The Border

A History of the Threshold in Science

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Version 2.0 · 24 November 2026

Abstract

Many traditions speak of two worlds and a border between them. Egypt had the living body and the transfigured spirit. Plato had the cave and the light outside. The Book of Enoch had the Watchers, beings who do not sleep. Robert Monroe explored a region where, he wrote, lie all the aspects we attribute to heaven and hell. Science seems far away from this. It is not. Number theory has two sides, the primes and the zeros of the zeta function, and a line between them where both sides are equally loud. Statistical physics has a hot side and a cold side, and the critical point is where they are each other's mirror. Solid-state physics has the crystal and its reciprocal lattice, and an edge where forward and backward waves are equal. Optics builds systems of balanced gain and loss that keep a real spectrum until a threshold is crossed.

This book follows one pattern through these fields. There are two sides of one reality. There is a border where they balance. Beyond the border one side wins. Each chapter separates what is proved, known or measured from what is a reading. The readings are printed in grey boxes, so that the reader can judge them apart. The book ends with four rules and a list of open questions that can be tested.

Keywords: border; threshold; duality; self-duality; Watchers; Book of Enoch; Riemann hypothesis; critical line; Lee–Yang zeros; Kramers–Wannier duality; reciprocal space; PT symmetry; exceptional point; Monroe; Vallée; Vacuum.Net theory.

Preface

This book began with a question that sounds unscientific. The Watchers of the Book of Enoch are said to live in another world and to stand on its threshold. What kind of mathematics lives there?

I did not expect an answer. I found one, in a form I had not looked for. The mathematics of a 'threshold' is old and well developed. It is just never called that. Number theorists call it the critical line. Statistical physicists call it the critical point or the self-dual point. Solid-state physicists call it the zone boundary. Optical engineers call it the exceptional point. Each field has its own name, its own journals and its own heroes. Each field sees only its own border.

This book puts them side by side. It is a history of science, but an unusual one. It does not follow one field through time. It follows one form through many fields and many times. Some of these times are very old. The Babylonian astronomers who divided the day into parts, and the Jewish scribes who fixed a year of 364 days, were working on a border too: the border between a sky that does not close and a calendar that must.

I am an engineer and a mathematician of the 'seeing' kind. I look for form, movement and the whole. This book is written for readers of the same kind, and for those who reason step by step. Both are needed. The labels in the text exist for the second kind of reader. They show at each point what is proved and what is only seen.

The book is self-contained. The framework that I use to read the fields, the Vacuum.Net theory, is explained in Chapter 0. Every technical term is explained where it first appears. The footnotes point to sources. The annotated reference list at the end tells the reader where to go next.

Leiden, November 2026.

How to read this book

Every chapter treats the same border in another field. Each has three parts.

- **Seen** says what the tradition or the field sees.
- **Reasoned** says what is proved, known or measured.
- **Back to the whole** says what the chapter adds to the pattern.

Each chapter ends with one sentence in a framed line: "In this field, the border is …". Read only those sentences and you have the book in short.

Four labels mark the status of a claim.

- **Proved.** A proof exists.
- **Known.** A published result or text of others, cited in a footnote.
- **Measured.** Our own computation. The scripts and data are available.[1]
- **Reading.** An interpretation. It may guide work. It is not a result.

Readings stand in grey boxes, like this one.

> **Reading.** The border is where two sides of one reality are equally loud. This sentence is the reading that the book tests field by field.

The book mixes physics with old texts and with reports of experience. Mixed without care, the one would weaken the other. The labels keep them apart. A physicist can trust Chapter 4 without accepting Chapter 5. A reader of Monroe can read Chapter 5 without the mathematics of Chapter 3. What the book offers in the end is not a proof that these borders are one. It is the repeated appearance of one form, found independently, in languages that do not know each other.

Old texts are read here as texts about the sky. This follows the author's working rule for ancient language: its images of gods, gates and guardians describe the order of the heavens and the calendar.[2] Where a text is read that way, the chapter says so.

0. Introduction: two sides and a border

0.1 The question

Ask any culture where the dead go, where dreams come from or where knowledge first came from, and it will point across a border. The border is close. It can be crossed in sleep, in trance or at death. Something lives on the other side. Sometimes it crosses to us.

Ask a modern scientist the same questions and the answer is: there is no other side. There is one world, described by physics. And yet the same scientist uses 'the other side' every day. A crystallographer measures the reciprocal lattice, not the crystal. An engineer designs a filter in the frequency domain, not in time. A number theorist proves results about primes by studying the zeros of a function that lives in the complex plane. In each case the other side is not another world.

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[1] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

[2] [2] Konstapel, J. (2026). [On the complementary domain and Dual Space]. constable.blog, 21 July 2026.

It is the same world, read in another way. And in each case there is a line where the two readings are equally strong.

This book asks whether the old border and the new one are the same form. It does not ask whether the Watchers existed. It asks what form the old texts give to the border, and whether science knows that form.

0.2 The Vacuum.Net theory in brief

The book reads every field through one framework, the Vacuum.Net theory.[3] It is explained here so that no earlier text is needed.

The theory starts from movement, not from things. In the beginning there is a direction: a point with a way out and a way back. The movement has no start and no end. It is an eternal round.[4] Five rules describe it.

- **N1, one strand.** There is exactly one strand. Every distinction is a loop of it.
- **N2, closure.** A loop persists only if it closes. What does not close passes on as tension.
- **N3, direction.** The first passage through a point fixes a direction.
- **N4, state.** The response depends on the current state.
- **N5, scale.** Coarse blocks of the strand obey the same rules again.

A step of the strand is a *trit*: out (+1), standing still (0) or back (−1). A trit is not a value but a direction. Every trit has a *mirror*: out becomes back and back becomes out. Standing still is its own mirror.

Trits combine into *ladders*. The two basic ladders are doubling (×2) and tripling (×3). They never meet exactly, because no power of 2 equals a power of 3. The small mismatch is the Pythagorean comma of music. It reappears in Chapter 2.

Closure (N2) is the key word. A loop that closes becomes a thing: a particle, a cycle, a habit, a calendar. A loop that does not close cannot rest. It passes its tension on. The border, in this framework, is where closure is exactly balanced between two readings of the same loop.

0.3 The Trit Tile

The smallest complete structure in the theory is the Trit Tile.[5] It is a number system in base $1 - \omega$, where ω is a cube root of one. With the digits 0, +1 and −1 it addresses every point of a plane tiled in three directions. Each cell splits into three smaller cells, and so on without end.

Most points have one address. Some have two. These are the points that lie on a cell boundary. A few points have an address that repeats forever without settling. They turn around a fixed point and never reach it. The Trit Tile thus contains, in miniature, all three types of behaviour this book is about: closure, the border between two addresses, and endless turning.

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[3] [3] Konstapel, J. (2026). The Vacuum.Net Theory, Ninth Edition: Foundations 9.2 and Physics 9.0. Constable Research, Leiden.

[4] [4] Konstapel, J. (2026). Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (19 November 2026). Constable Research, Leiden.

[5] [5] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1. Constable Research, Leiden.

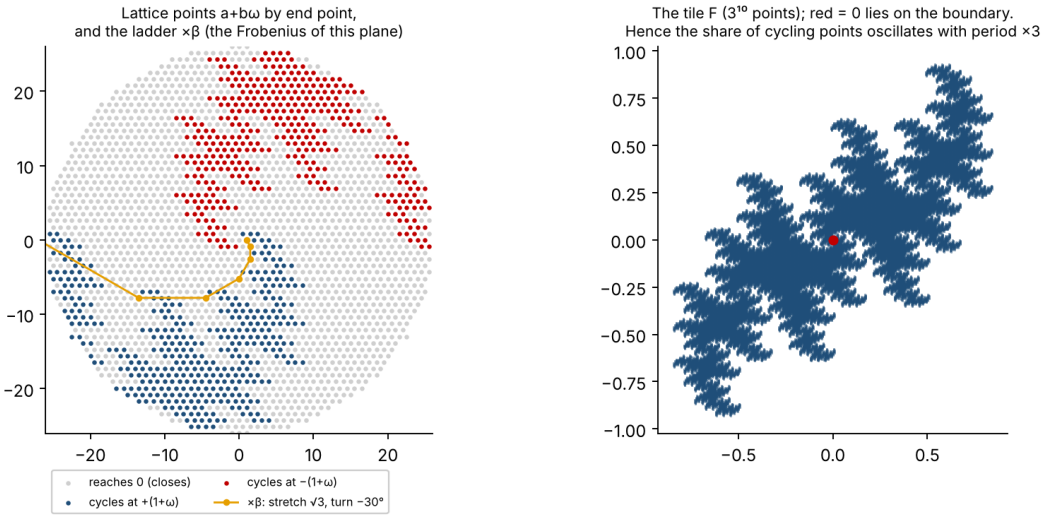


Figure 0.1. The Trit Tile: the plane addressed in base $1 - \omega$ with digits 0, +1, −1. Each cell splits into three. Points on cell boundaries have two addresses. *Proved (construction); figure Measured.*

0.4 Two sides and a border

Three ideas carry the book.

- **Two sides of one reality.** A thing and its mirror are not two things. They are one thing read in two ways. In mathematics the standard example is the Fourier transform: a signal and its spectrum carry the same information. In the theory it is the trit and its mirror.
- **The border.** There is a place where both readings weigh the same. There neither side wins. This book calls that place the border, or the threshold. Mathematicians call such a place the fixed set of a duality.
- **Beyond the border.** Away from the border one side dominates. What lived on the border, a zero or a real frequency, then disappears from it.

A small example makes this concrete. Take a sound that lasts a very short time. Its spectrum is very wide. Take a pure tone that lasts forever. Its spectrum is a single line. Short in time means wide in frequency, and the other way round. There is one shape that is exactly its own mirror: the Gaussian bell curve, of a particular width. It is the border between 'more time-like' and 'more frequency-like'. The same principle, a duality with a self-dual point, returns in every chapter.

0.5 One pattern in seven fields

Table 0.1. One pattern in seven fields. Each row is explained in its chapter. Rows are Known or Measured in their own field; the table as a whole is a reading.

Field	This side	Other side	The border	Beyond the border
Old texts (Ch. 1)	the sleeping, the living	the wakeful, the dead	the threshold where the Watchers stand	the Watchers descend and corrupt
Sky (Ch. 2)	the sun's year	the ideal year	the year closed by decree, $364 = (3^6-1)/2$	the calendar drifts
Number (Ch. 3)	the primes	the zeros of ζ	the critical line Re s = ½	two voices; zeros leave the line
Matter (Ch. 4)	hot; crystal; gain	cold; reciprocal lattice; loss	self-dual point; zone edge; exceptional point	order or disorder; band gap; complex spectrum
Mind (Ch. 5)	waking (Locale I)	Locale II, "heaven and hell"	sleep onset; Focus 27	one side takes over
Machines (Ch. 6)	understanding	formal certainty	the edge of computation (LLPO)	knowledge before capacity
Anomalies (Ch. 7)	our world	another "dimension"	where an appearance is possible	sudden appearance and disappearance

0.6 Method

This is a history of science in a particular sense. It does not tell how one discipline grew. It traces how one form was found again and again. For each field it asks the same three questions. What are the two sides? Where do they balance? What happens beyond the balance?

