map $\mathcal{M}$

Axiom OVM. *The Nakanishi–Lautrup field B and the vacuum dual $\bar{\Psi}$ are two representations of the same BRST-closed, non-exact structure in $\mathrm{Cl}(3,1)$ in bounded geometries where the doublet decoupling theorem fails.*

The explicit algebraic map $\mathcal{M}: \bar{\Psi} \to B$ is defined by

$$\mathcal{M}(\bar{\Psi}) = -\frac{1}{2\hbar\omega_0}\left(\bar{\Psi} + \bar{\Psi}^\dagger\right) \cdot\sqrt{\varepsilon_0\mathcal{E}_\mathrm{vac}},$$

where $\omega_0 = mc^2/\hbar$ is the electron Compton frequency and $\mathcal{E}_\mathrm{vac}$ the vacuum energy density. This map preserves:

- **Ghost number**: both B and $\bar{\Psi}$ carry ghost number 0
- **Cohomological class**: $\mathcal{M}$ sends BRST-closed elements to BRST-closed elements
- **Conserved charge**: the zero-totality $\mathcal{E}{\hat{\Psi}} + \mathcal{E}{\bar{\Psi}} = 0$ maps to $\mathcal{E}_\mathrm{transverse} + \mathcal{E}B = \mathcal{E}\mathrm{total}$, the statement that total Casimir energy (transverse plus B-sector) equals the regularised zero-point energy of the cavity

A full Hilbert-space isomorphism requires specifying the operator representation of $\hat{\Psi}$ on $\mathcal{H}_\mathrm{vac}$, which is deferred. The map $\mathcal{M}$ is a structural correspondence sufficient to motivate the physical identification and the Casimir prediction.

3. The B-Field Casimir Contribution

3.1 Failure of the Doublet Theorem at Asymmetric Boundaries

The doublet decoupling theorem requires global nilpotency $Q_B^2 = 0$ as an operator on the full cavity Hilbert space. In a Casimir geometry with a conductor plate at $z = 0$ and a dielectric plate at $z = d$, this fails by the following argument.

For a conductor at $z = 0$, the physical boundary condition is Dirichlet: $A_i|{z=0} = 0$ for tangential components $i = 1,2$. The BRST transformation $\delta_B A_i = \partial_i c$ maps this to the ghost condition $\partial_i c|{z=0} = 0$, and the Nakanishi–Lautrup equation of motion $B = -\partial^\mu A_\mu/\xi$ with the Dirichlet condition gives $B|{z=0} = 0$ (Dirichlet on B). The doublet transformation $\delta_B\bar{c} = B$ then requires $\bar{c}|{z=0}$ to satisfy Dirichlet as well.

For a dielectric at $z = d$ with permittivity κ, the physical boundary condition on the tangential electric field components is Robin: varying the gauge-fixed action with respect to B at the boundary yields (see Section 3.2)

$$\partial_z B\big|{z=d} + \beta B\big|{z=d} = 0, \qquad \beta = \frac{\kappa - 1}{\kappa\lambda_e},$$

where λ_e is the Compton wavelength setting the material response scale. The BRST transformation of this Robin condition is:

$$\delta_B\!\left(\partial_z B + \beta B\right)\big|_{z=d} = \partial_z(\delta_B B) + \beta(\delta_B B) = 0$$

identically, since $\delta_B B = 0$. This means the BRST transformation of the B-field Robin condition is trivially satisfied — but the Robin condition on $\bar{c}$, obtained from $\delta_B\bar{c} = B$ applied at the boundary, requires $\bar{c}|_{z=d}$ to satisfy

$$\partial_z\bar{c}\big|{z=d} + \beta\bar{c}\big|{z=d} = \text{(source from B-Robin mode)},$$

which is a **different** Robin parameter from the Dirichlet condition inherited from the conductor plate at $z = 0$. The ghost field c and anti-ghost $\bar{c}$ therefore satisfy boundary conditions on the two plates that are incompatible under the doublet transformation $(\bar{c}, B) \to (B, 0)$.

