

The Standard Reference of the Maze

From the First Distinction to E8, with the Occupants of Every Ring

J. Konstapel, Constable Research, Leiden, 28-8-2026

© 2026 J. Konstapel, Constable Research, Leiden. All rights reserved. Quotation with attribution is permitted; reproduction of the whole requires written permission.

1. Purpose and rules of this document

This document is self-contained. Every definition, every rule, every table and every number needed to fill the maze with concepts stands in this text. Nothing requires an external list. All counts and identities in this document have been computed and verified in exact arithmetic; the verification list stands in Section 10.

Status discipline, used throughout: **counted** = an exact, checkable equality; **choice** = an assignment made by rule, stated as such; **read** = an interpretation placed on a count; **open** = not yet demonstrated.

2. The maze

Trit. A turn: +1, 0 or -1. Written +, 0, -.

Route. A finite sequence of trits, deepest turn first.

Value. Route $t(d-1)\dots t(1)t(0)$ has value $\sum t(i) \cdot 3^i$. Weights: 1, 3, 9, 27, 81, ...

Ring d. All routes of depth d: 3^d rooms. Values run from $-(3^d-1)/2$ to $+(3^d-1)/2$. Ring ceilings: 1, 4, 13, 40, 121, 364, ...

Address. Ring plus value. The empty route is rest: address 0.

Mirror rule. The mirror of an address is its negative value. Everywhere in this document, mirror = reversal: of direction, of parity, of matter.

Theorem (unique address). Every integer has exactly one route. *Existence:* divide by 3; remainder 0 or 1 is the trit; remainder 2 gives trit - with carry $(n+1)/3$; the value shrinks, so the process stops. *Uniqueness:* two different routes with equal value differ first at some depth; the difference there is ± 1 or ± 2 times a power of 3, all deeper terms are multiples of the next power; such a sum is never zero. ■

Hand calculation: $43 \rightarrow$ remainders +, -, -, -, + $\rightarrow$ route +---+ = $81-27-9-3+1 = 43$.

Addition. Position by position: (+)(+) = write -, carry +; (-)(-) = write +, carry -; all other pairs write their sum, no carry. Hand calculation: $43+1$: lowest + + + $\rightarrow$ -, carry; next - + + $\rightarrow$ 0; result + --0- = 44.

The first distinction. Ring 0 is rest. Ring 1 holds three rooms: +1 (the mark), −1 (its mirror), 0 (the boundary). The first distinction gives two rooms; the maze keeps the boundary as a third. Everything below grows from these three.

3. The zero-free principle

This one rule places every binary classification system exactly, with no margins.

Theorem (zero-free rooms). On ring d , the routes that use only + and − number exactly 2^d . The routes that carry at least one 0 number $3^d - 2^d$. *Proof:* each of the d positions independently takes 2 or 3 values. ■

The counts, verified:

Ring d	Rooms 3^d	Zero-free 2^d	Zero-carrying $3^d - 2^d$
1	3	2	1
2	9	4	5
3	27	8	19
4	81	16	65
5	243	32	211
6	729	64	665
8	6.561	256	6.305

Counted: every binary system of d signs occupies exactly the 2^d zero-free rooms of ring d . The binary world is the zero-free sub-maze. Its share $(2/3)^d$ shrinks with depth: the zero wins.

Counted, twice: the zero-carrying counts of rings 3 and 4 are 19 and 65 — and $19 = 3^3 - 2^3$, $65 = 3^4 - 2^4$. The maze depth 19 and the convergent numerator 65 are both the difference between the two towers, read at one ring. The companion identities hold too: $12 = (3^3 - 3)/2$ and $41 = (3^4 + 1)/2$. The convergent pairs (12, 19) and (41, 65) of $\log_2 3 = [1; 1, 1, 2, 2, 3, \dots]$ — the ladder behind twelve-tone and 41-tone closure, with the Pythagorean comma $3^{12}/2^{19} = 531.441/524.288$ at the first rung — live on rings 3 and 4 as plain counts. **Read:** the comma of Pythagoras, carried to infinity, is the growth of the zero.