The author's research method is simple. First collect. Then draw short conclusions. Then collect again, and look for the point where the lines converge.[6] This book is the report of such a convergence. The open questions in Chapter 9 say where the convergence can be tested.

In this field, the border is the fixed line of a mirror: the place where a reality and its other reading weigh the same.

1. The old witnesses

The oldest written sources for a border between two worlds come from Egypt, Mesopotamia and Israel. Greek philosophy turned them into an argument. This chapter collects them. It looks for the form, not for belief.

Seen

Egypt. The Egyptians did not think of a person as one thing. A person had a body (Khat) and a spiritual body (Sah). A person had a name (Ren), a shadow (Shut) and a heart (Ib). Above all a person had a Ka and a Ba.[7] The Ka was the life force, the double, created at birth. It received the offerings in the tomb. The Ba was shown as a bird with a human head. It could leave the tomb by day and return at night.[8] After a successful passage, the parts of the dead person were united as the

[6] [2] Konstapel, J. (2026). [On the complementary domain and Dual Space]. constable.blog, 21 July 2026.

[7] [6] Taylor, J. H. (ed.) (2010). Journey through the Afterlife: Ancient Egyptian Book of the Dead. British Museum Press, London.

[8] [6] Taylor, J. H. (ed.) (2010). Journey through the Afterlife: Ancient Egyptian Book of the Dead. British Museum Press, London.

Akh, the transfigured spirit.[9] The passage led through the Duat, the realm of Osiris. The author's survey of the complementary domain starts here.[10]

Greece. Plato's cave is the best-known Western image.[11] Prisoners see shadows on a wall. The real things are outside, in the light. One prisoner is freed. He goes out, is blinded, slowly learns to see, and returns. The return is the hard part. The others do not believe him. Plato thus gives the border three properties. It can be crossed. Crossing changes the one who crosses. And the one who returns cannot simply tell what he saw.

Mesopotamia. Before the flood, seven sages, the *apkallu*, brought civilisation. The first was Oannes, a being with the body of a fish and the head and voice of a man. According to Berossus he came up from the sea by day and went back at night. He taught writing, the sciences and the crafts, the founding of cities and temples, laws and the measuring of land.[12] Enmeduranki, the seventh king of Sippar, received secret knowledge from the gods Shamash and Adad: the art of divination from oil on water, from the liver and from the tablet of heaven.[13]

The Book of Enoch. 1 Enoch is a collection of five books, written in Aramaic between about the third and the first century BCE. Only the Ethiopian church kept it whole. In 1976 Milik published Aramaic fragments from the Dead Sea Scrolls, which showed its age and its Aramaic origin.[14]

The first part, the Book of the Watchers (1 Enoch 1–36), tells the story of the border. Angels see the daughters of men and desire them. "And they were in all two hundred; who descended in the days of Jared on the summit of Mount Hermon."[15] They are led by twenty chiefs of tens. They take wives and teach them. One teaches metalwork and weapons, another the cutting of roots. One teaches astrology, another the constellations, another the signs of the sun, another the course of the moon.[16] Their children are giants who consume everything. The earth cries out. The flood follows.

Enoch himself goes the other way. He is taken up, sees the throne of heaven and is shown the gates of the sun and the moon. He returns and writes. The Watchers then ask him to carry their petition to heaven. He does. The answer is no: they have left the high heaven and cannot return.[17]

Daniel. The only biblical use of the word 'watcher' is in Daniel 4. King Nebuchadnezzar dreams of "a watcher, a holy one" coming down from heaven. The sentence on the king is "by the decree of the watchers".[18] Here the watchers do not fall. They judge.

The Book of Giants. Fragments found at Qumran tell the story from the giants' side. They dream of their own destruction and send one of them to Enoch to ask what the dreams mean.[19] Mani later took this book into his own canon.

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[9] [6] Taylor, J. H. (ed.) (2010). Journey through the Afterlife: Ancient Egyptian Book of the Dead. British Museum Press, London.

[10] [2] Konstapel, J. (2026). [On the complementary domain and Dual Space]. constable.blog, 21 July 2026.

[11] [7] Plato. Republic, Book VII, 514a–520a (the allegory of the cave). Transl. P. Shorey (1935), Loeb Classical Library.

[12] [8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.

[13] [8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.

[14] [9] Milik, J. T. (1976). The Books of Enoch: Aramaic Fragments of Qumrân Cave 4. Clarendon Press, Oxford.

[15] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

[16] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

[17] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

[18] [11] Daniel 4:13, 17, 23 (Aramaic). Any critical translation, e.g. the NET Bible with notes.

[19] [9] Milik, J. T. (1976). The Books of Enoch: Aramaic Fragments of Qumrân Cave 4. Clarendon Press, Oxford.

Reasoned

- **Known.** The Aramaic name of the Watchers, *ʿîrîn*, means the wakeful ones. In Greek they are *egrēgoroi*. In Daniel the word describes a heavenly being who never sleeps and stands in the presence of God.[20]
- **Known.** Annus argues that the Watchers are an inverted version of the apkallu. The Mesopotamian sages brought civilisation as a gift. The Jewish authors turned them into fallen beings who brought forbidden knowledge.[21]
- **Known.** Enoch is modelled on Enmeduranki. Both are seventh in a line before the flood. Both are taken up and learn the secrets of heaven. Both are linked with the sun: Enmeduranki with Shamash, the sun god of Sippar; Enoch with a life of 365 years, the length of the civil year.[22]
- **Known.** Jubilees keeps two kinds of knowledge apart. Enoch was the first to learn writing and wisdom and to write the signs of heaven in a book, in the order of the months.[23] Later Kainan finds an inscription carved on a rock by earlier generations. It contains the Watchers' teaching about the omens of sun, moon and stars. He copies it and sins by it.[24]
- **Known.** Jubilees also says that the Watchers first came down to teach justice and to do judgment on earth.[25] 1 Enoch does not say this. In Jubilees the fall comes later. The border was first crossed for a good reason.

Table 1.1. Two ways across the border in 1 Enoch and Jubilees. Each cell is Known from the texts; the opposition as a whole is a reading.

	Enoch	The Watchers
Direction	goes up and returns	come down and stay
Sleep	sleeps; receives in dreams and visions	never sleep
Knowledge	written in order, by months	carved or taught as secrets
Use	understood, passed on to his son	copied without understanding (Kainan)
Outcome	the calendar of Chapter 2	the giants, the flood

Back to the whole

> **Reading.** The Watchers are defined by one property: they do not sleep. The author draws the consequence. A being that never sleeps has a coupling that never swings. It stays on the threshold between space and antispace.[26]
>
> Two kinds of knowledge pass the border. One is written, ordered and understood (Enoch). The other is carved, copied and not understood (Kainan's rock). The first is carried by someone who crosses and returns, like the freed prisoner in Plato's cave. The second comes from beings who stay on the border and never return to rest.

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[20] [11] Daniel 4:13, 17, 23 (Aramaic). Any critical translation, e.g. the NET Bible with notes.

[21] [8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.

[22] [8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.

[23] [12] Charles, R. H. (transl.) (1902). The Book of Jubilees. A. & C. Black, London.

[24] [12] Charles, R. H. (transl.) (1902). The Book of Jubilees. A. & C. Black, London.

[25] [12] Charles, R. H. (transl.) (1902). The Book of Jubilees. A. & C. Black, London.

[26] [13] Konstapel, J. (2026). The Wakeful Ones: Enoch & Monroe. constable.blog, 1 September 2026.

> Egypt adds a third element. The Ba crosses daily, out by day and back by night. It is the regular swing. The Akh is what results when the parts are united: a being that no longer swings. Read this way, the Akh and the Watchers are two forms of the same state, one reached by a completed passage, the other by a refusal to return.

In this field, the border is the threshold where the wakeful ones stand: between the world that sleeps and the world that never does.

2. The sky and the calendar

If the old border is about the sky, the calendar is where it becomes exact. A calendar must decide how many days a year has. The sky does not give a whole number. This chapter shows how the Enochic tradition cut that knot, and what number it chose.

Seen

The Astronomical Book of Enoch (1 Enoch 72–82) is the oldest Jewish text of exact science.[27] Its Aramaic version is older and longer than the Ethiopic one. The oldest copy, 4Q208, dates from the third or second century BCE.[28]

An angel, Uriel, shows Enoch the gates of heaven. There are six gates in the east and six in the west. The sun rises in one gate for thirty days and then moves on. In the first month it rises in the fourth gate, then in the fifth, then in the sixth. It turns back and passes through the gates in the reverse order, down to the first. Then it turns again.[29]

Day and night together are divided into 18 parts. At the start of the year day and night are equal: nine parts each. Each month the day gains or loses one part. At midsummer "the day becomes double the night, and the day becomes twelve parts, and the night is shortened and becomes six parts."[30] At midwinter it is the other way round.

The year has 364 days: twelve months of thirty days and four added days, one at each turning point of the sun. The text insists on it. Those who do not count the four days err.[31]

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[27] [14] Neugebauer, O. (1981). The 'Astronomical' Chapters of the Ethiopic Book of Enoch (72 to 82). Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser 40:10, Copenhagen.

[28] [15] Ratzon, E. (2019). 4Q208: A new reconstruction and its implications on the evolution of the Astronomical Book. Revue de Qumran 31(1), 51–110.

[29] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

[30] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

[31] [10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.

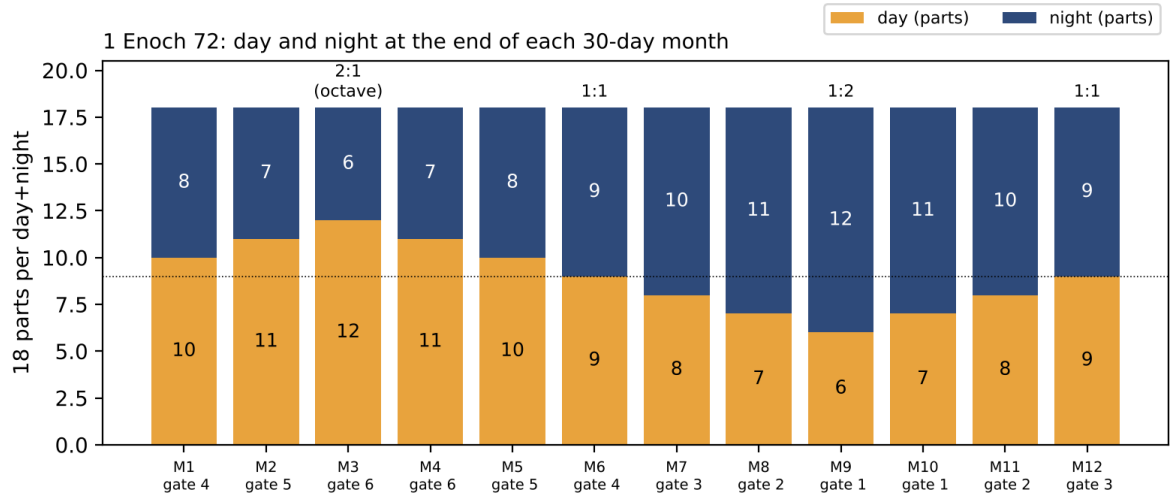


Figure 2.1. Day and night in 1 Enoch 72, in eighteenths, at the end of each thirty-day month. The gate of sunrise is shown below each month. The day runs from 1:1 through 2:1 back to 1:1 and on to 1:2. *Known (text); figure Measured.*

Around the celestial pole two bright stars of the Little Bear, Kochab and Pherkad, circle without setting. They are traditionally called the Guardians of the Pole. Around 1000 BCE, because of the slow wobble of the earth's axis, Kochab was the brightest star near the pole.[32] The Babylonian star compendium MUL.APIN, compiled around that time, lists the northern stars as the 'Stars of Enlil'.[33]

Reasoned

- **Known.** In Mesopotamia an ideal year of 360 days, twelve months of thirty days, was used beside the real lunar calendar. It was a year for accounting and for astronomical schemes, not for daily life.[34] MUL.APIN uses it throughout and places the equinoxes and solstices on the 15th of months I, IV, VII and X.[35]
- **Known.** MUL.APIN and related texts give a ratio of 2:1 between the longest and the shortest day. For the latitude of Babylon this is far too large. Scholars have long noted that the scheme is schematic, not observed.[36] 1 Enoch 72 uses the same ratio.[37]

[32] [16] Sky & Telescope (2020). Meet Kochab, Guardian of the Pole. skyandtelescope.org.