The eigenvalue spectra of B and $\bar{c}$ in the cavity are distinct, and their zero-point energy contributions do not cancel.

Proposition 1 (Boundary BRST Incompleteness). *In an asymmetric Casimir cavity with conductor ($z = 0$, Dirichlet) and dielectric ($z = d$, Robin with $\beta \neq 0$), the BRST doublet cancellation between the B-field and ghost sectors is incomplete. The residual zero-point energy is:*

$$\mathcal{E}_\mathrm{res}(\kappa, d) = \mathcal{E}B(\kappa, d) - \mathcal{E}\mathrm{ghost}(\kappa, d) \neq 0 \quad \text{for } \kappa \neq \infty.$$

In the symmetric limit $\kappa \to \infty$ (conductor on both plates), $\beta \to 0$ and the cancellation is restored: $\mathcal{E}_\mathrm{res} \to 0$.

This result is consistent with the recent literature on BRST-extended Casimir calculations (Dudal et al., arXiv:2412.04270; Stouten et al., arXiv:2605.10662), which demonstrates that asymmetric boundary conditions in general require new boundary fields to restore BRST invariance, and that these boundary fields modify the Casimir spectrum.

3.2 Derivation of the Robin Boundary Condition

Varying the gauge-fixed action $S = \int_\mathcal{C} d^4x,[\ldots + B(\partial^\mu A_\mu) + \frac{\xi}{2}B^2]$ with respect to B at the boundary $z = d$ yields the boundary term

$$\delta S_\partial = \int_{\partial\mathcal{C}} d^3x, \delta B \cdot n^\mu A_\mu\bigl_{z=d}.$$

For this to vanish independently of δB, the dielectric boundary condition on the normal component of the displacement field $D^z = \kappa(\partial_z A_0 - \partial_0 A_z)$ gives, after eliminating A_0 via the Lorenz gauge:

$$n^\mu\partial_\mu B\bigl{z=d} + \frac{\kappa - 1}{\kappa\lambda_e} B\bigl{z=d} = 0.$$

This is the Robin condition with $\beta = (\kappa-1)/(\kappa\lambda_e)$. It vanishes for $\kappa = 1$ (vacuum, no interface effect) and approaches a pure Neumann condition for $\kappa \to \infty$ (perfect conductor). The asymmetry between the two plates is fully captured by the parameter β.

3.3 Mode Spectrum and Force Correction

The B-field eigenvalue equation in the cavity with Dirichlet at $z = 0$ and Robin at $z = d$ is:

$$k_n\cos(k_n d) + \beta\sin(k_n d) = 0, \qquad k_n > 0.$$

For $\beta = 0$ this gives $k_n = (n + \tfrac{1}{2})\pi/d$ (standard Casimir-like spectrum); for $\beta \neq 0$ the modes are shifted by $\delta k_n \sim \beta d/(1 + \beta^2 d^2)$ per mode. The ghost field satisfies a distinct spectrum with a different effective Robin parameter $\tilde{\beta} \neq \beta$ (arising from the doublet transformation incompatibility of Section 3.1), so the residual energy

$$\mathcal{E}\mathrm{res}(\kappa, d) = \frac{\hbar c}{2}\sum_n\left[\sqrt{k_n^2 + k\perp^2} - \sqrt{\tilde{k}n^2 + k\perp^2}\right]$$

is non-zero for $\kappa \neq \infty$. After zeta-function regularisation (the explicit evaluation of which determines α_B), the leading-order contribution to the force is

$$\frac{\delta F_B}{F_\mathrm{QED}} = \alpha_B \cdot \frac{\kappa - 1}{\kappa + 1} \cdot \left(\frac{d}{\lambda_e}\right)^{-1} + O!\left[\left(\frac{d}{\lambda_e}\right)^{-2}\right],$$

where α_B is a dimensionless constant of order α set by the ratio $(\beta - \tilde{\beta})/k_1$ at $d \sim \lambda_e$. The factor $(\kappa-1)/(\kappa+1)$ follows from the Robin spectrum asymptotics and is independent of α_B.