4. The ladder of symmetries, rung by rung

Each family occupies its own wing. Within a wing, addresses follow the rules below. Three rungs leave exactly three rooms open, and those are always the constant routes: pure +, pure −, rest.

Rung 0 — rest. Address 0, ring 0.

Rung 1 — first distinction. $Z/2$. Ring 1: +1 and −1.

Rung 2 — the trit. $Z/3$. Ring 1 full: −1, 0, +1. The register itself.

Rung 3 — the triangle, S_3 (6 elements). Verified: order 6, closed, non-commutative, 3 even + 3 odd. Address rule (choice): sign = parity. Ring 2:

Value	Route	Permutation
1	0	identity
2	+-	rotation
3	0	rotation
-1	0	reflection
-2	-+	reflection
-3	0	reflection

Open rooms: ++ (4), 00 (0), -- (-4): the three constants.

Rung 4-5 — the four worldviews and the centre (Paths of Change). Two independent distinctions give the Klein group $\mathbb{Z}/2 \times \mathbb{Z}/2$ (verified: every element self-inverse). Ring 2, own wing (choice):

Value	Route	Concept
4	++	Unity (knowledge)
2	+-	Sensory
-2	-+	Mythic
-4	--	Social
1	0	centre up (observer, P)
-1	0	centre down (in-the-world, J)

Rung 6-7 — the 24 dyads. Six unordered pairs $\times$ two directions $\times$ two centre stands = 24. Address rule (choice): value = pair number (1-6) + 6 if stand is down; sign = direction. Verified: the 24 values are exactly $\pm 1 \dots \pm 12$, and mirror address = reversed direction. Ring 3, in full:

Pair	up A→B	up B→A	down A→B	down B→A
U-S	+1 (00+)	-1 (00-)	+7 (+++)	-7 (---)
U-M	+2 (0+-)	-2 (0-+)	+8 (+0-)	-8 (-0+)
U-So	+3 (0+0)	-3 (0-0)	+9 (+00)	-9 (-00)
S-M	+4 (0++)	-4 (0--)	+10 (+0+)	-10 (-0-)
S-So	+5 (++-)	-5 (-+)	+11 (+++)	-11 (---)
M-So	+6 (+-0)	-6 (-+0)	+12 (+++0)	-12 (---0)

Open rooms: +++ (13), 000 (0), --- (-13).

Counted identities of the 24. The 24 Hurwitz unit quaternions ($\pm 1, \pm i, \pm j, \pm k$ and $(\pm 1 \pm i \pm j \pm k)/2$) are 24, all of norm 1, closed, with all inverses (verified). They form a 24-cell: every vertex has 8 neighbours at 60° (verified). The 24 roots of D_4 ($\pm e_i \pm e_j, i < j \leq 4$) form the same shape: identical angle spectrum (verified). **Correction, from the verification itself:** the unit group is the binary tetrahedral group $2T$, not S_4 — $2T$ has one element of order 2, S_4 has nine (counted). The identity dyads—units—24-cell— D_4 holds as set and shape, not as group operation.

Rung 8 — triality. The Cartan matrix of D_4 (simple roots $e_1 - e_2, e_2 - e_3, e_3 - e_4, e_3 + e_4$) admits exactly 6 symmetries fixing the central node: S_3 on the three outer nodes (verified). Triality lives in the three open rooms of ring 3 (choice): the three axes it permutes are the three constants.

Rung 9 — F_4 (48). The 24 long roots ($\pm e_i \pm e_j$, norm² 2) plus 24 short (8 of type $\pm e_i$, 16 of type $(\pm \frac{1}{2})^4$, norm² 1). Verified: 48, two lengths in ratio $\sqrt{2}$, crystallographic. Concepts (choice): long = the 24 movements (dyads, by the rung-7 table); short = the 24 states: 8 pure stances $U_\pm, S_\pm, M_\pm, So_\pm$ and 16 full profiles (a sign on all four at once). F_4 = movement plus state, 24 + 24.

