The Successor of the Holy Grail

What the proof of the Riemann hypothesis leaves open, seen through the Trit Tile and the Vacuum.Net theory

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Abstract

On 20 November 2026 a computer-checked proof of the Riemann hypothesis was announced. It is correct by the standards of formal verification. Nobody understands it. This essay asks what remains to be understood, and what the successor of this "holy grail" of mathematics is. We approach the question from a small object: the Trit Tile, a number system in base $1 - \omega$ on the triangular lattice. Its rhythm is exactly periodic. The rhythm of the zeta zeros is not. We measure, in eight pictures, how the rhythm of the zeros arises from the rhythms of the primes. Each prime is a ladder with period ln p. One ladder gives a regular fence. Many loose ladders give chance. Ladders coupled by the functional equation, with a direction, give the repelling rhythm of random matrices. Finite sets of ladders create zeros off the line, near their own poles. These disappear only when all ladders act together. Over the finite field F_3, the field of the trits, a single operator explains everything: Frobenius. Over the integers that operator is missing. We conclude that the successor of the grail is this operator: one object that contains all prime ladders at once and has the zeros as its eigenvalues. Much of the mathematics here is known. What is new is the synthesis, the measurements as pictures, and the link to the Trit Tile.

Keywords: Riemann hypothesis; Hilbert–Pólya; random matrix theory; Euler product; functional equation; complex dimensions; finite fields; Trit Tile; Vacuum.Net theory.

1. Introduction

1.1 What happened

The Riemann hypothesis (RH) has been open since 1859. It is one of the seven Millennium Prize Problems. On 20 November 2026 Levent Alpöge announced a proof, written with the AI system Claude.[1] The proof exists only as a library in the proof assistant Lean. It has about 17 million lines. On 21 November the Lean and Mathlib community stated that both the statement and the proof are correct. Two independent Lean kernels accepted it.[2] The proof also covers the generalised RH for all Dirichlet L-functions.

The proof uses many known human ingredients in a new combination.[3] A readable version would run to some ten thousand pages. Nobody, including its author, can follow it as a whole. The Clay Mathematics Institute still lists the problem as open. It waits two years before it decides.

1.2 The question

Many mathematicians reacted in the same way. A proof should tell us *why*. This one tells us only *that*. The question of this essay follows from that reaction. If the statement is now settled, what is

[1] [1] Van den Brandhof, A. (2026). Article on the Riemann proof of L. Alpöge. NRC, 22 November 2026.

[2] [2] Lean FRO and Mathlib community (2026). Lean verification of the Riemann Hypothesis. leanprover-community.github.io/blog, 21 November 2026.

[3] [3] Nasqret (2026). riemann-rev: a map of the Lean proof of the Riemann Hypothesis. github.com/nasqret/riemann-rev.

the next grail? We answer: the mechanism. More precisely: the object that explains why the zeros lie on one line and why they keep their distance.

We reach this answer from an unusual direction. The starting point is the Trit Tile, a small object from the Vacuum.Net theory.[4] Its rhythm is exactly periodic, and we know why. The rhythm of the zeta zeros is irregular, and we do not know why. The essay walks from the one to the other.

1.3 Method: seeing, reasoning, and back to the whole

Dutch has an old word for mathematics: *wiskunde*. It means "knowledge of what is certain". The formal proof delivers exactly that: certainty. It does not deliver insight. Thurston wrote that mathematics is about increasing human understanding, not only about proving theorems.[5] We follow that view. The essay has three movements.

- **Seeing.** Eight pictures, each from a computation that anyone can repeat.[6] They show the shape of the problem.
- **Reasoning.** What in each picture is proved, by whom, and what is only measured.
- **Back to the whole.** What the pictures say together, and what they point to.

Every claim carries one of four labels. **Proved** means a proof exists. **Known** means a published result of others. **Measured** means our own computation. **Reading** means an interpretation in terms of the Vacuum.Net theory; it may guide work but it is not a theorem.

2. Background: the Vacuum.Net theory

This section explains the theory as far as this essay needs it. No earlier text is required.[7]

The Vacuum.Net theory starts from movement, not from sets or numbers. There is exactly one strand. It moves and returns. Every distinction is a loop of this strand. Five rules fix how it behaves.

- **N1.** There is exactly one strand.
- **N2, closure.** A loop persists only if it closes. What does not close passes on as tension.
- **N3.** The first passage through a point fixes a direction.
- **N4.** The response depends on the current state.
- **N5.** Coarse blocks of the strand obey the same rules again.

Each step of the strand is a *trit*: forward (+1), standing still (0) or backward (−1). On a crossing there is no value, only a direction. Trits combine into *ladders*. The two basic ladders are doubling (×2) and tripling (×3). No power of 2 equals a power of 3. The ladders come close at certain depths: 1, 2, 5, 12, 41, 53 and so on. At depth 12 the miss is the Pythagorean comma.

Closure is the central idea of this essay. A loop that closes stays. In number theory, a prime p will turn out to behave like a closed loop of length ln p. The *step* (±1, adding) and the *ladder* (×p, multiplying) are the two movements that the essay keeps apart and then joins.