Outstanding calculation. The explicit evaluation of α_B via mode-sum zeta-function regularisation is the principal remaining step to fully close Prediction 1. The functional form $(\kappa-1)/(\kappa+1)\cdot d^{-1}$ and the falsification criterion are independent of this value.

4. Quantum-Scale Predictions

4.1 Prediction 1: Anomalous Casimir Force in Conductor–Dielectric Geometries

Statement. The Casimir force in asymmetric conductor–dielectric cavities deviates from the QED prediction by

$$\frac{\delta F}{F_\mathrm{QED}} = \alpha_B \cdot \frac{\kappa - 1}{\kappa + 1} \cdot \left(\frac{d}{\lambda_e}\right)^{-1},$$

with $\alpha_B \sim \alpha \approx 1/137$. The correction vanishes identically in symmetric conductor–conductor geometries ($\kappa = \infty$ on both plates) and grows monotonically with κ. This functional dependence on $(\kappa-1)/(\kappa+1)$ is absent from all known systematic QED corrections (finite conductivity, surface roughness, thermal effects), which depend on d and material properties through different functional forms.

Experimental protocol. Measure Casimir forces in conductor–dielectric geometries (e.g. gold sphere against a dielectric flat) over separations 50–500 nm with $\kappa \in {1, 2, 4, 7, 10}$. After subtraction of finite-conductivity (Drude model) and roughness corrections, test for a residual systematic with the form $(\kappa-1)/(\kappa+1)\cdot d^{-1}$ beyond the QED Lifshitz baseline. Map the coefficient α_B from the joint (κ, d)-dependence and compare against α.

Falsification. No systematic $(\kappa-1)/(\kappa+1)$-dependent deviation beyond 5×10^{-4} across separations 50–500 nm after all known QED corrections are subtracted.

4.2 Prediction 2: Algebraic Relationships Among Higgs Yukawa Couplings

Statement. If fermion masses arise from the harmonic resonance spectrum of $\mathrm{Cl}(3,1)$, the ratios of squared Yukawa couplings $\lambda_f^2 = 2m_f^2/v^2$ satisfy

$$\frac{\lambda_f^2}{\lambda_{f'}^2} = \frac{n_f}{n_{f'}}, \qquad n_f, n_{f'} \in \mathbb{Z},$$

where n_f labels resonance modes. The observed ratios $m_e : m_\mu : m_\tau \approx 1 : 207 : 3477$ are not satisfied for low-order integers, indicating either multi-mode interference or failure of the prediction. The mass-hierarchy calculation — identifying ${n_f}$ from the $\mathrm{Cl}(3,1)$ resonance structure while accounting for chirality, CKM mixing, renormalisation running, and electroweak symmetry breaking — is the principal open calculation of the OVM at the particle physics level.

Falsification. No integer assignment ${n_f}$ reproduces the observed mass ratios to within 1% for the first 100 resonance modes including two-mode interference.

5. The Dynamical Bridge: Construction of Φ

The category mismatch between $\mathrm{Cl}(3,1)$ (linear, invertible, Hamiltonian) and macroscopic dissipative systems (non-invertible, stochastic) is bridged by $\Phi = \Phi_2 \circ \Phi_1$.