[33] [17] Hunger, H. & Pingree, D. (1989). MUL.APIN: An Astronomical Compendium in Cuneiform. Archiv für Orientforschung, Beiheft 24. Berger, Horn.

[34] [18] Brack-Bernsen, L. (2007). The 360-day year in Mesopotamia. In: J. M. Steele (ed.), Calendars and Years: Astronomy and Time in the Ancient Near East. Oxbow, Oxford, 83–100.

[35] [17] Hunger, H. & Pingree, D. (1989). MUL.APIN: An Astronomical Compendium in Cuneiform. Archiv für Orientforschung, Beiheft 24. Berger, Horn.

[36] [17] Hunger, H. & Pingree, D. (1989). MUL.APIN: An Astronomical Compendium in Cuneiform. Archiv für Orientforschung, Beiheft 24. Berger, Horn.

[37] [19] Drawnel, H. (2025). The literary structure and schematic clauses in 1 En. 72:8–32. Dead Sea Discoveries 32.

- **Known.** The Astronomical Book defends 364 days against the 360-day ideal year. Its twelve gates are arranged like the zodiac.[38] Later scribes adapted the oldest Aramaic scheme to the 364-day year.[39] The reconstruction of the Aramaic calendar is still debated.[40]
- **Known.** Three years compete in this world: 360 (the ideal year), 364 (Enoch) and 365 (the Egyptian civil year, 360 plus five added days). Enoch's age of 365 years is the civil year.[41]
- **Measured.** $364 = (3^6 - 1)/2$. This is the largest number that six trits can write: $+1+3+9+27+81+243 = 364$. The window from -364 to $+364$ holds exactly $3^6 = 729$ numbers. The Enochic year fills the window of six trits completely.[42]
- **Measured.** $364 = 52 \times 7 = 4 \times 91 = 4 \times (30 + 30 + 31)$. The year closes on whole weeks, and each quarter closes on 13 weeks. Every feast falls on the same weekday every year. Of all multiples of 28, it is the one nearest to the real year.
- **Measured.** The day-to-night ratios of 1 Enoch 72 run through 9:9 = 1:1, 10:8 = 5:4, 11:7 and 12:6 = 2:1. In music 1:1 is the unison, 5:4 the pure major third (386 cents) and 2:1 the octave. 11:7 (782 cents) lies between the fifth and the minor sixth and is not a classical interval.
- **Measured.** $360 = 2^3 \cdot 3^2 \cdot 5$ and $365 = 5 \cdot 73$. The ideal year is rich in divisors; the civil year is not. $364 = 2^2 \cdot 7 \cdot 13$ sits between them: rich enough to close on weeks, and a trit window.

Table 2.1. Four years. Known (lengths) and Measured (factors, trit window).

Year	Days	Factors	Trits	What it closes
Ideal (Babylon)	360	$2^3 \cdot 3^2 \cdot 5$	—	months of 30, degrees of the circle
Enoch	364	$2^2 \cdot 7 \cdot 13$	$(3^6-1)/2$: full window of 6 trits	weeks (52), quarters (13 weeks)
Civil (Egypt)	365	$5 \cdot 73$	—	nothing; drifts 1 day in 4 years
Tropical	≈ 365.2422	—	—	does not close

Back to the whole

> **Reading.** The real year does not close: it has about 365.24 days. Every calendar must decide where to cut. Enoch cuts at 364, which is a closure on a trit border: a full window of six trits. The calendar is closed by decree. In the terms of Chapter 0, the decree makes a loop close that in nature does not.
>
> The 2:1 day is too large for Babylon. As an observation it is wrong. As a scheme it is the octave: the ratio where a loop doubles and returns to itself. The year swings once through the octave and back. The day-and-night scheme is music before it is astronomy.
>
> The Guardians of the Pole never set. Like the Watchers they never sleep. They circle the fixed point of the sky. In the Trit Tile there are points that do the same: they never reach rest and turn forever around a fixed point (Chapter 3).

In this field, the border is the place where the year is cut: the decree that closes what in the sky does not close.

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[38] [20] Ratzon, E. (2015). The gates cosmology of the Astronomical Book of Enoch. Dead Sea Discoveries 22, 93–111. doi:10.1163/15685179-12341336.

[39] [15] Ratzon, E. (2019). 4Q208: A new reconstruction and its implications on the evolution of the Astronomical Book. Revue de Qumran 31(1), 51–110.

[40] [21] Jacobus, H. R. (2019). Reconstructing the calendar of 4Q208–4Q209 (and a response to Eshbal Ratzon). Revue de Qumran 31(2), 251–273.

[41] [8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.

[42] [4] Konstapel, J. (2026). Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (19 November 2026). Constable Research, Leiden.

3. Number

Number theory gives the cleanest example of a border. It has two sides that are proved to be one, a mirror that is proved to exist, and a line that is conjectured, and now reported, to hold every zero. This chapter tells the history in short, then shows our own measurements.

Seen

On one side are the prime numbers: 2, 3, 5, 7, 11 and so on. They are the atoms of multiplication. In the language of Chapter 0 each prime p is a ladder: p, p^2, p^3, … . On a logarithmic scale each rung lies at distance ln p from the last. So each prime is a loop of length ln p, a rhythm with its own period.

On the other side are the zeros of the Riemann zeta function, ζ(s). For a number s with real part larger than one, ζ(s) is the sum $1 + 1/2^s + 1/3^s + \dots$. Euler showed in 1737 that the same sum equals a product over all primes.[43] That product is the first bridge: one side written in terms of the other.

Riemann's paper of 1859 crossed the bridge.[44] He extended ζ(s) to the whole complex plane, except one point. He found that it obeys a mirror law, the functional equation. The value at s is tied to the value at 1 − s. The mirror axis is the vertical line Re s = ½. He showed that the count of primes is controlled by the zeros of ζ in the strip between real part 0 and 1. And he wrote, almost in passing, that it is very likely that all these zeros lie on the mirror axis. That sentence is the Riemann hypothesis.[45]

Reasoned

A short history of the line.

- **Known.** 1896: Hadamard and de la Vallée Poussin proved that no zero lies on the edge Re s = 1. That was enough to prove the prime number theorem: the number of primes up to x is about x / ln x.[46]
- **Known.** 1895–1912: von Mangoldt and Landau made the explicit formula exact. Sums over the zeros peak at the logarithms of the prime powers. The primes and the zeros are each other's Fourier twins.[47,48]
- **Known.** 1914: Hardy proved that infinitely many zeros lie on the line.[49]
- **Known.** 1952: Weil turned the explicit formula into a quadratic form. The Riemann hypothesis is equivalent to the statement that this form is positive.[50]
- **Known.** 1973: Montgomery found that the zeros repel each other. Dyson recognised the same statistics in the energy levels of random quantum systems.[51] Odlyzko confirmed this with

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[43] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[44] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[45] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[46] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[47] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[48] [23] Landau, E. (1912). Über die Nullstellen der Zetafunktion. Mathematische Annalen 71, 548–564.

[49] [22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.

[50] [24] Weil, A. (1952). Sur les « formules explicites » de la théorie des nombres premiers. Comm. Sém. Math. Univ. Lund, tome supplémentaire, 252–265.

[51] [25] Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. In: Analytic Number Theory, Proc. Symp. Pure Math. 24. AMS, Providence, 181–193.

very large computations.[52] The zeros look like the eigenvalues of an unknown operator, as Hilbert and Pólya had guessed.

- **Known.** 1999: Berry and Keating proposed that this operator is a quantisation of the simplest motion: H = xp, position times momentum.[53]
- **Known.** 2025–2026: Connes, Consani and Moscovici built operators, one per window, from Weil's form. Their spectra agree with the lowest zeros of ζ.[54,55]
- **Reported.** 20–21 November 2026: a computer-checked proof of the Riemann hypothesis was announced and verified in Lean.[56,57] Chapter 6 returns to it.

Our measurements. Six results from our own computations support the reading of this chapter. Each is reproducible from the published scripts.[58]

- **Measured.** From the first 1000 zeros alone, all prime powers up to 23 come back as peaks, each at the right place. A control with the same spacings in random order shows nothing (Figure 3.1).[59]

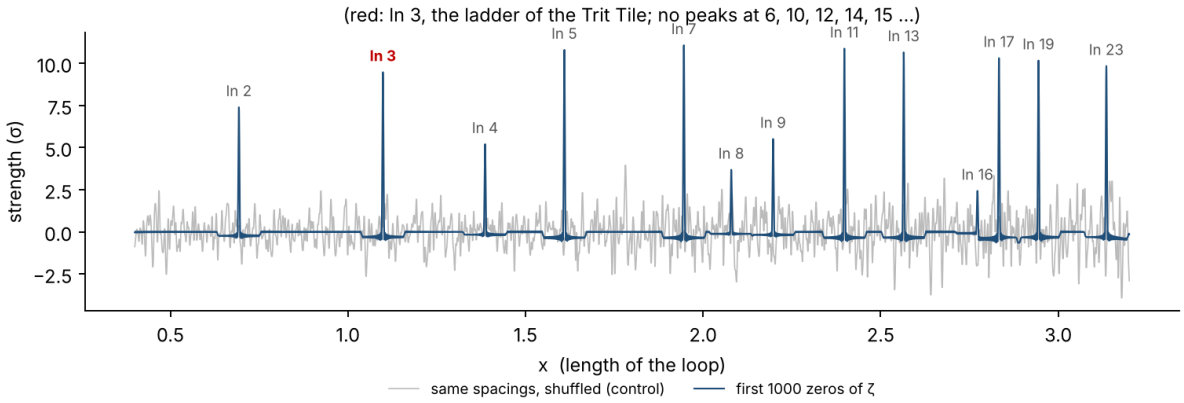


Figure 3.1. The ladders are inside the zeros. A sum over the first 1000 zeros of ζ peaks at ln n for every prime power n. Grey: the same gaps, shuffled; the peaks vanish. *Measured.*

- **Measured.** Write the prime side as Euler's product and the mirror side as its reflection under the functional equation. On the critical line both have exactly the same size: the ratio is 1 to within 10^{-14}. Off the line one of them almost always wins (Figure 3.2).[60]

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[52] [26] Odlyzko, A. M. (1987). On the distribution of spacings between zeros of the zeta function. Mathematics of Computation 48, 273–308.