Rung 10 — the division-algebra ceiling. $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$: dimensions 1, 2, 4, 8, and nothing after (Hurwitz 1898; cited). The 16 unit octonions $\pm e_0 \dots \pm e_7$ are closed under multiplication, non-associative ($(e_1 e_2) e_4 \neq e_1 (e_2 e_4)$), with multiplicative norm (verified). Addresses: value $\pm(i+1)$ on ring 3, own wing.

Rung 11 — E_8 (240). Construction: all $\pm e_i \pm e_j$ ($i < j \leq 8$): 112 roots; all $(\pm \frac{1}{2})^8$ with an even number of minus signs: 128 roots. Verified: 240 total, every norm² = 2, every root's inner-product spectrum is $1 \times 2, 56 \times 1, 126 \times 0, 56 \times (-1), 1 \times (-2)$, mirrors complete, crystallographic.

Two address systems, both verified collision-free:

Ordinal (ring 5): the 120 positive roots (each root or its mirror, by lexicographic rule) get $+1 \dots +120$; mirrors get $-1 \dots -120$. Open rooms: $+++++$ (121), 00000 (0), $-----$ (-121).

Coordinate (16 trits): double the root; every coordinate lies in $\{-2 \dots 2\}$; write each as two trits ($-2 = -+, -1 = 0-, 0 = 00, +1 = 0+, +2 = +-)$. Highest root $(1, 1, 0^6) \rightarrow +-+000000000000 = 10.628.820$. Half-vector $(\frac{1}{2})^8 \rightarrow 0+0+0+0+0+0+0+0+ = 5.380.840$.

The concept filling of the 240 (choice of names, counted structure). Split the 8 coordinates into an inner four (U, S, M, So) and an outer four (U', S', M', So'). Then, verified: $240 = 24 + 24 + 64 + 64 + 64$:

- 24 inner dyads (last four coordinates zero): the inner wing, named by the rung-7 table.
- 24 outer dyads (first four zero): a second, outer wing.
- 64 on channel 0 (pair): one inner stance coupled to one outer stance, e.g. $(1, 0, 0, 0, 0, -1, 0, 0) = U_+ \times S'_-$.
- 64 on channel + and 64 on channel - (profile): a full inner profile coupled to a full outer profile; channel sign = parity of the inner profile. Each half carries norm 1 (verified).

Reading: E_8 = two complete worlds plus everything that can couple them, along three channels — and the three channels sit on the three rooms that ring 3 left open.

Rung 12 — the end. E_8 is the last exceptional system (Cartan–Killing; cited). Its lattice is the densest packing in 8 dimensions (Viazovska 2016; cited). Above the coupling there is, proven, nothing.

5. Ifá: the 256 on the maze, with the full key

The sixteen meji with their traditional signatures (I = single mark = +, II = double mark = −; positions 1–4; source: the standard table after Bascom):

Meji	Signature	Parity	Meji	Signature	Parity
Ogbe	++++	even	Ogunda	+++−	odd
Oyeku	----	even	Osa	−+++	odd
Iwori	−++−	even	Ika	−+−−	odd
Odi	+−−+	even	Oturupo	−−+−	odd
Irosun	++−−	even	Otura	+−++	odd
Owonrin	−−++	even	Irete	++−+	odd
Ose	+−+−	even	Obara	+−−−	odd
Ofun	−+−+	even	Okanran	−−−+	odd

Eight even, eight odd (counted). An odu is a pair of legs (left, right): $16 \times 16 = 256$. Placement rule (choice): odu $\rightarrow$ the 8-coordinate vector (left/2, right/2), every entry $\pm\frac{1}{2}$.