3. Background: the Trit Tile

Take a point $a + b\omega$ of the triangular lattice. Here ω is a cube root of one, $\omega = (-1 + i\sqrt{3})/2$. Subtract a digit −1, 0 or +1 so that the result is divisible by $\beta = 1 - \omega$. Divide by β. Repeat. Each step stretches

[4] [4] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1 (21 November 2026). Constable Research, Leiden.

[5] [5] Thurston, W. P. (1994). On proof and progress in mathematics. Bulletin of the AMS 30(2), 161–177.

[6] [6] Konstapel, J. with Claude (2026). Research log 'Riemann hypothesis and the Trit Tile', version 0.23, with Python scripts and data (zeros.py, ritme.py, graaf.py, schreier.py, euler_theta.py, landschap.py, buiten_lijn.py, twee_stemmen.py, familie.py and others). Constable Research, Leiden (unpublished).

[7] [7] Konstapel, J. (2026). Vacuum.Net Theory, Ninth Edition: Foundations 9.2 and Physics 9.0. Constable Research, Leiden.

distances by √3 and turns the plane by −30 degrees. Some points reach 0: they close. All others end in one of two points that turn forever.[8]

Proved. The share of turning points does not settle. It oscillates with exact period: it repeats each time the norm grows by a factor 3. Its mean is 30.87%.[9]

Known. This oscillation is a case of the theory of complex dimensions.[10] The tile has complex dimensions on one vertical line, at 1 + 2πik/ln 3 for integers k. They form an arithmetic progression with spacing 2π/ln 3. Such objects are called *lattice* objects. Lapidus and van Frankenhuijsen showed that lattice objects behave like varieties over a finite field.[11]

Known. The residue field of the tile is the field of the trits: $\mathbb{Z}[\omega]/(1-\omega) \cong F_3$. The digits −1, 0, +1 are exactly its three elements, with addition and multiplication modulo 3. So the trit is an algebraic field. This fact matters in Section 5.7.

Figure 2 — The Trit Tile: one ladder (×3) over the field $F_3 = \mathbb{Z}[\omega]/(1-\omega)$

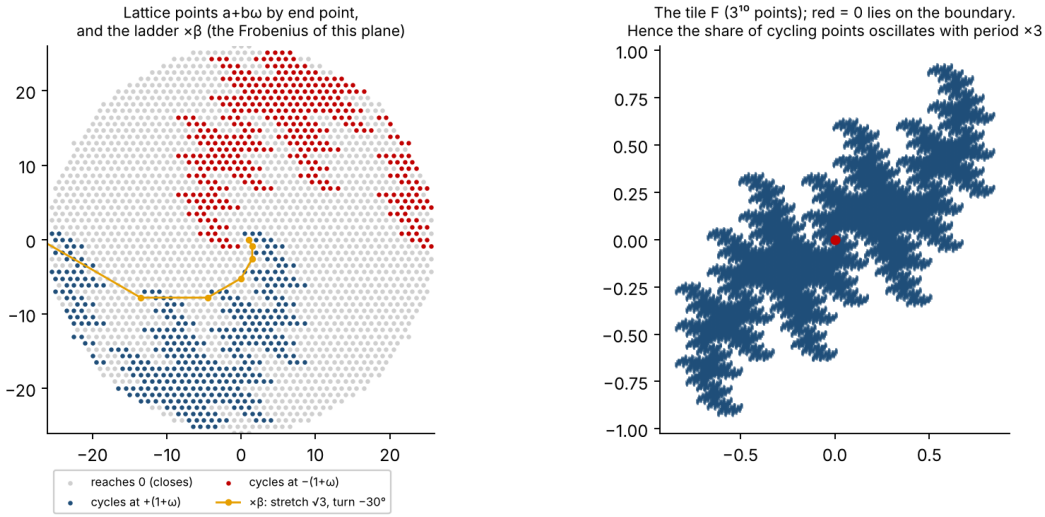


Figure 2. The Trit Tile. Left: lattice points coloured by where they end (grey: reach 0; blue and red: turn forever), with the ladder ×β in orange (stretch √3, turn −30°). Right: the tile F, built from 3^{10} digit strings. The origin (red) lies on its boundary. That is why the share of turning points keeps oscillating with period ×3. *Measured* after the Trit Tile reference.[12]

4. Background: the zeta function in two descriptions

The Riemann zeta function ζ(s) can be written in two ways. Both matter for everything that follows.

The ladders (Euler product). ζ(s) = Π_p (1 − p^(−s))^(−1), over all primes p. Each factor belongs to one prime. On a vertical line each factor repeats with period 2π/ln p. So each factor is a lattice rhythm, like the Trit Tile. The factor for p = 3 has exactly the tile's period. In this sense the Trit Tile is one factor of ζ.

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[8] [4] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1 (21 November 2026). Constable Research, Leiden.

[9] [4] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1 (21 November 2026). Constable Research, Leiden.

[10] [8] Lapidus, M. L. & van Frankenhuijsen, M. (2013). Fractal Geometry, Complex Dimensions and Zeta Functions, 2nd ed. Springer.

[11] [9] Lapidus, M. L. & van Frankenhuijsen, M. (2004). Fractality, self-similarity and complex dimensions. Proceedings of Symposia in Pure Mathematics 72 (arXiv:math/0401156).