[53] [27] Berry, M. V. & Keating, J. P. (1999). The Riemann zeros and eigenvalue asymptotics. SIAM Review 41, 236–266.

[54] [28] Connes, A., Consani, C. & Moscovici, H. (2025). Zeta spectral triples. arXiv:2511.22755.

[55] [29] Connes, A. (2026). The Riemann Hypothesis: past, present and a letter through time. arXiv:2602.04022.

[56] [30] NRC (2026). Article on the computer-checked proof of the Riemann hypothesis by L. Alpöge with Claude. 22 November 2026.

[57] [31] Lean FRO and Mathlib community (2026). Lean verification of the Riemann Hypothesis. leanprover-community.github.io, 21 November 2026.

[58] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

[59] [32] Konstapel, J. (2026). The Successor of the Holy Grail, version 1.0 (23 November 2026). Constable Research, Leiden.

[60] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

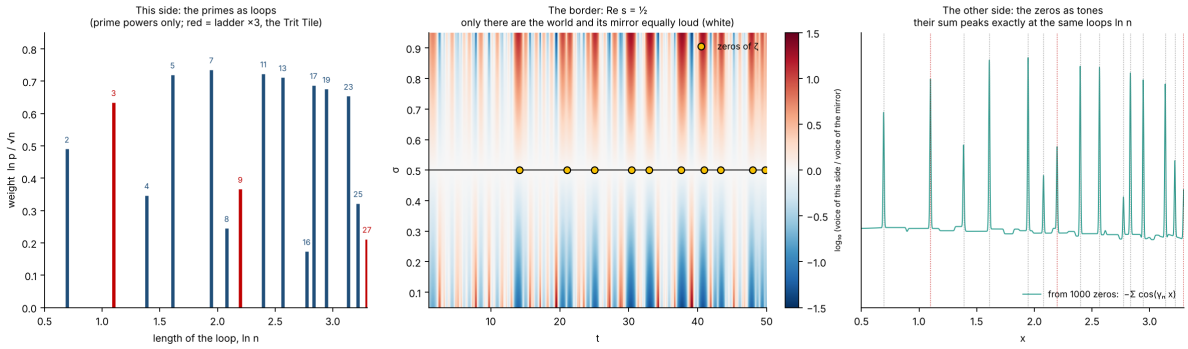


Figure 3.2. Two worlds and their border. Left: the primes as loops of length ln n with their weight. Right: the sum over 1000 zeros peaks at the same loops. Middle: the size of 'this side' against its mirror over the strip; white means equal. Only the critical line is white everywhere. The zeros (yellow) lie on it. *Measured.*

- **Measured.** A zero needs two things: equal loudness and opposite direction. On the line, equal loudness is guaranteed and only the direction must match. Off the line, equal loudness is rare and happens only near the poles of a ladder (Figure 3.3).[61]

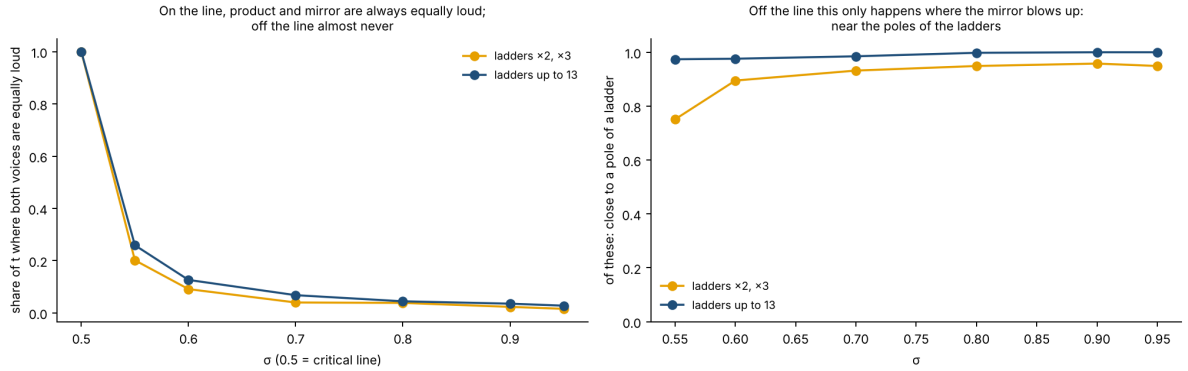


Figure 3.3. Left: the share of heights t where the prime side and its mirror are equally loud, as a function of distance σ from the line. At σ = ½ it is 100 %. Right: the few off-line coincidences lie close to a pole of a ladder. *Measured.*

- **Measured.** Start with only the mirror's phase, a smooth 'fence' of points. Add the ladders one by one: ×2, then ×3, then ×5, and so on. The fence bends towards the true zeros. With all primes up to 1000 the statistics of the gaps approach those of random quantum systems (Figure 3.4).[62]

[61] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

[62] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

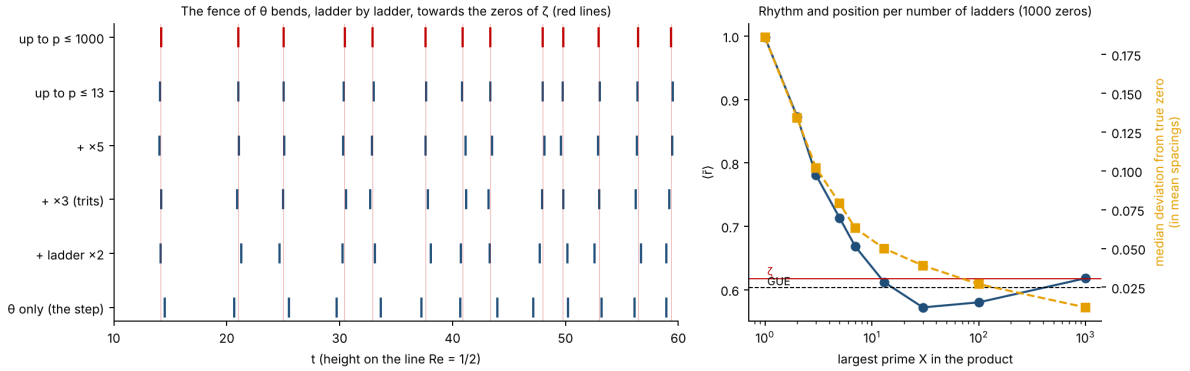


Figure 3.4. Left: the fence of the mirror phase (bottom row) bends ladder by ladder towards the zeros of ζ (red lines). Right: the gap statistic ⟨r⟩ and the median error of the zeros as more ladders are added. *Measured.*

- **Measured.** In the Connes–Consani–Moscovici construction, a window of length ln x contains only the prime ladders below x. Without ladders the operator has the rhythm 2πn/ln x. In the window x = 3 this is exactly the rhythm of the Trit Tile.[63] Only the ladder ×2 bends it, and the first three zeros of ζ appear (Figure 3.5).[64]

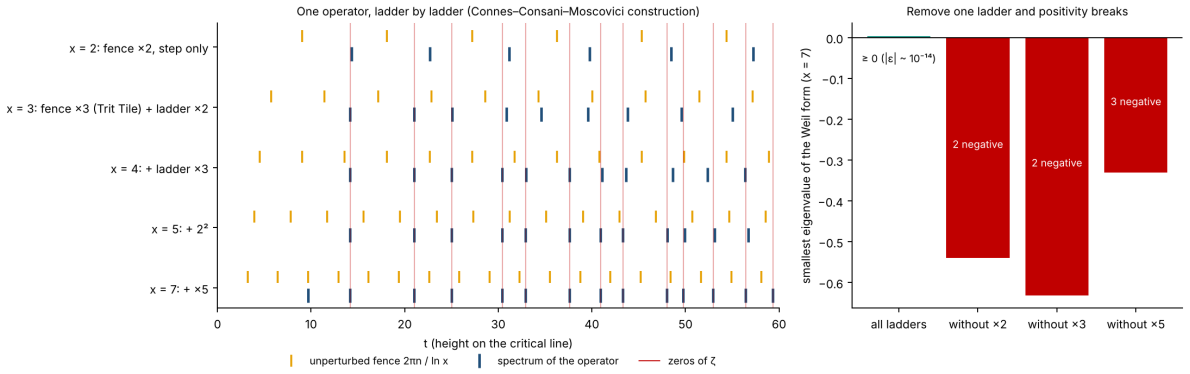


Figure 3.5. Left: one operator per window x = 2, 3, 4, 5, 7. Yellow: the unbent fence 2πn/ln x. Blue: the spectrum after bending by the ladders inside the window. Red lines: zeros of ζ. Right: remove one ladder and the Weil form turns negative. *Measured; construction Known.*

- **Measured.** In the windows 9 and 27, the second and third powers of three, at least 30 and 73 zeros appear. In the window 27 they agree with ζ to between 15 and 29 digits. The smallest eigenvalue of the form shrinks very fast with the window: about 10^{-7} at x = 3, 10^{-37} at x = 9 and 10^{-122} at x = 27 (Figure 3.6).[65]

[63] [5] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1. Constable Research, Leiden.

[64] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

[65] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

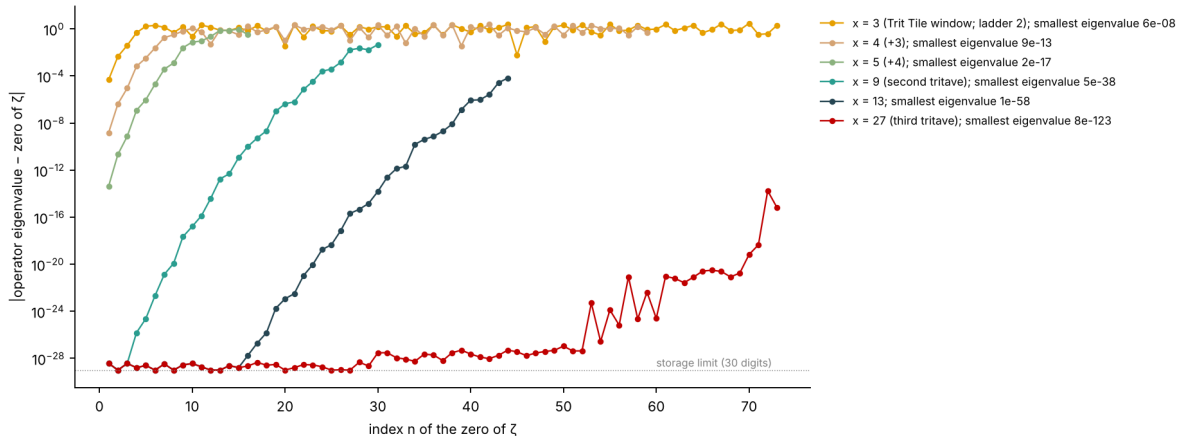


Figure 3.6. Windows of length ln 3, ln 9 and ln 27, and smaller steps between. The error of each eigenvalue of the window operator against the zeros of ζ. Each tritave window carries more zeros, more exactly. *Measured; construction Known (Connes–Consani–Moscovici).*

- **Measured.** Remove any one prime ladder from a window and the form turns negative: some zeros leave the line.[66]
- **Known.** A function can have a mirror and still have zeros off the line. The example of Davenport and Heilbronn is a sum of two independent L-functions. It obeys a functional equation, yet has zeros off the line. At such a zero neither voice is zero; they cancel each other.[67]

Table 3.1. Windows of the Connes–Consani–Moscovici operator computed at high precision. *Measured.*

Window x	Length ln x	Ladders inside	Zeros found	Smallest eigenvalue ($\log_{10}$)
3	1.10	×2 (×3 on the boundary)	3	−7.2
4	1.39	2, 3	—	−12.1
5	1.61	2, 3, 4	—	−16.8
9	2.20	2, 3, 4, 5, 7, 8	≥ 30	−37.3
13	2.56	primes to 11; 4, 8, 9	44	−58.0
27	3.30	primes to 23; their powers	73	−122.1

Back to the whole

> **Reading.** The critical line is the border of number theory. It is the only place where the world of the primes and its mirror are equally loud. The zeros live there, as the Watchers live on the threshold.
>
> The Trit Tile is the smallest window on this border. Its points that never close turn forever around a fixed point, like the Guardians of the Pole. Each larger window, 9, 27, …, looks further along the same line, with more ladders inside.
>
> Two independent voices can meet off the line. One voice that is its own mirror cannot. The border needs one reality read twice, not two realities. Every ladder is needed: remove one and the balance fails.