Counted results, all verified:

1. The 128 odu whose legs have equal parity land exactly, bijectively, on the 128 half-vector roots of E8. The 16 meji (left = right) are the 16 self-couplings. Ogbe Meji $\rightarrow$ 0+0+0+0+0+0+0+0+.
2. The 128 odu with unequal parity fill the other half-spinor of Cl(16). The 256 odu are, cell for cell, the full spinor space of Cl(16): $128 + 128$; E8 keeps one half.
3. Ifá's 240 mixed pairs split 112 equal-parity + 128 unequal. E8's 240 roots split 112 integer-type + 128 half-type. The same pair (112, 128) on both sides, independent of any assignment choice.
4. Gap arithmetic: $256 - 240 = 16$ (the meji); $243 - 240 = 3$ (the constants); $16 - 3 = 13$: the gap between the two rims. Also: $256 =$ the zero-free rooms of ring 8 (2^8), so Ifá as a whole lives in the zero-free sub-maze at depth 8.

Open: an element-for-element rule from the 112 equal-parity mixed odu to the 112 integer-type roots (counts match; the forms differ).

6. The occupants: old systems, placed exactly

By the zero-free principle, every binary system has an exact home — no margins:

System	Count	Exact home	Concept and status
I Ching trigrams	$8 = 2^3$	zero-free rooms of ring 3	three lines, each $\pm$; the 19 zero-carrying rooms are its
I Ching	$64 = 2^6$	zero-free rooms of ring 6	six lines; identical in count and doubling to the 64
Ifá meji / Llull points / MBTI	$16 = 2^4$	zero-free rooms of ring 4	four signs; the 65 zero-carrying rooms of ring 4 remain (counted)

Tree of Life (10 22)	$32 = 2^5$	zero-free rooms	five signs (counted as 2^5 ; the 10+22 split is read)
Ifá odu	$256 =$ 28	zero-free rooms	eight signs = two legs of four; E8-map in Section 5
Nakshatras	$27 = 3^3$	ring 3, full	the only old system on the ternary ladder itself:
Elder Futhark runes	24	the dyad wing, ring 3	exactly the 24; the three constants stay open (counted as count; rune-to-dyad map open)
Zodiac	12	the up-dyads	$12 = 6 \text{ pairs} \times 2 \text{ directions}; \times 2 \text{ stands} = 24 \text{ (read)}$
Lunar mansions	$28 =$ 6(8,2)	grade 2 of Cl(8)	the bivectors (counted as identity; read as meaning)
Tarot minor	$56 =$ 6(8,2)	grade 3 of Cl(8)	the trivectors; also the 56 sixty-degree neighbours of
Tarot major	22	with the 22 paths	part of the $32 = 2^5$ (counted)
Lull lines	$120 =$ C(16,2)	the 120 mirror pairs of ring 5	
Chinese 60-cycle	$60 =$ 10,12	ring 4	stems $\times$ branches (counted factorisation; placement
Tzolkin	$260 =$ 13,20	just above ring 5	$260 = 243 + 17$ (counted)
Enoch calendar	$364 =$ $4 \times 7 \times 13$	ring 6	carries 13, the ceiling of ring 3 (counted factorisation)
Go board	$361 =$ 19	—	the square of the zero-count of ring 3 (counted

The grade row of Cl(8), for reference, with the old occupants marked: 1, **8**, **28**, **56**, 70, 56, 28, 8, 1 (sum $256 = \sum C(8,k)$, verified). Four of the nine grades carried old names before Clifford wrote the algebra.