[12] [4] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1 (21 November 2026). Constable Research, Leiden.

The step (functional equation). $\zeta(s) = \chi(s)\,\zeta(1-s)$, with an explicit factor χ. It mirrors the function in the line Re s = 1/2. It comes from additive Fourier analysis (the theta function and Poisson summation).[13] On the critical line it defines a phase $\theta(t)$ that grows smoothly with height t.

The non-trivial zeros of ζ lie in the strip $0 < \mathrm{Re}\, s < 1$. RH says they all lie on the line Re s = 1/2. Their heights are irregular. They never form an arithmetic progression (**Known**, Putnam).[14] So the primes, taken together, never have the regular rhythm that one prime has alone.

Throughout, we measure rhythm with one number, $\langle \tilde{r} \rangle$. Take three consecutive levels. Divide the smaller gap by the larger. Average. Reference values (**Known**):[15] a regular fence gives 1; independent random levels (Poisson) give 0.386; eigenvalues of a random real symmetric matrix (GOE) give 0.536; eigenvalues of a random complex Hermitian matrix (GUE) give 0.603. The first 1000 zeros of ζ give 0.617 in our computation (**Measured**). This is GUE: the zeros repel each other.

5. Seeing: eight pictures

5.1 The ladders are inside the zeros

Take the first 1000 zeros γ_n. Add up cosines $\cos(\gamma_n x)$, weighted gently, and remove the smooth background. **Known** (Landau, 1912): peaks must appear at x = ln n, but only when n is a power of a prime.[16] Their height follows $\Lambda(n)/\sqrt{n}$.

Measured. All peaks appear at the right place, within 0.0002: ln 2, 3, 4, 5, 7, 8, 9, 11, 13, 16, 17, 19 and 23. There are no peaks at 6, 10, 12, 14 or 15. The peak at ln 3 has strength 9.5σ. The control, with the same gaps shuffled, shows only noise.

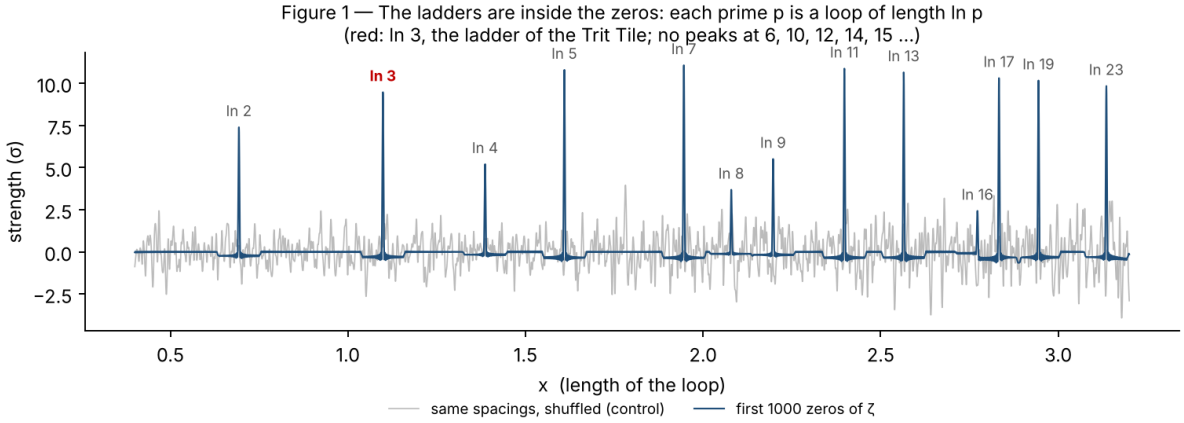


Figure 1. Each prime p is a loop of length ln p, and each loop leaves a peak in the zeros. Red: ln 3, the ladder of the Trit Tile. Grey: the same gaps in random order (control). *Measured; Known theory (Landau).*

Reading. Seen from far away, the zeros carry every prime ladder as a hidden period. The Trit Tile is one of them. Seen from close by, the zeros repel. The next pictures ask how both can be true.

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[13] [10] Tate, J. (1950/1967). Fourier analysis in number fields and Hecke's zeta-functions. In Cassels & Fröhlich (eds.), Algebraic Number Theory. Academic Press.

[14] [11] Putnam, C. R. (1954). On the non-periodicity of the zeros of the Riemann zeta-function. American Journal of Mathematics 76, 97–99.

[15] [12] Atas, Y. Y., Bogomolny, E., Giraud, O. & Roux, G. (2013). Distribution of the ratio of consecutive level spacings in random matrix ensembles. Physical Review Letters 110, 084101 (arXiv:1212.5611).

[16] [13] Landau, E. (1912). Über die Nullstellen der Zetafunktion. Mathematische Annalen 71, 548–564.

5.2 The staircase of rhythm

If each prime is a ladder, perhaps the zeros are just all ladders stacked. We tested this directly. The union of the progressions $2\pi k/\ln p$ for all $p \le P$ is exactly the zero set of a finite Euler product on a vertical line.

Measured. Stacking fails. One ladder gives a fence. Two or three give an intermediate form. More ladders move steadily towards Poisson, not towards GUE. With all primes up to 5000 the gaps are indistinguishable from chance. **Known:** this is a general law; superposing independent spectra gives Poisson (Berry–Robnik).