5.1 Step 1: Lindblad Extension and Spectrum Preservation

The nilpotent operator $\hat{\Psi}$ generates the Hamiltonian $\hat{H}_\Psi = \sum_n \hbar\omega_n \hat{a}_n^\dagger\hat{a}_n$. Coupling to an environment via the GKSL master equation:

$$\frac{d\rho}{dt} = \mathcal{L}[\rho] = -\frac{i}{\hbar}[\hat{H}*Psi, \rho] + \sum*\alpha! \left(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}{L_\alpha^\dagger L_\alpha, \rho}\right).$$

The physically meaningful claim is not the trivially vacuous $[L_\alpha, \hat{\Psi}^2] = [L_\alpha, 0] = 0$. The non-trivial result is:

Proposition 2 (Spectrum Preservation). *Let the jump operators be energy-diagonal: $[L_\alpha, \hat{H}\Psi] = \lambda\alpha L_\alpha$ for real λ_α (the standard Born–Markov assumption). Then by the quantum regression theorem, the steady-state power spectral density $S(\omega) = \int e^{i\omega t}\langle\hat{A}(t)\hat{A}^\dagger(0)\rangle_\mathrm{ss}dt$ has peaks at the nilpotent resonance frequencies ω_n, broadened but not shifted to leading order. The rational ratios ω_n/ω_m are preserved in the power spectrum.*

The first mapping is $\Phi_1: \mathrm{Cl}(3,1) \to \mathcal{L}(\mathcal{H}*mathrm{vac} \otimes\mathcal{H}*\mathcal{E})$, adding dissipation while preserving ${\omega_n}$.

5.2 Step 2: RG Coarse-Graining and Emergent Irreversibility

A renormalization group coarse-graining map $\mathcal{C}_b$ at blocking scale b gives $ \mathcal{L}_b = b^{-z}\mathcal{C}_b\mathcal{L}\mathcal{C}_b^{-1}$. Modes with scaling dimension $y_n > 0$ (relevant) survive; those with $y_n < 0$ (irrelevant) are suppressed.

The category shift follows from a structural theorem (Kobayashi, 2025): when microscopic time evolution forms a group but the induced macroscopic evolution under a non-injective coarse-graining map forms only a semigroup, macroscopic time-reversal symmetry is necessarily broken. Since $\mathcal{C}_b$ is non-injective by construction, the macroscopic dynamics is automatically irreversible — irreversibility is a structural consequence of information loss, not an additional postulate.

The second mapping is $\Phi_2: \mathcal{L}(\mathcal{H}*mathrm{vac}) \to \mathcal{D}* \mathrm{macro}(\mathbf{x},t)$.

5.3 Step 3: Arnold-Tongue Selection Principle

For a system driven at frequency $\omega_\odot$, a vacuum resonance mode ω_n is macroscopically observable if it lies within an Arnold tongue:

$$|\omega_n - \omega_\odot\cdot p/q| < K^{\max(p,q)}\omega_\odot, \qquad Q_n = \omega_n/\gamma_n > Q_\mathrm{min},$$

where p/q is a low-order rational, K the effective coupling, and γ_n the mode dissipation rate.

Proposition 3 (Selection Principle). *The macroscopically observable vacuum resonances are precisely those satisfying the above Arnold-tongue condition. This is not spectral curve-fitting: the base frequency ($\omega_\odot$), tolerance function ($K^{\max(p,q)}\omega_\odot$), and integer range (bounded by Q_min) are all determined by measured system parameters, not fitted to observations.*

The full mapping is $\Phi = \Phi_2\circ\Phi_1: \mathrm{Cl}(3,1)\to\mathcal{D}_\mathrm{macro}$, with properties: (i) rational period ratios preserved when the RG fixed point has discrete scaling symmetry; (ii) irreversibility structural via Kobayashi; (iii) thermodynamic consistency via NUCRS and Lindblad trace-preservation; (iv) reduction to quantum vacuum in zero-coupling limit.

5.4 Open Calculations

Three calculations remain to fully close the macro predictions: (a) explicit RG scaling dimensions y_n for climate and grid systems; (b) the value of Q_min for the Earth's climate from observed dissipation rates; (c) whether the RG fixed point has discrete scaling symmetry.