7. The occupants: modern systems, placed exactly

Syste	Co	Home	Concept and status
MBTI	$16 = 2^4$	zero-free ring 4	four binary axes E/I, S/N, T/F, J/P; the J/P axis is the centre stand of the worldview wing (counted as 2^4 ; the axis identification is a choice)
Genetic	$4 = 2^2$	zero-free ring 2	base = two signs: purine/pyrimidine and strong/weak — A = (+,-)? assignment is a choice; the 2-bit structure is counted
Genetic code:	$64 = 2^6$	zero-free ring 6	three bases $\times$ two signs = six signs; same home as the hexagrams (counted)
Amino acids	21 = 20	read as selection from the	$20 = C(6,3)$: the middle grade of the six-sign algebra (counted identity; read as meaning)
Periodic	118	ring 5, value = Z	element Z at address +Z; mirror -Z read as the antimatter twin. $118 = 121 - 3$: the table ends three below the ring-5 ceiling (counted). Period lengths
Linnaeus	8	ring 2 / rung 8	domain to species: eight (counted)
SI base	7	ring 2	$7 = 8 - 1$ (counted)
Dewey	1.000	ring 7 (2.187)	the decimal library in the ternary building (counted)
ICD-11	~55.00	ring 10 (59.049)	the disease classification fills ring 10 with ~4.000 vacancies (count circa)
ISO countr	~249	$243 + 6$	six countries over the rim of ring 5 (count circa)

Wikipedia, round	65 fields	the zero-carrying rooms of	$65 = 3^4 - 2^4$, exactly the complement of the 16 profiles: the fields fill what the meji leave open (counted identity; placement read)
Wikipedia, census	~3, 6 M piec	ring 14 ($3^{14} = 4.782.969$)	loaded store; ~1,2 M vacancies (count circa)

Read, from the two tables together: the old systems occupy the low rings and the grades of Cl(8); the modern systems fill the high rings (7, 10, 14). Ring 5, with E8, lies between them: Ifá (256), the country list (249) and the periodic system (118) stand around it without landing on it. The margins carry the story: 0 at the nakshatras, 3 at the runes and at the end of the periodic table, 6 over the rim at the countries, 13 between the two rims of Ifá and the net.

8. The generation rules, complete

Everything above regenerates from six rules. (1) Address: balanced ternary, Section 2. (2) Mirror = negation. (3) Binary system of d signs → zero-free rooms of ring d, sign for sign. (4) Dyads: value = pair + 6·[down], sign = direction (table in Section 4). (5) E8: the two root types of Section 4; ordinal addresses by mirror-pairing; coordinate addresses by doubling and the two-trit code. (6) Concepts: inner/outer wings U S M So / U' S' M' So'; channel 0 = one-to-one, channels ± = profile-to-profile by inner parity; Ifá legs by the signature table of Section 5. These six rules, plus the tables printed here, are sufficient to rebuild every list in this document.

9. Status

Counted: the unique-address theorem; the zero-free theorem and the counts $19 = 3^3 - 2^3$ and $65 = 3^4 - 2^4$; the identities $12 = (3^3 - 3)/2$ and $41 = (3^4 + 1)/2$; all rung counts (6, 24, 48, 16, 240 = 112+128 = 24+24+64+64+64); the 24-cell checks; $2T \neq S_4$; triality = S_3 on D_4 ; the E8 spectrum 1/56/126/56/1; the Ifá parity theorem, the 128-bijection and the (112,128) double count; the gap chain $16 - 3 = 13$; the Cl(8) grade row; $118 = 121 - 3$; all occupancy counts not marked circa.

Choice: sign conventions (parity for S_3 , direction for dyads, I = + for Ifá, axis order U S M So, R/Y–S/W for bases, J/P = centre stand).

Read: worldview names on the Klein rooms; channels as coupling kinds; antimatter as mirror; the comma as the growth of the zero; the field-complement placement of Wikipedia round 1.

Open: element-for-element maps inside equal counts (rune↔dyad, hexagram↔codon, the 112↔112 of Ifá and E8); and the carry R as a complete derivation rule.

10. Verification

All counted claims in this document were computed in exact rational arithmetic and passed: T7 round-trip (–121...121 and 1.000 random integers to 10^{12}); group and closure tests for S_3 , Klein, 2T, octonion units; 24-cell neighbour and angle-spectrum tests; D_4 Cartan triality; F_4 and E8 root counts, norms, spectra and crystallographic tests; dyad and E8 address bijections with mirror rule; the 16-trit code round-trip on all 240; the Ifá parity split and 128-bijection; the zero-free counts per ring; and the 21 arithmetic identities of Sections 3–7. The executor of every verification named here is the computation performed for this document; the tables printed above are its output.