So the rhythm of ζ is not in the ladders, and not in their sum. It is in their coupling. We tested coupled loop models, after Kottos and Smilansky.[17] Loops have lengths $\ln p$. They meet at vertices that scatter the movement from one loop into the others.

Table 1. The staircase of rhythm (Measured).

Step	Model	$\langle \tilde{r} \rangle$	Class
1	One ladder (×3, the Trit Tile)	1.000	fence
2	Many loose ladders ($p \le 1000$)	0.388	Poisson
3	Coupled loops, forth = back	0.530	GOE
4	Coupled by adding and multiplying ($x \to 2x$, $x \to 3x$, $x \to x+1$ on $\mathbb{Z}/199$)	0.588	≈ GUE
—	Zeta: first 1000 zeros	0.617	GUE

Three further results sharpen the picture (**Measured**). First, a direction on each loop is not enough. Directed loops with a symmetric coupling still give GOE. GUE needs a coupling in which going from loop a to loop b differs from going back. Second, in a coupling through a Dirichlet character this asymmetry sits in a single sign, $\chi(-1)$. Odd characters gave GUE and even characters GOE, in ten cases out of ten. Third, adding alone or multiplying alone gives Poisson. The commutative symmetry splits the system into independent blocks. Only adding and multiplying together give repulsion.

Known. This is the structure of the Bost–Connes system, which joins addition on Q/Z with multiplication by the positive integers.[18] It is also the structure of Connes' adele class space: additive adeles modulo the multiplicative group.[19] It fits the physics: the Riemann dynamics must break time reversal, because each prime orbit counts only once.[20,21]

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[17] [14] Kottos, T. & Smilansky, U. (1999). Periodic orbit theory and spectral statistics for quantum graphs. Annals of Physics 274, 76–124 (arXiv:chao-dyn/9812005).

[18] [15] Bost, J.-B. & Connes, A. (1995). Hecke algebras, type III factors and phase transitions with spontaneous symmetry breaking in number theory. Selecta Mathematica 1, 411–457.

[19] [16] Connes, A. (1999). Trace formula in noncommutative geometry and the zeros of the Riemann zeta function. Selecta Mathematica 5, 29–106 (arXiv:math/9811068).

[20] [17] Sierra, G. (2010). A physics pathway to the Riemann hypothesis. arXiv:1012.4264.

[21] [18] Berry, M. V. & Keating, J. P. (1999). The Riemann zeros and eigenvalue asymptotics. SIAM Review 41(2), 236–266.

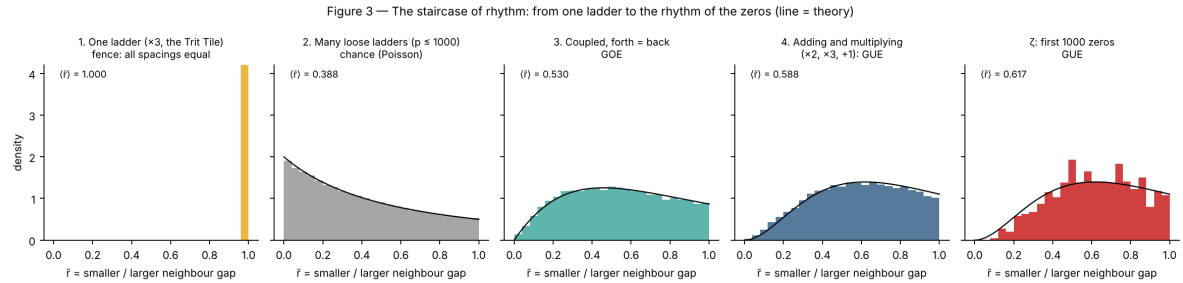


Figure 3. The staircase of rhythm. Histograms of r̃ for the five steps of Table 1. Black lines: theoretical distributions.[22] *Measured.*

5.3 The fence bends, ladder by ladder

The coupling models mix adding and multiplying inside one operator. That creates mixed loops of length ln 6, ln 10 and so on. Zeta has none: Figure 1 shows no peak at 6 or 10. Zeta keeps the two apart. Multiplying lives in the Euler product. Adding lives in the functional equation. So we built a model that keeps them apart too.[23]

The model is Z_X(t) = Re(e^(iθ(t)) · Π_(p≤X) (1 − p^(−1/2−it))^(−1)). The product holds the loose ladders up to X. The phase θ comes from the functional equation: the step. This is in essence Gonek's family of symmetrised finite Euler products.

Table 2. Step plus ladders, over the first 1000 zeros (Measured).

Ladders in the model	Median distance to the true zero (in mean gaps)	Within ¼ gap	⟨r̃⟩
none (θ only, the step)	0.186	64%	0.998 (fence)
×2	0.134	78%	0.873
×2, ×3 (the trit ladders)	0.102	92%	0.781
all p ≤ 13	0.050	99.8%	0.612
all p ≤ 1000	0.012	100%	0.618
ζ itself	—	—	0.617

Measured. The step alone gives exactly the right number of zeros, 1000, as a regular fence. **Known:** these are the Gram points, and their number follows from the Riemann–von Mangoldt formula. Each added ladder bends the fence. With only ×2 and ×3, 92% of the zeros lie within a quarter gap of the true ones. With all primes up to 1000 the rhythm is that of ζ.