6. Macroscopic Predictions

6.1 Prediction 3: Climate Periodicities Near Solar Forcing Harmonics

Statement. Dominant paleoclimate periodicities cluster near rational multiples of $\omega_\odot$ (Schwabe cycle, $T \approx 11$ yr) with widths $K^{\max(p,q)}\omega_\odot$ at solar–climate coupling $K \sim 0.1$. Pairwise period ratios of dominant spectral peaks satisfy

$$\frac{T_i}{T_j} = \frac{p_i/q_i}{p_j/q_j} + \delta_{ij}, \qquad |\delta_{ij}| < K^{\max(p_i,q_i)},\quad p, q \leq 12.$$

This differs from the generic claim that "climate cycles look rational": it specifies the base frequency ($\omega_\odot$, not arbitrary), the tolerance function (Arnold tongue width, not a free parameter), and the integer range.

Protocol. Morlet wavelet transform of EPICA Dome C record (800 kyr, $\omega_0 = 6$); KS test of clustering near $\omega_\odot$-multiples against red-noise null hypothesis.

Falsification. Dominant climate periodicities cluster near arbitrary rationals but not specifically near rational multiples of $\omega_\odot$.

6.2 Prediction 4: Grid Desynchronisation Threshold

Statement. The critical renewable penetration at which the transverse Lyapunov exponent changes sign is

$$f_R^* = \frac{2\gamma_\mathrm{grid}}{\pi\,g(\omega_\mathrm{grid})} \cdot \frac{\sigma_\mathrm{grid}^2}{\sigma_\mathrm{met}^2},$$

derived from the Kuramoto bifurcation condition $K_\mathrm{eff}(f_R^*) = K_c$ with $K_\mathrm{eff}(f_R) = f_R\cdot\sigma_\mathrm{met}^2/\sigma_\mathrm{grid}^2$ and $K_c = 2\gamma_\mathrm{grid}/[\pi g(\omega_\mathrm{grid})]$. All quantities are dimensionless and measurable from ENTSO-E and meteorological records. For representative European grid values ($\gamma_\mathrm{grid} \approx 0.2$ rad/s, $g(\omega_\mathrm{grid}) \approx 10$ (rad/s)$^{-1}$, $\sigma_\mathrm{met}/\sigma_\mathrm{grid} \approx 0.3$), the formula gives $f_R^* \approx 0.14$, consistent with early stability concerns at 15–20% renewable penetration.

Protocol. ENTSO-E grid frequency time series (2015–2025); Rosenstein-algorithm Lyapunov exponent estimation ($m = 10$) as function of instantaneous renewable fraction; test for sign change at predicted f_R^*.

Falsification. Observed threshold deviates from predicted f_R^* by more than 10% after substituting measured inputs.

7. Relationship to Existing Work

BRST / Nakanishi–Lautrup theory. The OVM's contribution is the identification of the boundary-localized B-field zero-point energy with the nilpotent vacuum dual, motivated by the failure of the doublet theorem in asymmetric Casimir geometries (Stouten et al. 2026; Dudal et al. 2024).

Dressed photon theory (Ojima, Kyoto). The Nakanishi–Lautrup formalism identifies that unphysical longitudinal and scalar modes, while absent as free particles, appear physically in non-particle forms: as infrared Coulomb tails and macroscopic condensate wave functions. The OVM Casimir prediction is a related phenomenon: boundary-confined B-field modes that are unphysical as free particles but contribute to vacuum energy in confined geometries.

Stochastic electrodynamics (Boyer 1975). SED retains the zero-point field as a classical stochastic background. The OVM structures it deterministically via the nilpotent constraint; QED is recovered in the limit $B \to 0$.

Open quantum systems / Kuramoto. Prediction 4 derives from standard Kuramoto bifurcation theory. The OVM adds the identification of f_R^* with the Arnold-tongue structure of the climate resonance spectrum.

Scale relativity (Nottale 2011). Both frameworks predict hierarchical resonance; the OVM derives it algebraically, Nottale geometrically.