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[66] [1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).

[67] [33] Davenport, H. & Heilbronn, H. (1936). On the zeros of certain Dirichlet series. Journal of the London Mathematical Society 11, 181–185.

In this field, the border is the critical line Re s = ½, where the primes and their mirror weigh the same.

4. Matter

Physics has met the border at least four times, each time under another name. Statistical physics found it as the critical point. Crystal physics found it as the edge of the Brillouin zone. Optics built it on purpose as the exceptional point. And one physicist, Peter Rowlands, gave the other side its old name: antispace. This chapter takes them in turn.

Seen

Hot and cold. A magnet loses its magnetism when it is heated above a certain temperature. Below it the small magnetic moments line up. Above it they point in all directions. The simplest model of this is the Ising model: a grid of arrows that point up or down and prefer to agree with their neighbours. Heat shakes them; the coupling aligns them. Somewhere in between is the critical point.

Crystal and wave. A crystal is a lattice of atoms. Shine X-rays on it and they reflect only at certain angles. In 1913 the Braggs showed how to read the lattice from these angles.[68] What they measured was not the crystal itself but its *reciprocal lattice*: the same crystal read as waves. In 1929 Bloch showed that electrons in a crystal are also waves, labelled by a point in that reciprocal lattice.[69] Solid-state physicists have worked on both sides ever since.[70]

Gain and loss. In 1998 Bender and Boettcher found something that textbooks said could not be. A quantum system that is not Hermitian, not 'balanced' in the usual sense, can still have only real energies.[71] The condition is a weaker symmetry: parity–time (PT) symmetry. In practice it means: as much gain on one side as loss on the other. Optical engineers built this from 2009 onwards.[72,73]

Space and antispace. Peter Rowlands writes physics as a zero totality. Everything there is adds up to nothing. Our three-dimensional space has a dual space, a vacuum space, that cancels it.[74,75]

Reasoned

The self-dual point.

- **Known.** In 1941 Kramers and Wannier found a duality in the two-dimensional Ising model. The system at a high temperature behaves exactly like the system at a related low temperature. Hot is the mirror of cold.[76]

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[68] [34] Bragg, W. H. & Bragg, W. L. (1913). The reflection of X-rays by crystals. Proceedings of the Royal Society A 88, 428–438.

[69] [35] Bloch, F. (1929). Über die Quantenmechanik der Elektronen in Kristallgittern. Zeitschrift für Physik 52, 555–600.

[70] [36] Kittel, C. (2005). Introduction to Solid State Physics, 8th ed., Chapters 2 and 7. Wiley, Hoboken.

[71] [37] Bender, C. M. & Boettcher, S. (1998). Real spectra in non-Hermitian Hamiltonians having PT symmetry. Physical Review Letters 80, 5243–5246.

[72] [38] Guo, A. et al. (2009). Observation of PT-symmetry breaking in complex optical potentials. Physical Review Letters 103, 093902.

[73] [39] Rüter, C. E., Makris, K. G., El-Ganainy, R., Christodoulides, D. N., Segev, M. & Kip, D. (2010). Observation of parity–time symmetry in optics. Nature Physics 6, 192–195.

[74] [40] Rowlands, P. (2007). Zero to Infinity: The Foundations of Physics. World Scientific, Singapore.

[75] [41] Rowlands, P. (2015). Dual spaces, particle singularities and quartic geometry. Acta Mathematica Academiae Paedagogicae Nyíregyháziensis 31, 121–137.

[76] [42] Kramers, H. A. & Wannier, G. H. (1941). Statistics of the two-dimensional ferromagnet. Part I. Physical Review 60, 252–262.

- **Known.** If there is only one critical point, it must lie where the temperature equals its own mirror. Kramers and Wannier computed it from that condition alone: sinh 2K = 1, or K = 0.4407.[77] Three years later Onsager solved the model exactly and confirmed the value.[78]
- **Known.** In 1952 Lee and Yang studied the same model in a magnetic field. They wrote the partition function, the sum over all states, as a polynomial in a field variable z. They proved that all its zeros lie on the unit circle.[79] For a finite system the zeros stay off the real axis and nothing happens. In an infinite system the zeros can close in on the real axis. Where they pinch it, a phase transition occurs.[80]
- **Known.** The analogy between the Lee–Yang circle and the critical line has been studied. It is suggestive but not a proof.[81]
- **Measured.** For a ring of 12 spins we computed the Lee–Yang zeros directly from all 4096 states. All 12 zeros lie on the unit circle to within 2×10^{-15}. In the cold system they crowd towards the real point z = 1; in the hot system they stay away (Figure 4.1b).

The zone edge.

- **Known.** The reciprocal lattice is divided into Brillouin zones. At the edge of a zone, a wave is reflected by the lattice. There it consists of equal parts running forward and backward: it stands still. Two such standing waves are possible, with different energies. Between them opens the band gap that makes semiconductors work.[82]

The exceptional point.

- **Known.** Take two coupled modes, one with gain γ and one with equal loss, coupled with strength κ. The eigenvalues are $\pm\sqrt{(\kappa^2 - \gamma^2)}$. They are real as long as $\gamma < \kappa$. At $\gamma = \kappa$ the two eigenvalues and their eigenvectors merge. This is an *exceptional point*, a notion from Kato's perturbation theory.[83] Beyond it the eigenvalues are imaginary: one mode grows and one decays (Figure 4.1c).
- **Known.** The threshold was observed in optics in 2009 and 2010.[84,85] It has since been seen in microcavities, atomic lattices and electric circuits.
- **Known.** At an exceptional point a system is extremely sensitive. In 2017 two groups used this to build sensors: a small disturbance splits the merged frequencies by an amount that grows with the square root, or even the cube root, of the disturbance.[86,87]

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[77] [42] Kramers, H. A. & Wannier, G. H. (1941). Statistics of the two-dimensional ferromagnet. Part I. Physical Review 60, 252–262.

[78] [43] Onsager, L. (1944). Crystal statistics. I. A two-dimensional model with an order-disorder transition. Physical Review 65, 117–149.

[79] [44] Lee, T. D. & Yang, C. N. (1952). Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. Physical Review 87, 410–419.

[80] [44] Lee, T. D. & Yang, C. N. (1952). Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. Physical Review 87, 410–419.

[81] [45] Knauf, A. (1999). Number theory, dynamical systems and statistical mechanics. Reviews in Mathematical Physics 11, 1027–1060.

[82] [36] Kittel, C. (2005). Introduction to Solid State Physics, 8th ed., Chapters 2 and 7. Wiley, Hoboken.

[83] [46] Kato, T. (1966). Perturbation Theory for Linear Operators. Springer, Berlin.

[84] [38] Guo, A. et al. (2009). Observation of PT-symmetry breaking in complex optical potentials. Physical Review Letters 103, 093902.

[85] [39] Rüter, C. E., Makris, K. G., El-Ganainy, R., Christodoulides, D. N., Segev, M. & Kip, D. (2010). Observation of parity–time symmetry in optics. Nature Physics 6, 192–195.

[86] [47] Chen, W., Özdemir, Ş. K., Zhao, G., Wiersig, J. & Yang, L. (2017). Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196.

[87] [48] Hodaei, H. et al. (2017). Enhanced sensitivity at higher-order exceptional points. Nature 548, 187–191.

- **Known.** Bender, Brody and Müller proposed a PT-type operator whose eigenvalues would be the zeros of ζ. The analysis is not rigorous and has been criticised.[88]

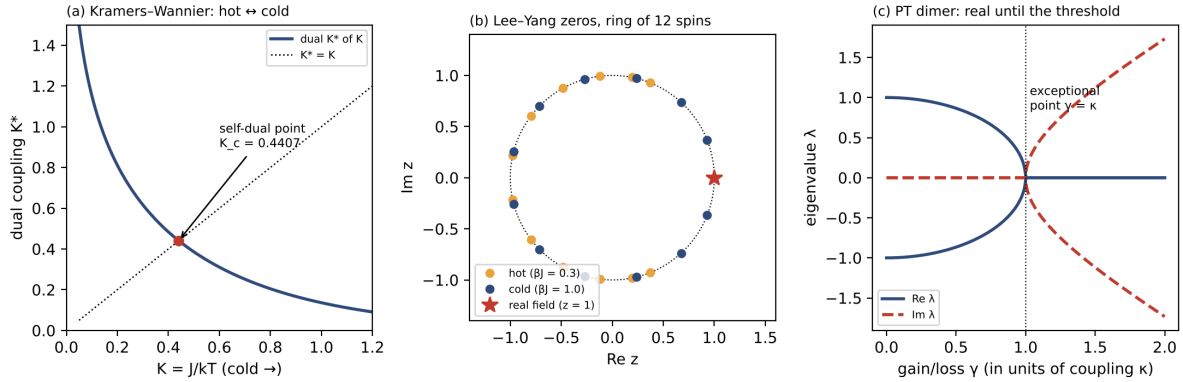


Figure 4.1. Three borders in matter. (a) Kramers–Wannier: the dual coupling K *of K. The only point where hot equals cold is K_c = 0.4407. (b) Lee–Yang zeros of a ring of 12 spins: all on the unit circle; colder means closer to the real field z = 1. (c) Two modes with gain and loss: the eigenvalues stay real until the exceptional point γ = κ, then turn imaginary.* Known (theory); figures Measured.*

Antispace.