[22] [12] Atas, Y. Y., Bogomolny, E., Giraud, O. & Roux, G. (2013). Distribution of the ratio of consecutive level spacings in random matrix ensembles. Physical Review Letters 110, 084101 (arXiv:1212.5611).

[23] [19] Gonek, S. M. (2012). Finite Euler products and the Riemann Hypothesis. Transactions of the AMS 364(4), 2157–2191 (arXiv:0704.3448).

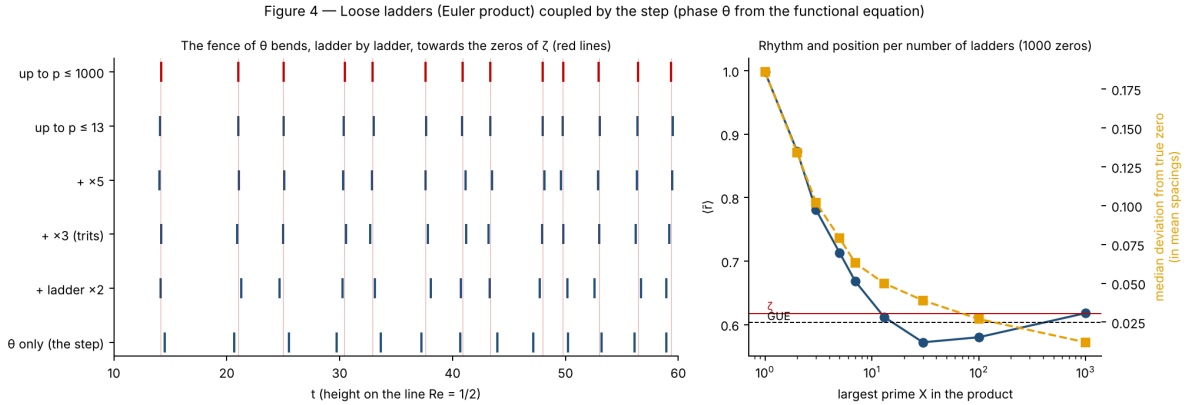


Figure 4. Left: the zeros of the model for growing sets of ladders (blue), against the true zeros of ζ (red lines). Right: rhythm and median position error as functions of the largest prime X. *Measured.*

Reading. The Trit Tile is one ladder without the step: a fence that does not bend. Zeta is the step with all ladders. **Known:** Gonek noted that number theorists expect a proof of RH to use both the Euler product and the functional equation, since functions with only one of the two can violate RH.[24]

5.4 The landscape: why the zeros keep their distance

Write the product of the ladders as P = |P| e^(i arg P). On the critical line the model is zero where the total phase θ + arg P crosses π/2 + kπ. **Known** (Cauchy–Riemann): the speed of arg P along the line equals the slope of log|P| to the right. We confirmed this numerically (correlation 1.000000).

Measured (X = 1000). All zeros of the model lie in valleys of the landscape log|P|. The median |P| at a zero is 0.23, against 1.08 overall. Between zeros lie ridges. There the phase slows down or even runs back. It runs back up to 32% of the time, but never near a level where a zero could form.