8. Discussion

The OVM has a four-layer epistemic structure:

Layer 1 (algebraic): The identification $B \leftrightarrow \bar{\Psi}$ via $\mathcal{M}$ in $\mathrm{Cl}(3,1)$ is a structural correspondence preserving ghost number, cohomological class, and conserved charge. It is not refuted by the doublet theorem, which applies to the bulk theory where global BRST nilpotency holds.

Layer 2 (quantum prediction 1): The Casimir correction follows from the boundary-induced BRST incompleteness of Section 3. Its functional form $(\kappa-1)/(\kappa+1)\cdot d^{-1}$ is determined by Robin spectrum asymptotics, independently of α_B. The outstanding step is the zeta-function computation of α_B.

Layer 3 (quantum prediction 2): The Yukawa relation is explicitly contingent on the mass-hierarchy calculation, named as the framework's principal open problem.

Layer 4 (macro predictions): Predictions 3 and 4 follow from Φ with three identified remaining calculations. Prediction 4 is parameter-free in its threshold formula.

The framework is falsified at the quantum level if the asymmetric Casimir correction is not observed: this eliminates the B-field identification and removes the physical basis for all subsequent structure. Confirmation would elevate the macro predictions from physically grounded conjectures to open empirical questions.

9. Conclusions

The OVM identifies the Nakanishi–Lautrup B-field in BRST-extended QED with the nilpotent vacuum dual $\bar{\Psi}$ of Rowlands' formalism via an explicit map $\mathcal{M}$ in $\mathrm{Cl}(3,1)$. The BRST doublet decoupling theorem, which would render this identification physically trivial, fails in asymmetric conductor–dielectric Casimir cavities because the Robin boundary condition at the dielectric plate is incompatible with the BRST transformation of the doublet, breaking global BRST nilpotency at the boundary. The resulting residual B-field zero-point energy produces a Casimir force correction with the specific form $(\kappa-1)/(\kappa+1)\cdot d^{-1}$, falsifiable at 5×10^{-4} precision. A second prediction concerns Higgs Yukawa coupling relations. An explicit three-step dynamical operator Φ, using Lindblad extension, RG coarse-graining, and Arnold-tongue selection, bridges the quantum algebraic structure to macroscopic dissipative systems. Spectrum preservation follows from the quantum regression theorem; irreversibility follows from the non-injectivity of coarse-graining; the selection principle specifies which vacuum resonances survive macroscopically without free parameters. Two macroscopic predictions are stated quantitatively with explicit falsification criteria.

Methods

Casimir: $\hbar = 1.055\times10^{-34}$ J·s, $c = 2.998\times10^8$ m/s, $\varepsilon_0 = 8.854\times10^{-12}$ F/m, $\lambda_e = 2.426\times10^{-12}$ m. Yukawa: $\lambda_f = m_f\sqrt{2}/v$, $v = 246.22$ GeV. Wavelet: Morlet, $\omega_0 = 6$. Lyapunov: Rosenstein, $m = 10$, τ at mutual-information minimum. Arnold tongue: $\Delta_{p/q}(K) = K^{\max(p,q)} \omega_\odot$, $K = 0.1$. Grid: $\gamma_\mathrm{grid} \approx 0.2$ rad/s, $g(\omega_\mathrm{grid}) \approx 10$ (rad/s)$^{-1}$, $\sigma_\mathrm{met}/\sigma_\mathrm{grid} \approx 0.3$ from ENTSO-E (2025).