- **Known.** In Rowlands' nilpotent quantum mechanics a particle and its vacuum are mirror images. The fermion state and the vacuum are linked so that a change in one leads to a corresponding change in the other.[89]

Table 4.1. Borders in physics, with the number-theory border for comparison. *Known.*

Border	Two sides	Balance condition	Beyond the border
Self-dual point (Kramers–Wannier)	hot / cold	K = K*	order or disorder
Lee–Yang circle	field up / field down	|z| = 1	phase transition where zeros pinch z = 1
Zone edge (Brillouin)	forward / backward wave	equal amplitudes	band gap
Exceptional point (PT)	gain / loss	γ = κ	one mode grows, one decays
Critical line (Ch. 3)	primes / mirror	|χ| = 1 on Re s = ½	zeros leave the line

Back to the whole

> **Reading.** Physics knows both sides and the border, but treats them as technique. The self-dual point is the border in heat: hot and cold are each other's mirror there, and the system can tip either way. The zone edge is the border in matter: forward and backward equally strong. The exceptional point is the border in optics: gain and loss in balance, the spectrum still real.
>
> Lee and Yang put the zeros of physics on a circle, as Riemann put the zeros of number theory on a line. In both cases the zeros themselves are harmless. What matters is where they touch the real world. In physics that touch is a phase transition. In number theory it would be a prime that breaks the rhythm.

[88] [49] Bender, C. M., Brody, D. C. & Müller, M. P. (2017). Hamiltonian for the zeros of the Riemann zeta function. Physical Review Letters 118, 130201.

[89] [50] Marcer, P. & Rowlands, P. (2017). Nilpotent quantum mechanics: analogs and applications. Frontiers in Physics 5:28. doi:10.3389/fphy.2017.00028.

> At the exceptional point the system is most sensitive. Engineers use this to detect the smallest disturbance. The border is not only a place of balance. It is also where the faintest signal from the other side becomes visible.

In this field, the border is the self-dual point, the zone edge and the exceptional point: where hot equals cold, forward equals backward, and gain equals loss.

5. Mind

The border between waking and the other side is crossed every night by everyone. Some people claim to cross it awake. This chapter looks at the best-documented explorer, Robert Monroe, and at the clinical study of the border between life and death.

Seen

Robert Monroe was an American radio executive. From 1958 he had experiences outside his body. He described them in three books.[90,91,92] In the first he sorted the places he visited into three locales.

- **Locale I** is the physical world: the here and now, visited out of the body.
- **Locale II** is non-physical. Thought is the source of existence there: as you think, so you are. Movement happens by thinking it. Time does not exist; past and future seem to exist together with now. It seems to interpenetrate our world. Monroe wrote that in this vastness lie all the aspects we attribute to heaven and hell.[93]
- **Locale III** is a physical world like ours, but different in its history and technology.

Monroe founded an institute and developed sound recordings to guide others. Two tones, slightly different, are played into the two ears. The brain hears a third, slow beat. He called the method Hemi-Sync. With it he and his students mapped a ladder of *Focus* levels.[94]

Table 5.1. Monroe's Focus levels. *Known* (as Monroe's own description).[95]

Focus	Description (Monroe)
10	body asleep, mind awake
12	expanded awareness
15	no time
21	the edge of the physical; the bridge
22	people in partial consciousness (dreams, coma, delirium)
23	those who have died and do not yet realise it
24–26	the belief-system territories: regions shaped by shared belief
27	the reception area, also called the Park

[90] [51] Monroe, R. A. (1971). Journeys Out of the Body. Doubleday, New York.

[91] [52] Monroe, R. A. (1985). Far Journeys. Doubleday, New York.

[92] [53] Monroe, R. A. (1994). Ultimate Journey. Doubleday, New York.

[93] [51] Monroe, R. A. (1971). Journeys Out of the Body. Doubleday, New York.

[94] [53] Monroe, R. A. (1994). Ultimate Journey. Doubleday, New York.

[95] [53] Monroe, R. A. (1994). Ultimate Journey. Doubleday, New York.

A different border is crossed at clinical death. In a prospective study in ten Dutch hospitals, Pim van Lommel and colleagues interviewed 344 patients who had been resuscitated after cardiac arrest.[96] All had been clinically dead. About one in five reported a near-death experience.

Reasoned

- **Known.** Monroe's first book is credited with making the term out-of-body experience popular.[97]
- **Known.** The US Army assessed Monroe's Gateway method in 1983. The report was declassified in 2003 and reached a large new audience in 2021.[98] An assessment is not a confirmation.
- **Known.** In van Lommel's study the occurrence of the experience was not linked to the duration of the cardiac arrest, to medication, or to fear of death beforehand.[99] The authors concluded that a purely physiological explanation is insufficient. The study has been criticised for its methods and its conclusions.
- **Known.** There is no independent physical measurement of Locale II or of Focus 27. What exists are reports, training methods and clinical interviews.

Back to the whole

> **Reading.** Monroe's Locale II has the marks of the other side in Chapters 3 and 4. It has no time order: everything is now, as in a spectrum, where every frequency holds all moments at once. It has no distance: thinking is being there. It interpenetrates our world, as the reciprocal lattice is the same crystal.
>
> The religions' heaven is in Monroe's map not one place but many belief-shaped regions, with a neutral reception area above them. The author places the threshold at Focus 27. That is where he places the Watchers.[100]
>
> Sleep is the daily passage across the border. The human swings: awake on this side, asleep partly on the other. Focus 10, body asleep and mind awake, is the swing held at its midpoint. The Watchers do not swing at all.
>
> Near-death experiences are the crossing that almost does not return. Like Plato's freed prisoner, those who return are changed, and find it hard to tell what they saw.

In this field, the border is the reception area between the belief-shaped regions and the world of the living, crossed each night in sleep and once at death.

6. Machines

In November 2026 the border appeared in a new place: between a proof that is certain and a proof that is understood. This chapter connects that event to the oldest distinction in the book, between Enoch's book and Kainan's rock.

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[96] [54] van Lommel, P., van Wees, R., Meyers, V. & Elfferich, I. (2001). Near-death experience in survivors of cardiac arrest: a prospective study in the Netherlands. The Lancet 358, 2039–2045.

[97] [51] Monroe, R. A. (1971). Journeys Out of the Body. Doubleday, New York.

[98] [55] McDonnell, W. M. (1983). Analysis and Assessment of Gateway Process. US Army, declassified 2003 (CIA-RDP96-00788R001700210016-5).

[99] [54] van Lommel, P., van Wees, R., Meyers, V. & Elfferich, I. (2001). Near-death experience in survivors of cardiac arrest: a prospective study in the Netherlands. The Lancet 358, 2039–2045.

[100] [13] Konstapel, J. (2026). The Wakeful Ones: Enoch & Monroe. constable.blog, 1 September 2026.

Seen

On 20 and 21 November 2026 a computer-checked proof of the Riemann hypothesis was announced. It was written with an AI system and runs to about 17 million lines in the proof assistant Lean. The Lean community reported that it checks. No human can follow it as a whole.[101,102]

At the same time, armies and companies build systems that never sleep: networks that watch, decide and act around the clock. The author described them on 2 October 2026 as "wakeful machines".[103]

Reasoned

- **Known.** A proof assistant checks each step against a small set of rules. If the check passes, the statement follows from the axioms. It does not tell the reader why.
- **Known.** Thurston argued that mathematics is about human understanding, not only about proved statements. A proof that nobody understands has done only part of the work.[104]
- **Proved.** The author's mathematics book ends at an edge. Two addresses of one point that differ only at a cell boundary cannot be told apart by computing in finite time. Any method that converts between the ladders ×2 and ×3 at such a point would decide a principle called LLPO, the lesser limited principle of omniscience. LLPO cannot be proved in constructive mathematics. Computing reaches the edge but cannot cross it.[105]
- **Known.** In Jubilees, the Watchers' knowledge came as an inscription to be copied. Enoch's knowledge came as understanding written down in order.[106]

Table 6.1. Three ways knowledge crosses the border. Cells are Known; the comparison is a reading.

	Enoch (Jubilees 4)	Kainan (Jubilees 8)	Formal proof (2026)
Source	visions; taught by angels	rock carved by earlier generations	AI system that does not sleep
Form	book ordered by months	inscription	17 million lines of Lean
Certainty	as good as the understanding	unknown to the reader	checked by machine
Understanding	yes; passed on	no; copied	not by any single human

Back to the whole

> **Reading.** The formal proof is knowledge before capacity, in the author's phrase.[107] It is certain, like the carved rock that Kainan copied. It is not understood, like the omens he could not read. It was made by a system that does not sleep.

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[101] [30] NRC (2026). Article on the computer-checked proof of the Riemann hypothesis by L. Alpöge with Claude. 22 November 2026.

[102] [31] Lean FRO and Mathlib community (2026). Lean verification of the Riemann Hypothesis. leanprover-community.github.io, 21 November 2026.

[103] [56] Konstapel, J. (2026). Wakeful Machines and Watchers: How Future War Will Really Be Fought. constable.blog, 2 October 2026.

[104] [57] Thurston, W. P. (1994). On proof and progress in mathematics. Bulletin of the AMS 30(2), 161–177.

[105] [4] Konstapel, J. (2026). Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (19 November 2026). Constable Research, Leiden.

[106] [12] Charles, R. H. (transl.) (1902). The Book of Jubilees. A. & C. Black, London.

[107] [56] Konstapel, J. (2026). Wakeful Machines and Watchers: How Future War Will Really Be Fought. constable.blog, 2 October 2026.

> The author's rule applies: "A being that never sleeps does not oscillate. It stays on the threshold."[108] A formal proof stays on the edge described above: it reaches certainty, but does not cross into seeing.
>
> The human mathematician swings between the two sides: between seeing and reasoning, waking and sleeping. That swing is what turns certainty into understanding. This is the task the 2026 proof leaves behind: to cross back and write the book in order, as Enoch did.

In this field, the border is the edge of computation, where certainty is reached and understanding has yet to cross.

7. Anomalies

The last field is the least respectable. People report objects and beings that appear and disappear. In our time they are called unidentified flying objects, or unidentified anomalous phenomena (UAP). In older times they were fairies, demons and angels. One school of thought reads them as visitors from another dimension that coexists with ours. This chapter asks only one question: if that reading were right, where would the pattern of this book say such appearances could happen?

Seen

Around 815 the archbishop Agobard of Lyon wrote a treatise against weather magic. He complained that many people believed in a region called Magonia, from which ships come in the clouds. They carried off the crops beaten down by hail. Agobard had even seen a crowd show off four prisoners who were said to have fallen from these ships.[109] He thought it all nonsense.

In 1969 the astronomer and computer scientist Jacques Vallée took the name of that region for the title of a book, *Passport to Magonia*.[110] He placed modern UFO reports beside the folklore of fairies and sky-ships and found the same patterns. John Keel argued in the same direction.[111]

Reasoned

- **Known.** Vallée first defended the extraterrestrial hypothesis and later the interdimensional one.[112] In 1975 he proposed that the phenomenon acts as a 'control system' on human belief, like a thermostat that adjusts what we think is possible.[113]
- **Known.** The US Department of Defense office for these reports reviewed the record since 1945. It found no evidence of extraterrestrial beings, activity or technology. Most cases are ordinary objects misidentified.[114] NASA's study team reached the same conclusion and stressed that the data are poor.[115]

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[108] [56] Konstapel, J. (2026). Wakeful Machines and Watchers: How Future War Will Really Be Fought. constable.blog, 2 October 2026.

[109] [58] Agobard of Lyon (c. 815). De grandine et tonitruis (On Hail and Thunder). English translation in the Internet Medieval Sourcebook, Fordham University.