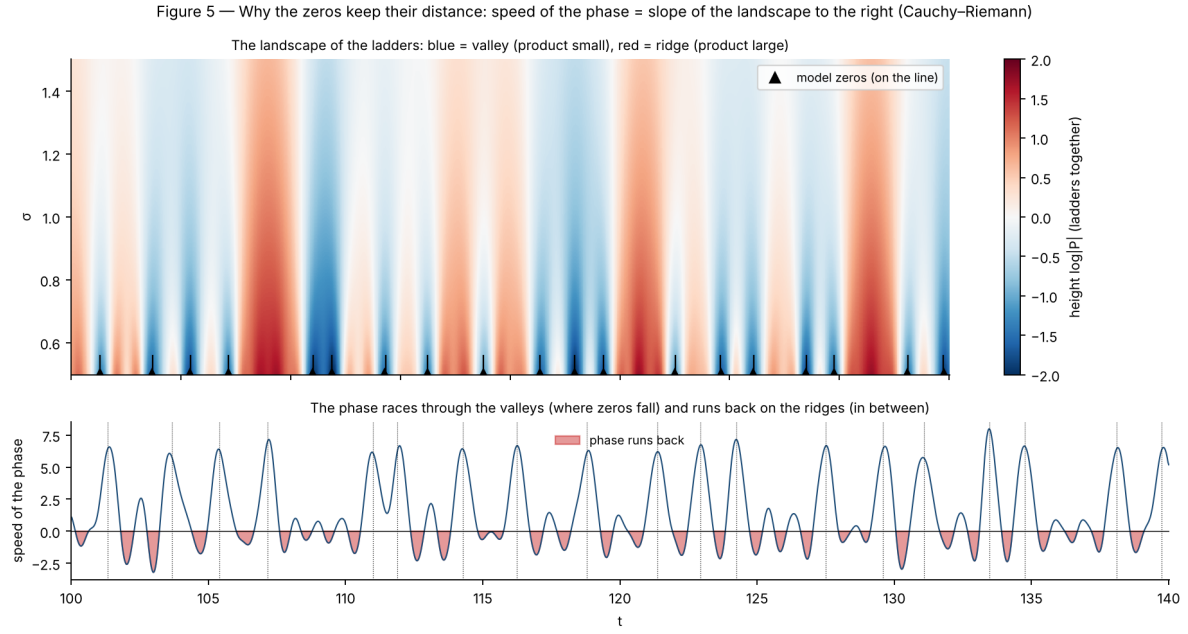


Figure 5. Top: the landscape log|P| of the ladders up to 1000 above the critical line (blue valleys, red ridges), with the model zeros (triangles) and the true zeros (ticks). Bottom: the speed of the phase. It races through the valleys, where the zeros fall, and runs back on the ridges. *Measured; Known mechanism (Cauchy–Riemann).*

[24] [19] Gonek, S. M. (2012). Finite Euler products and the Riemann Hypothesis. Transactions of the AMS 364(4), 2157–2191 (arXiv:0704.3448).

Reading. Two wells cannot merge unless the ridge between them disappears. Repulsion is geometry. **Known:** this is Dyson's picture of random-matrix eigenvalues as charges on a line with logarithmic repulsion, the log-gas.[25]

5.5 Poles pull wells off the line

Figure 5 explains repulsion *on* the line. Does it also forbid zeros *off* the line? That would be RH itself. We continued the model off the line as G_X(s) = P_X(s) + χ(s) P_X(1 − s) and searched the strip.

Measured. It does not forbid them. With only ×2 and ×3 there are 41 zeros off the line between heights 100 and 250. With primes up to 13 there are 167, up to 100 there are 313. Zeta has none there. All of these off-line zeros, in every case, lie within 0.3 of a pole of one of the ladders. These poles sit on σ = 1, at heights 2πk/ln p. For p = 3 they are exactly the complex dimensions of the Trit Tile. Random heights come that close only 6–66% of the time.

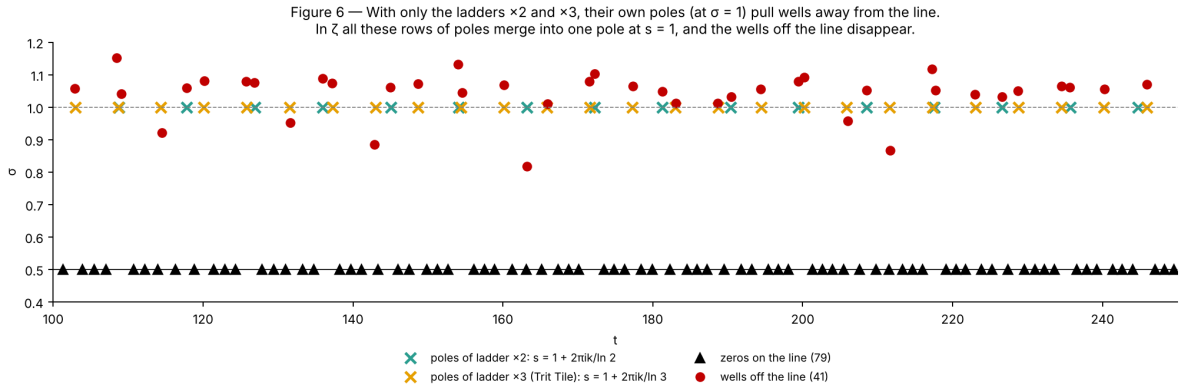


Figure 6. With only the ladders ×2 and ×3, the zeros on the line are all present (triangles), but extra wells appear off the line (red dots). Each sits next to a pole of a ladder (crosses on σ = 1). *Measured.*

Measured. When more ladders are added, the off-line wells drift towards σ = 1, not towards the critical line. Their median distance to σ = 1 falls from 0.083 to 0.026. None comes closer than σ ≈ 0.72. **Known:** for ζ itself exactly this region is empty. That is the classical zero-free region of de la Vallée Poussin and its later widenings.[26] **Known:** with the additive form (partial sums plus mirror) the picture is different. There most zeros stay on the line.[27]

Reading. Each ladder is a lattice object with its own row of poles, like the Trit Tile. As long as ladders are few, their own rhythms pull wells away from the line. In ζ all these rows merge into a single pole at s = 1. The ladders must give up their own rhythm to make the rhythm of the zeros together. RH is a property of the complete whole, not of any finite part.

5.6 Two voices

G_X is a sum of two terms: the product and its mirror. A zero needs both to be equally loud and exactly opposite. **Known, exact:** on the critical line the two are always equally loud, because |χ(1/2 + it)| = 1 and the mirror is the complex conjugate. So on the line a zero only has to find the right phase. That makes the line the natural home of zeros.

Measured. Off the line the two voices are rarely equally loud: in 2–26% of heights, falling quickly with distance from the line. Where they are, it is in 75–100% of cases close to a pole of a ladder: the mirror blows up there.

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[25] [20] Dyson, F. J. (1962). Statistical theory of the energy levels of complex systems I. Journal of Mathematical Physics 3, 140–156.

[26] [21] Ford, K. (2019). Zero-free regions for the Riemann zeta function. arXiv:1910.08205.