11. Annotated references

Knuth, D. E. (1997). The Art of Computer Programming, Vol. 2, §4.1. Addison-Wesley. Why read it? The standard statement of balanced ternary: unique representation and the carry rules of Section 2.

Stifel, M. (1544). Arithmetica Integra. Nuremberg. Why read it? The oldest use of the unique-address theorem: weights 1, 3, 9, 27 in either pan weigh every load to 40.

Baez, J. C. (2002). "The Octonions." Bulletin of the AMS 39, 145–205. Why read it? The clearest account of the division-algebra ceiling, triality, F_4 and E_8 — the proofs behind Rungs 8–11.

Conway, J. H., and Sloane, N. J. A. (1999). Sphere Packings, Lattices and Groups, 3rd ed. Springer. Why read it? The 240 roots of E_8 and their neighbour structure (Chapter 4) — the spectrum verified in Section 4.

Coxeter, H. S. M. (1973). Regular Polytopes, 3rd ed. Dover. Why read it? The 24-cell: the shape shared by the Hurwitz units and the D_4 roots.

Viazovska, M. (2017). "The sphere packing problem in dimension 8." Annals of Mathematics 185, 991–1015. Why read it? The proof that the E_8 lattice is the densest packing in 8 dimensions: the end of the ladder.

Hurwitz, A. (1898). "Über die Composition der quadratischen Formen." Göttinger Nachrichten. Why read it? Dimensions 1, 2, 4, 8 and nothing after: the ceiling of Rung 10.

Abimbola, W. (1976). Ifa: An Exposition of Ifa Literary Corpus. Oxford University Press Nigeria. Why read it? The 256 odu, the sixteen meji and the verse corpus: the Ifá side of Section 5.

Bascom, W. (1969). Ifa Divination. Indiana University Press. Why read it? The casting apparatus and the signature marks (Table 1): the I/II table used in Section 5.

Smith, F. D. (Tony), Jr. (2008). Physics of E_8 and $Cl(16) = Cl(8) \otimes Cl(8)$; and (2019) $Cl(16)$: Science, Art, Music. Why read them? The 256 of $Cl(8)$, $248 = 120 + 128$, the three 64-blocks, and Smith's own line $Ifá \rightarrow Llull \rightarrow Cl(16)$ — the frame that Section 5 makes exact.

Wilhelm, R., and Baynes, C. F. (1951). The I Ching or Book of Changes. Princeton University Press. Why read it? The 8 trigrams and 64 hexagrams placed on the zero-free rooms in Section 6.

Schönberger, M. (1973). The I Ching and the Genetic Code: The Hidden Key to Life. ASI Publishers. Why read it? The first systematic $64 \leftrightarrow 64$ correspondence between hexagrams and codons; Section 7 restates it as a shared home on ring 6.

Llull, R. (c. 1305). Ars Brevis. Why read it? The wheel of 16 points and 120 lines: $C(16,2)$, verified in Section 6.

Myers, I. B., and Myers, P. B. (1980). Gifts Differing. Consulting Psychologists Press. Why read it? The four binary axes of MBTI: the modern 2^4 placed beside the meji in Section 7.

IUPAC Periodic Table of the Elements (current release, iupac.org). Why read it? The 118 elements and the period lengths 2, 8, 8, 18, 18, 32, 32 used in Section 7.

World Health Organization (2022). ICD-11 Reference Guide. WHO. Why read it? The scale of the modern disease classification placed on ring 10.

Konstapel, J. (2026). "Everything Has Its Number in the Maze"; "The Comma of Pythagoras, Carried to Infinity"; "Ifá as the Anchor"; "E8, the Bronze Mean and the Vacuum.Net Theory." Constable Research, Leiden, constable.blog. Why read them? The address convention, the comma ladder, and the Ifá frame that this reference consolidates. Ordinary source references.