Appendix A: BRST Analysis in Bounded Geometries

The doublet decoupling theorem (Kugo–Ojima 1979) requires a proof in three steps: (i) the physical Hilbert space is the cohomology of Q_B; (ii) any state in a BRST doublet (u, Qu) has zero inner product with all physical states; (iii) therefore doublet states decouple from all S-matrix elements. Step (ii) requires $Q_B^2 = 0$ globally. In a Casimir cavity, $Q_B^2 \neq 0$ as a global operator because the Robin boundary condition at $z = d$ is not preserved under δ_B: applying δ_B to the B-field Robin condition $\partial_z B + \beta B|_{z=d} = 0$ gives $0 = 0$ trivially, but applying δ_B to the associated $\bar{c}$ boundary condition requires a first-order Robin condition on $\bar{c}$ that contradicts the Dirichlet condition inherited from the conductor plate via $\delta_B\bar{c} = B$. The incompatibility is:

$$\text{Conductor plate: } \bar{c}|_{z=0} = 0 \quad \text{(Dirichlet)},$$ $$\text{Dielectric plate: } \delta_B\bar{c}|_{z=d} = B|_{z=d} \neq 0 \quad \text{(Robin mode)}.$$

A single function $\bar{c}$ cannot simultaneously satisfy Dirichlet at $z = 0$ and have a non-zero Robin-mode value at $z = d$ unless the Robin mode has zero projection onto the Dirichlet-compatible sector — which is generically false for $\kappa \neq \infty$. This is the precise statement of global BRST incompleteness. The approach of Stouten et al. (2026) for DEM boundary conditions introduces new edge fields to restore BRST invariance; the OVM predicts instead that this incompleteness is the physical signal, not an artefact to be repaired.

Appendix B: The Algebraic Map $\mathcal{M}$

The map $\mathcal{M}(\bar{\Psi}) = -\frac{1}{2\hbar\omega_0}(\bar{\Psi} + \bar{\Psi}^\dagger) \sqrt{\varepsilon_0\mathcal{E}_\mathrm{vac}}$ sends the anti-Hermitian $\bar{\Psi} \in \mathrm{Cl}(3,1)$ to a real scalar field B satisfying:

1. $\mathcal{M}(\bar{\Psi})^2 = B^2 \geq 0$, consistent with $(\hat{\Psi}+\bar{\Psi})^2 = 0 \Rightarrow$ the complement carries positive energy
2. $\delta_B[\mathcal{M}(\bar{\Psi})] = 0$ since $\delta_B B = 0$ by construction
3. $\mathcal{E}_B = \varepsilon_0 B^2/2 = \mathcal{E}_{\bar{\Psi}} = \hbar\omega_0/2$ at $d = \lambda_e$, enforcing the zero-totality energy conservation $\mathcal{E}_{\hat{\Psi}} + \mathcal{E}_{\bar{\Psi}} = 0$ as $\mathcal{E}_\mathrm{transverse} + \mathcal{E}_B = \mathcal{E}_\mathrm{total}$

A full Hilbert-space isomorphism would require specifying a coherent-state or semiclassical identification between the operator algebra of $\hat{\Psi}$ on $\mathcal{H}_\mathrm{vac}$ and the B-field Fock space. This is an open problem whose resolution would strengthen the identification from a structural correspondence to a rigorous isomorphism.

References

1. Maxwell, J.C. (1865). A dynamical theory of the electromagnetic field. *Phil. Trans. R. Soc. Lond.* **155**, 459–512.
2. Maxwell, J.C. (1873). *A Treatise on Electricity and Magnetism*. Clarendon Press, Oxford.
3. Rowlands, P. (2007). *Zero to Infinity: The Foundations of Physics*. World Scientific.
4. Rowlands, P. & Marcer, P. (2017). Nilpotent quantum mechanics: Analogs and applications. *Front. Phys.* **5**, 28.
5. Rowlands, P. (2010). Physical interpretations of nilpotent quantum mechanics. *arXiv:1004.1523*.
6. Marcer, P. & Rowlands, P. (2010). The grammatical universe and the laws of thermodynamics. *AIP Conf. Proc.* **1303**, 161–167.
7. Heaviside, O. (1893). *Electromagnetic Theory*, Vol. 1. The Electrician, London.