[27] [22] Gonek, S. M. & Montgomery, H. L. (2013). Zeros of a family of approximations of the Riemann zeta-function. International Mathematics Research Notices 2013(20), 4712–4733.

Figure 7 — Two voices: a zero needs equal loudness and opposite direction. On the line "equally loud" is guaranteed.

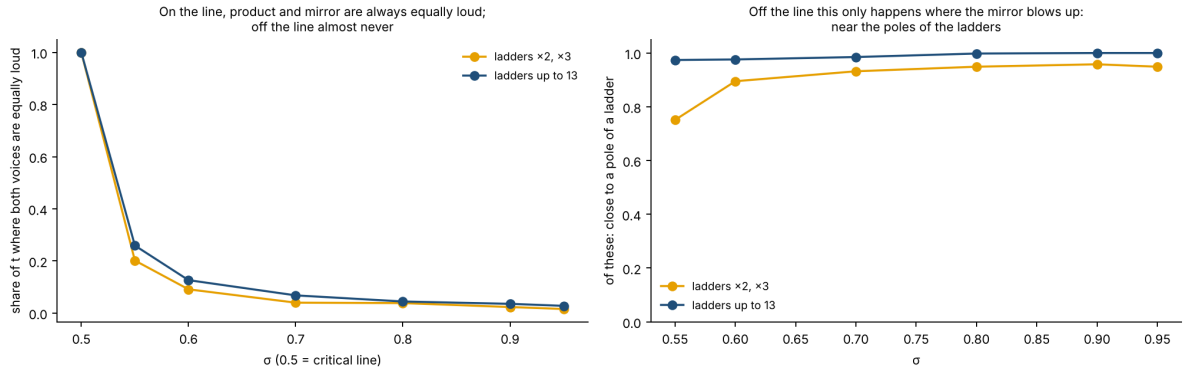


Figure 7. Left: share of heights where product and mirror are equally loud, as a function of σ. Right: of these, the share near a pole of a ladder. *Measured.*

We recomputed the classical Davenport–Heilbronn function.[28] It satisfies a functional equation but has no Euler product. It is a sum of two Dirichlet L-functions. **Measured:** at four of its off-line zeros, each L-function separately is far from zero (|L| between 0.42 and 0.70). They cancel because they are equally loud and 180° apart. **Proved** (if the new proof stands): each of them alone has no zeros off the line.

Reading. Two independent voices can cancel each other off the line. One voice cannot. For ζ the mirror is not a second voice: $\zeta(s) = \chi(s)\zeta(1 - s)$ is an identity. Zeta is one song that is its own mirror. **Known:** the Selberg class captures "one voice" in four axioms, including an Euler product. A *primitive* function cannot be factored inside the class.[29] A product of primitive functions is a chord; its zeros are those of the factors. Sums of voices fall outside the class.[30]

5.7 One operator

Why does one voice keep its zeros on the line? Over finite fields the answer is known. We return to the field of the trits, F_3.

Measured. The curve $y^2 = x^3 - x$ over F_3 has 4 points. Its zeta function has all its zeros exactly on $\sigma = 1/2$. They form two arithmetic progressions with spacing $2\pi/\ln 3 = 5.7192$: the period of the Trit Tile. **Known** (Weil):[31] here the one voice is literally one operator, Frobenius acting on cohomology. The zeros are its eigenvalues. RH says that these eigenvalues all have size $\sqrt{q}$. This is the Hilbert–Pólya idea, realised.

One curve gives a fence, step 1 of the staircase. Repulsion appears in families. **Known** (Katz–Sarnak):[32] over finite fields, families of curves have zero statistics of random matrices. **Measured.** For 600 random curves of genus 2 over F_{101} we computed the two Frobenius angles of each curve. Weil's RH held for all 600, to 10^{-8}. Small gaps (below 0.2) occur in 0.5% of curves. The eigenvalues of one random symplectic matrix USp(4) give 0.43%. Two independent angles give 18.8%.

[28] [23] Davenport, H. & Heilbronn, H. (1936). On the zeros of certain Dirichlet series. Journal of the London Mathematical Society 11, 181–185 and 307–312.

[29] [24] Kaczorowski, J. & Perelli, A. (1999). On the structure of the Selberg class I: 0 ≤ d ≤ 1. Acta Mathematica 182, 207–241.

[30] [25] Bombieri, E. & Hejhal, D. A. (1995). On the distribution of zeros of linear combinations of Euler products. Duke Mathematical Journal 80(3), 821–862.

[31] [26] Weil, A. (1948). Sur les courbes algébriques et les variétés qui s'en déduisent. Hermann, Paris.

[32] [27] Katz, N. M. & Sarnak, P. (1999). Zeroes of zeta functions and symmetry. Bulletin of the AMS 36, 1–26.