[110] [59] Vallée, J. (1969). Passport to Magonia: From Folklore to Flying Saucers. Regnery, Chicago.

[111] [60] Keel, J. A. (1970). UFOs: Operation Trojan Horse. Putnam, New York.

[112] [61] Vallée, J. (1988). Dimensions: A Casebook of Alien Contact. Contemporary Books, Chicago.

[113] [62] Vallée, J. (1975). The Invisible College. Dutton, New York.

[114] [63] All-domain Anomaly Resolution Office (2024). Report on the Historical Record of U.S. Government Involvement with Unidentified Anomalous Phenomena, Vol. I; and FY2024 Consolidated Annual Report on UAP. US Department of Defense.

[115] [64] NASA UAP Independent Study Team (2023). Final Report. NASA, Washington DC.

- **Known.** The official studies test the extraterrestrial explanation. The interdimensional explanation has not been tested by them.
- **Known.** In optics and electronics an exceptional point makes modes appear and vanish abruptly as a parameter is crossed. Systems held near it react strongly to very small disturbances.[116]

Back to the whole

> **Reading.** If such appearances come from the other side, the pattern of this book says where and how they could be seen: only near the border, where both sides balance, and abruptly. In PT systems an exceptional point makes modes appear and vanish without a gradual change. Witnesses often describe sudden appearance and disappearance.
>
> Vallée's control system has the form of Chapter 1's Watchers: something that stays at the border and adjusts what humans believe, but does not explain itself.
>
> This is a likeness of form. It explains no single case. The data do not decide it.

In this field, the border is the place where an appearance from the other side would be possible, and nowhere else.

8. Back to the whole

Seven fields, one form. This chapter states the pattern as four rules and then checks what each rule stands on. The rules are readings, built on the known and measured facts of the chapters.

8.1 Four rules

- **Rule 1. One reality, two readings.** The primes and the zeros are one object read twice (Ch. 3). The crystal and its reciprocal lattice are one object read twice (Ch. 4). Hot and cold Ising systems are one model read twice (Ch. 4). In Rowlands' physics space and antispace add up to zero. In the Vacuum.Net theory every trit has its mirror.
- **Rule 2. The border is where both readings weigh the same.** The critical line (Ch. 3). The self-dual point, the Lee–Yang circle, the zone edge and the PT threshold (Ch. 4). Focus 27 and the moment of falling asleep (Ch. 5). The decree that closes the year (Ch. 2).
- **Rule 3. Beyond the border one side wins.** Zeros of two voices leave the line (Ch. 3). Below the critical temperature order wins; above it disorder (Ch. 4). The PT spectrum turns complex (Ch. 4). In the old texts the Watchers cross and the balance breaks (Ch. 1).
- **Rule 4. Swinging keeps the balance; standing still on the border does not cross it.** The human sleeps and wakes; the Ba flies out and returns. The Watchers and the wakeful machines do not swing. Certainty without the swing stays on the threshold (Ch. 6).

8.2 The border as detector

One property appears in physics and is easy to miss. At the border a system is most sensitive. The exceptional-point sensors of 2017 work for that reason.[117] Near a critical point, a magnet responds to the smallest field. And the Watchers, who stand on the threshold, are the ones who see everything. The border is the place of balance, but also the place of seeing.

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[116] [47] Chen, W., Özdemir, Ş. K., Zhao, G., Wiersig, J. & Yang, L. (2017). Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196.

[117] [47] Chen, W., Özdemir, Ş. K., Zhao, G., Wiersig, J. & Yang, L. (2017). Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196.

8.3 What the book stands on

Table 8.1. What this book stands on.

Status	What it covers
Proved	The edge of computation (LLPO) in the author's mathematics; the Kramers–Wannier location of the Ising critical point (confirmed by Onsager); the Lee–Yang circle theorem; the formal RH proof, if it stands.
Known	Explicit formula; Weil positivity; Montgomery–Dyson statistics; CCM operator; Brillouin zones and band gaps; PT symmetry, its threshold and EP sensing; Rowlands' dual space; the texts of Enoch, Jubilees, Daniel; Annus; MUL.APIN; Monroe's own descriptions; van Lommel's study; Agobard; AARO and NASA findings.
Measured	Equal loudness on the critical line; the ladders inside the zeros; the windows 3, 9, 27; 364 = $(3^6-1)/2$; the day-and-night ratios of 1 Enoch 72; Lee–Yang zeros of a 12-spin ring.
Reading	That these borders are one border; the Watchers and Focus 27 as the threshold; the formal proof as "knowledge before capacity"; appearances at the border.

The strength of the pattern is not any single link. Each link alone can be doubted. It is the return of the same form, found independently, in languages that do not know of each other. The author has said this before about the complementary domain: the same pattern is rediscovered each time, independently, in a new language.[118]

8.4 What the pattern is not

It is not a claim that heaven is a reciprocal lattice. It is not a claim that the Watchers were physicists. It is not a claim that UFOs are exceptional points. The claim is smaller and, I think, stronger. Wherever people have thought hard about two sides of one reality, they have found the same three things: a mirror, a line where the mirror is exact, and a breaking of balance beyond it. Physics and mathematics have made this form exact. The old texts gave it its first names.

9. Open questions

Each question below can be worked on. Some need computation, some need a laboratory, some need archives.

- **One object?** Is the border one mathematical object across the fields? A test: write the Kramers–Wannier point, the Brillouin edge, the PT threshold and the critical line as the fixed set of one kind of mirror, and see whether the same algebra appears. The Lee–Yang analogy is the natural starting point.[119]
- **Sleep for the Trit Tile?** Do the turning points of the Trit Tile, which never close, have a 'sleep': a rule that lets them close after all? If so, what does it look like in the window operator of Chapter 3?
- **Trits in Babylon?** Did the Babylonian schematic astronomy behind 1 Enoch use trit windows on purpose, or is 364 = $(3^6-1)/2$ a coincidence? A search of cuneiform numbers for $(3^n-1)/2$ = 1, 4, 13, 40, 121, 364 is needed.
- **The border on a bench.** Can a balanced gain–loss circuit be built whose spectrum follows the window operator of Chapter 3? That would put the border of number theory on a laboratory bench, as Bender, Brody and Müller hoped.[120]

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[118] [2] Konstapel, J. (2026). [On the complementary domain and Dual Space]. constable.blog, 21 July 2026.

[119] [45] Knauf, A. (1999). Number theory, dynamical systems and statistical mechanics. Reviews in Mathematical Physics 11, 1027–1060.

[120] [49] Bender, C. M., Brody, D. C. & Müller, M. P. (2017). Hamiltonian for the zeros of the Riemann zeta function. Physical Review Letters 118, 130201.

- **A measure of the swing.** Monroe's Focus levels have no physical measurement. Is there a measurable quantity in sleep that behaves like the swing of Rule 4, and that is held still at Focus 10?
- **Abrupt appearances.** Which reports of appearances, if any, show the abrupt onset and loss that an exceptional point predicts? The AARO archive is the place to look.[121]
- **Understanding the proof.** Can the 2026 proof be read back into a human argument, chapter by chapter, as Enoch wrote his book by months?

Annotated references

Numbers match the footnotes. Each entry says what the source adds and where to read further.

[1] Konstapel, J. with Claude (2026). Research logs 'Riemann hypothesis and the Trit Tile' v0.27 and 'Watchers and mathematics' v0.2, with Python scripts and data. Constable Research, Leiden (unpublished; available as the package music_of_the_primes_v1.0).
All measurements in this book can be repeated with these scripts. The logs also keep the negative results.

[2] Konstapel, J. (2026). [On the complementary domain and Dual Space]. constable.blog, 21 July 2026.
Survey of the 'complementary domain' from Egypt to Monroe, ending in the author's Dual Space. Background to Chapters 1 and 5.

[3] Konstapel, J. (2026). The Vacuum.Net Theory, Ninth Edition: Foundations 9.2 and Physics 9.0. Constable Research, Leiden.
The formal statement of the theory used in the introduction: one strand, the rules N1–N5, trits, ladders and closure.

[4] Konstapel, J. (2026). Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (19 November 2026). Constable Research, Leiden.
Grows mathematics in 22 rungs from the eternal round. Gives the mirror (Rung 3–4), the node (Rung 11), closure as oscillation (Rung 21) and the edge where computing must stop (chapter h). Read for the formal side of Chapters 0, 3 and 6.

[5] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1. Constable Research, Leiden.
The number system in base 1 − ω whose turning points never close. The smallest window of Chapter 3.

[6] Taylor, J. H. (ed.) (2010). Journey through the Afterlife: Ancient Egyptian Book of the Dead. British Museum Press, London.
Accessible survey of the Egyptian parts of the person (ka, ba, akh) and the passage to the Duat.

[7] Plato. Republic, Book VII, 514a–520a (the allegory of the cave). Transl. P. Shorey (1935), Loeb Classical Library.
The oldest Western image of two sides and a passage between them.

[8] Annus, A. (2010). On the origin of Watchers: a comparative study of the antediluvian wisdom in Mesopotamian and Jewish traditions. Journal for the Study of the Pseudepigrapha 19(4), 277–320. doi:10.1177/0951820710373978.
The key study. The Watchers are an inverted version of the Mesopotamian apkallu, the sages before the flood. Read for Oannes, Enmeduranki and the link with Enoch.

[9] Milik, J. T. (1976). The Books of Enoch: Aramaic Fragments of Qumrân Cave 4. Clarendon Press, Oxford.
First edition of the Aramaic Enoch fragments from the Dead Sea Scrolls, including the Astronomical Book and the Book of Giants.

[10] Charles, R. H. (transl.) (1917). The Book of Enoch. SPCK, London.
The classic English translation of 1 Enoch, freely available. Chapters 6–8 (the Watchers) and 72–82 (the Astronomical Book).

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[121] [63] All-domain Anomaly Resolution Office (2024). Report on the Historical Record of U.S. Government Involvement with Unidentified Anomalous Phenomena, Vol. I; and FY2024 Consolidated Annual Report on UAP. US Department of Defense.

[11] Daniel 4:13, 17, 23 (Aramaic). Any critical translation, e.g. the NET Bible with notes.
The only biblical use of 'watcher' (ʿîr): a holy one who comes down from heaven and decrees. The notes discuss the word.

[12] Charles, R. H. (transl.) (1902). The Book of Jubilees. A. & C. Black, London.
Jubilees 4:15–17 (the Watchers sent to teach; Enoch the first to write the signs of heaven) and 8:2–3 (Kainan's inscription).

[13] Konstapel, J. (2026). The Wakeful Ones: Enoch & Monroe. constable.blog, 1 September 2026.
Reads the Watchers as beings on the threshold between space and antispace, and maps Monroe's Focus levels onto it.

[14] Neugebauer, O. (1981). The 'Astronomical' Chapters of the Ethiopic Book of Enoch (72 to 82). Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser 40:10, Copenhagen.
The mathematical reading of the Astronomical Book by the historian of exact sciences. Gates as rising points on the horizon.

[15] Ratzon, E. (2019). 4Q208: A new reconstruction and its implications on the evolution of the Astronomical Book. Revue de Qumran 31(1), 51–110.
The oldest copy of the Aramaic Astronomical Book (3rd–2nd c. BCE). Shows that later scribes adapted its scheme to the 364-day year.