8. Becchi, C., Rouet, A. & Stora, R. (1976). Renormalization of gauge theories. *Ann. Phys.* **98**, 287–321.
9. Kugo, T. & Ojima, I. (1979). Local covariant operator formalism of non-Abelian gauge theories. *Prog. Theor. Phys. Suppl.* **66**, 1–130.
10. Nakanishi, N. (1966). Covariant quantization of the electromagnetic field. *Prog. Theor. Phys.* **35**, 1111–1116.
11. Lautrup, B. (1967). On the divergence structure of the photon self-energy. *Mat. Fys. Medd. Dan. Vid. Selsk.* **35**, 1.
12. Ojima, I. Nakanishi–Lautrup B-field, crossed product and duality. *RIMS Kokyuroku* **1524**, Kyoto University.
13. Moss, I.G. & Silva, P.J. (1996). BRST invariant boundary conditions for gauge theories. *arXiv:gr-qc/9610023*.
14. Dudal, D. et al. (2024). Dynamical edge modes in Maxwell theory from a BRST perspective, with an application to the Casimir energy. *arXiv:2412.04270*.
15. Stouten, S. et al. (2026). On the Casimir effect with mixed dynamical edge mode and perfect electromagnetic conducting boundary conditions. *arXiv:2605.10662*.
16. Casimir, H.B.G. (1948). On the attraction between two perfectly conducting plates. *Proc. Kon. Ned. Akad. Wetensch.* **51**, 793–795.
17. Klimchitskaya, G.L., Mohideen, U. & Mostepanenko, V.M. (2009). The Casimir force between real materials. *Rev. Mod. Phys.* **81**, 1827–1885.
18. Saharian, A.A. (2000). Casimir effect for scalar fields under Robin boundary conditions on plates. *arXiv:hep-th/0007242*.
19. Gorini, V., Kossakowski, A. & Sudarshan, E.C.G. (1976). Completely positive dynamical semigroups. *J. Math. Phys.* **17**, 821.
20. Lindblad, G. (1976). On the generators of quantum dynamical semigroups. *Commun. Math. Phys.* **48**, 119.
21. Kobayashi, S. (2025). From three-particle dynamics to the structural origin of the arrow of time. *arXiv:2511.00131*.
22. Hotz, R. & Schaller, G. (2021). Coarse-graining master equation for periodically driven systems. *arXiv:2102.03063*.
23. Verstraete, F. et al. (2005). Renormalization group transformations on quantum states. *arXiv:quant-ph/0410227*.
24. Arnol'd, V.I. (1983). *Geometrical Methods in the Theory of Ordinary Differential Equations*. Springer.
25. Acharyya, S. & Amritkar, R.E. (2011). Desynchronization bifurcation. *arXiv:1101.3130*.
26. Kuramoto, Y. (1984). *Chemical Oscillations, Waves, and Turbulence*. Springer.
27. Boyer, T.H. (1975). Random electrodynamics. *Phys. Rev. D* **11**, 790–808.
28. Nottale, L. (2011). *Scale Relativity and Fractal Space-Time*. Imperial College Press.
29. Holling, C.S. (2001). Understanding the complexity of economic, ecological, and social systems. *Ecosystems* **4**, 390–405.
30. Jack, P.M. (2003). Physical space as a quaternion structure. *arXiv:math-ph/0307038*.
31. Jouzel, J. et al. (2007). Orbital and millennial Antarctic climate variability over the past 800,000 years. *Science* **317**, 793–796.
32. ENTSO-E Transparency Platform (2025). transparency.entsoe.eu.
33. Rosenstein, M.T., Collins, J.J. & De Luca, C.J. (1993). A practical method for calculating largest Lyapunov exponents. *Physica D* **65**, 117–134.
34. Cattaneo, A.S., Mnev, P. & Reshetikhin, N. (2015). Perturbative quantum gauge theories on manifolds with boundary. *arXiv:1507.01221*.