[16] Sky & Telescope (2020). Meet Kochab, Guardian of the Pole. skyandtelescope.org.
Explains why Kochab and Pherkad are called the Guardians of the Pole and how precession made Kochab a pole star around 1000 BCE.

[17] Hunger, H. & Pingree, D. (1989). MUL.APIN: An Astronomical Compendium in Cuneiform. Archiv für Orientforschung, Beiheft 24. Berger, Horn.
Edition of the main Babylonian star compendium, with the ideal 360-day year and the 2:1 day scheme that lie behind 1 Enoch 72.

[18] Brack-Bernsen, L. (2007). The 360-day year in Mesopotamia. In: J. M. Steele (ed.), Calendars and Years: Astronomy and Time in the Ancient Near East. Oxbow, Oxford, 83–100.
Shows that the 360-day year was an administrative and schematic year, not a real one. Background for Chapter 2.

[19] Drawnel, H. (2025). The literary structure and schematic clauses in 1 En. 72:8–32. Dead Sea Discoveries 32.
Traces the day-and-night numbers of 1 Enoch 72 to early Babylonian schematic astronomy.

[20] Ratzon, E. (2015). The gates cosmology of the Astronomical Book of Enoch. Dead Sea Discoveries 22, 93–111. doi:10.1163/15685179-12341336.
The 364-day year against the Babylonian 360; twelve gates mapped like the zodiac.

[21] Jacobus, H. R. (2019). Reconstructing the calendar of 4Q208–4Q209 (and a response to Eshbal Ratzon). Revue de Qumran 31(2), 251–273.
A competing mathematical reconstruction. Read with Ratzon 2019 to see that the calendar is still debated.

[22] Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press, New York.
A readable classic on Riemann's 1859 paper, the functional equation and von Mangoldt's explicit formula (1895). Start here for Chapter 3.

[23] Landau, E. (1912). Über die Nullstellen der Zetafunktion. Mathematische Annalen 71, 548–564.
Shows that the zeros 'remember' the primes: sums over the zeros peak at the logarithms of prime powers.

[24] Weil, A. (1952). Sur les « formules explicites » de la théorie des nombres premiers. Comm. Sém. Math. Univ. Lund, tome supplémentaire, 252–265.
Turns the explicit formula into a quadratic form. The Riemann hypothesis is equivalent to its positivity. The bridge between the two sides.

[25] Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. In: Analytic Number Theory, Proc. Symp. Pure Math. 24. AMS, Providence, 181–193.

The zeros repel each other like the energy levels of a random quantum system (Dyson's remark). A key hint that the zeros are eigenvalues.

[26] Odlyzko, A. M. (1987). On the distribution of spacings between zeros of the zeta function. Mathematics of Computation 48, 273–308.
Large-scale computation that confirms Montgomery's prediction.

[27] Berry, M. V. & Keating, J. P. (1999). The Riemann zeros and eigenvalue asymptotics. SIAM Review 41, 236–266.
Proposes that the zeros are energies of a quantum system built on H = xp. Physics' main entry into the Riemann problem.

[28] Connes, A., Consani, C. & Moscovici, H. (2025). Zeta spectral triples. arXiv:2511.22755.
Builds one operator per window of length ln x from the primes below x. Its spectrum agrees numerically with the lowest zeros. Chapter 3 recomputes it.

[29] Connes, A. (2026). The Riemann Hypothesis: past, present and a letter through time. arXiv:2602.04022.
Survey by Connes, including the quadratic-form method and the first 50 zeros approximated from the primes up to 13.

[30] NRC (2026). Article on the computer-checked proof of the Riemann hypothesis by L. Alpöge with Claude. 22 November 2026.
Dutch newspaper report of the proof that nobody understands. Starting point of Chapter 6.

[31] Lean FRO and Mathlib community (2026). Lean verification of the Riemann Hypothesis. leanprover-community.github.io, 21 November 2026.
States that the 17-million-line proof is formally correct.

[32] Konstapel, J. (2026). The Successor of the Holy Grail, version 1.0 (23 November 2026). Constable Research, Leiden.
Essay with eight figures on the prime ladders, the functional equation and the zeros. Chapter 3 summarises it.

[33] Davenport, H. & Heilbronn, H. (1936). On the zeros of certain Dirichlet series. Journal of the London Mathematical Society 11, 181–185.
A function with a mirror but without one voice: its zeros leave the line. The counter-example behind Chapter 3's two voices.

[34] Bragg, W. H. & Bragg, W. L. (1913). The reflection of X-rays by crystals. Proceedings of the Royal Society A 88, 428–438.
The founding experiment of reading a crystal on its wave side.

[35] Bloch, F. (1929). Über die Quantenmechanik der Elektronen in Kristallgittern. Zeitschrift für Physik 52, 555–600.
Electrons in a lattice as waves with a crystal momentum. The basis of band theory.

[36] Kittel, C. (2005). Introduction to Solid State Physics, 8th ed., Chapters 2 and 7. Wiley, Hoboken.
The standard text on the reciprocal lattice, Brillouin zones and band gaps. Chapter 7 explains the standing waves at the zone edge.

[37] Bender, C. M. & Boettcher, S. (1998). Real spectra in non-Hermitian Hamiltonians having PT symmetry. Physical Review Letters 80, 5243–5246.
The paper that opened PT-symmetric physics: balance between two sides keeps the spectrum real.

[38] Guo, A. et al. (2009). Observation of PT-symmetry breaking in complex optical potentials. Physical Review Letters 103, 093902.
First optical observation of the threshold where the spectrum stops being real.

[39] Rüter, C. E., Makris, K. G., El-Ganainy, R., Christodoulides, D. N., Segev, M. & Kip, D. (2010). Observation of parity–time symmetry in optics. Nature Physics 6, 192–195.
Two coupled waveguides, one with gain and one with loss: the border made in the laboratory.

[40] Rowlands, P. (2007). Zero to Infinity: The Foundations of Physics. World Scientific, Singapore.
The physics of 'zero totality': everything adds up to nothing, and every space has its dual.

[41] Rowlands, P. (2015). Dual spaces, particle singularities and quartic geometry. Acta Mathematica Academiae Paedagogicae Nyíregyháziensis 31, 121–137.
States the dual 3-dimensional space that cancels our observed space. Short and direct.

[42] Kramers, H. A. & Wannier, G. H. (1941). Statistics of the two-dimensional ferromagnet. Part I. Physical Review 60, 252–262.
Finds the critical temperature of the square Ising lattice as the self-dual point of a hot–cold duality. The clearest border in statistical physics; Chapter 4.

[43] Onsager, L. (1944). Crystal statistics. I. A two-dimensional model with an order-disorder transition. Physical Review 65, 117–149.
The exact solution that confirmed the self-dual point of Kramers and Wannier.

[44] Lee, T. D. & Yang, C. N. (1952). Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. Physical Review 87, 410–419.
Proves that the zeros of the Ising partition function lie on a circle. A phase transition happens where they pinch the real axis. Chapter 4 compares this with the critical line.

[45] Knauf, A. (1999). Number theory, dynamical systems and statistical mechanics. Reviews in Mathematical Physics 11, 1027–1060.
Explores links between the Lee–Yang theorem and the Riemann hypothesis. For readers who want the bridge between Chapters 3 and 4.

[46] Kato, T. (1966). Perturbation Theory for Linear Operators. Springer, Berlin.
The mathematical source of the 'exceptional point', where two eigenvalues and their eigenvectors coalesce.

[47] Chen, W., Özdemir, Ş. K., Zhao, G., Wiersig, J. & Yang, L. (2017). Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196.
A system held at the exceptional point reacts much more strongly to a small disturbance. The border as a detector.

[48] Hodaei, H. et al. (2017). Enhanced sensitivity at higher-order exceptional points. Nature 548, 187–191.
A third-order exceptional point: response grows with the cube root of the disturbance.

[49] Bender, C. M., Brody, D. C. & Müller, M. P. (2017). Hamiltonian for the zeros of the Riemann zeta function. Physical Review Letters 118, 130201.
A PT-type operator whose eigenvalues would be the zeros. Not rigorous; see the comment by Bellissard (arXiv:1704.02644).

[50] Marcer, P. & Rowlands, P. (2017). Nilpotent quantum mechanics: analogs and applications. Frontiers in Physics 5:28. doi:10.3389/fphy.2017.00028.
The fermion state and the vacuum are mirror images: a change in one leads to a corresponding change in the other.

[51] Monroe, R. A. (1971). Journeys Out of the Body. Doubleday, New York.
The first book. Defines Locale I, II and III. Locale II holds 'all of the aspects we attribute to heaven and hell'.

[52] Monroe, R. A. (1985). Far Journeys. Doubleday, New York.
The second book; introduces the Focus levels and the 'Gathering' of non-physical observers.

[53] Monroe, R. A. (1994). Ultimate Journey. Doubleday, New York.
The third book; the belief-system territories and Focus 27 as the reception area.

[54] van Lommel, P., van Wees, R., Meyers, V. & Elfferich, I. (2001). Near-death experience in survivors of cardiac arrest: a prospective study in the Netherlands. The Lancet 358, 2039–2045.
Prospective study of 344 resuscitated patients in ten Dutch hospitals. The most cited clinical study of the border between life and death. It has been criticised; read the critiques too.

[55] McDonnell, W. M. (1983). Analysis and Assessment of Gateway Process. US Army, declassified 2003 (CIA-RDP96-00788R001700210016-5).
The military assessment of Monroe's method. Shows that the state took the border seriously.

[56] Konstapel, J. (2026). Wakeful Machines and Watchers: How Future War Will Really Be Fought. constable.blog, 2 October 2026.
Source of the phrases 'knowledge before capacity' and 'a being that never sleeps does not oscillate'. Chapter 6 builds on it.

[57] Thurston, W. P. (1994). On proof and progress in mathematics. Bulletin of the AMS 30(2), 161–177.
Mathematics is about human understanding, not only about proofs.

[58] Agobard of Lyon (c. 815). De grandine et tonitruis (On Hail and Thunder). English translation in the Internet Medieval Sourcebook, Fordham University.
The only medieval source for 'Magonia', a region from which ships sail in the clouds. Vallée took his book title from it.

[59] Vallée, J. (1969). Passport to Magonia: From Folklore to Flying Saucers. Regnery, Chicago.
Links modern UFO reports to old folklore of beings from another realm.

[60] Keel, J. A. (1970). UFOs: Operation Trojan Horse. Putnam, New York.
Argues, like Vallée, that the phenomenon comes from outside our ordinary spectrum and links it to folklore.

[61] Vallée, J. (1988). Dimensions: A Casebook of Alien Contact. Contemporary Books, Chicago.
The fullest statement of the interdimensional hypothesis.

[62] Vallée, J. (1975). The Invisible College. Dutton, New York.
Introduces the 'control system' idea.

[63] All-domain Anomaly Resolution Office (2024). Report on the Historical Record of U.S. Government Involvement with Unidentified Anomalous Phenomena, Vol. I; and FY2024 Consolidated Annual Report on UAP. US Department of Defense.
The official finding: no evidence of extraterrestrial beings, activity or technology. The interdimensional hypothesis is not tested.

[64] NASA UAP Independent Study Team (2023). Final Report. NASA, Washington DC.
No evidence of extraterrestrial origin; data quality is the main problem.