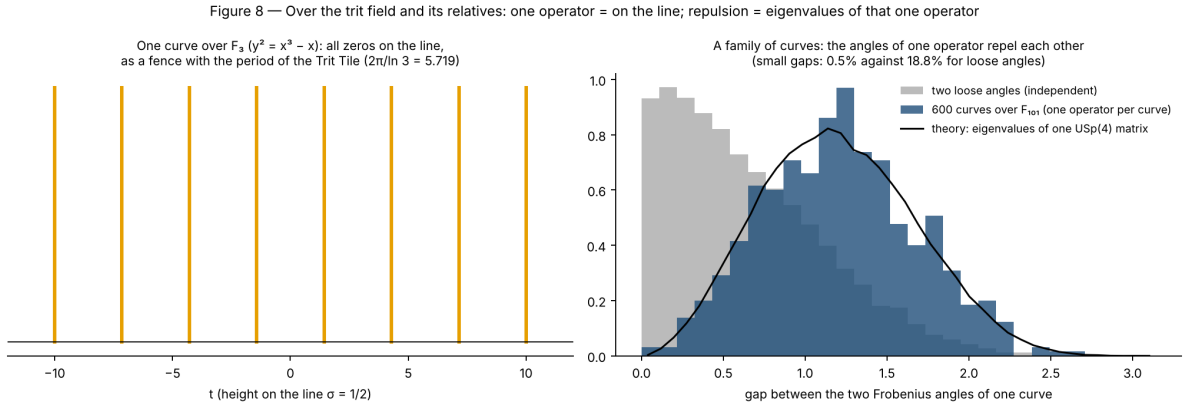


Figure 8. Left: one curve over F_3. All zeros lie on the line as a fence with the period of the Trit Tile. Right: in a family of curves over F_{101}, the two angles of one curve repel each other like the eigenvalues of one matrix (blue, black line), unlike two independent angles (grey). *Measured; Known theory (Weil, Katz–Sarnak).*

Reading. Repulsion means: eigenvalues of one and the same operator. Loose ladders are loose operators, and they give chance. One operator with many eigenvalues gives repulsion. The zeros of ζ repel. In this picture they are eigenvalues of one operator.

6. Reasoning: what is proved and what is not

The pictures rest on known mathematics. Here we sort it.

- **Proved, if the new proof stands.** RH for ζ, and the generalised RH for all Dirichlet L-functions. This includes $L(s, \chi_{-3})$, and with it RH for the Eisenstein plane $\mathbb{Q}(\omega)$ of the Trit Tile, since its zeta function is $\zeta(s)\cdot L(s, \chi_{-3})$.
- **Becomes unconditional.** Every theorem whose only assumption was RH. Examples: Montgomery's part of the pair correlation of zeros (the GUE behaviour for test functions with Fourier support in (−1, 1)), and that at least two thirds of the zeros are simple.[33] Also Gonek's convergence of the bent fence to the zeros of ζ.[34] Also Robin's inequality for σ(n)/n beyond 5040.[35] Each case must be checked: the assumption must be exactly RH, not a stronger form.
- **Still conjecture.** The full GUE hypothesis for the zeros, and RH for all automorphic L-functions (the Grand RH).
- **Known, independent of the new proof.** RH over finite fields (Weil); random-matrix statistics in families (Katz–Sarnak); local GUE statistics for all primitive L-functions (Rudnick–Sarnak);[36] the zero-free region near σ = 1; the Davenport–Heilbronn counterexample.
- **Measured here only.** All toy models: the loop and graph models, the role of the sign χ(−1), the wells near the poles, the two voices. They are illustrations that can be repeated. They prove nothing about ζ.

The route of the formal proof is known in outline.[37] It uses positivity on logarithmic windows, a one-sided bound on long windows [Y, Y^(3/2)], a slope inequality of Bost for lattices, auxiliary

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[33] [28] Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. Proceedings of Symposia in Pure Mathematics 24, 181–193.

[34] [19] Gonek, S. M. (2012). Finite Euler products and the Riemann Hypothesis. Transactions of the AMS 364(4), 2157–2191 (arXiv:0704.3448).

[35] [29] Robin, G. (1984). Grandes valeurs de la fonction somme des diviseurs et hypothèse de Riemann. Journal de Mathématiques Pures et Appliquées 63, 187–213.

[36] [30] Rudnick, Z. & Sarnak, P. (1996). Zeros of principal L-functions and random matrix theory. Duke Mathematical Journal 81, 269–322.

[37] [3] Nasqret (2026). riemann-rev: a map of the Lean proof of the Riemann Hypothesis. github.com/nasqret/riemann-rev.

number fields built per scale, Iwasawa theory and automorphic forms. In a footnote the author sets aside the route via the "field with one element". So the proof passes the gap without the geometry that would explain it. The gap remains.

7. Back to the whole: the successor of the grail

Put the eight pictures side by side. They give one consistent image.

- The zeros carry every prime ladder as a hidden period (Figure 1). The Trit Tile is the ladder ×3.
- One ladder gives a fence. Loose ladders give chance. Coupled ladders give repulsion, but only with a direction that cannot be mirrored back, and only when adding and multiplying work together (Figure 3).
- The step (the functional equation) gives the fence with the right number of zeros. The ladders bend it into the zeros of ζ (Figure 4).
- Repulsion on the line is geometry: a ridge between every two wells (Figure 5).
- Off the line, wells come from the own poles of finite sets of ladders. They vanish only in the complete whole (Figures 6 and 7).
- Over F_3 one operator, Frobenius, explains both the line and the rhythm (Figure 8).

Summary. The zeros of ζ behave like the eigenvalues of one operator that contains all prime ladders at once, coupled by the step, with a direction. Over F_3 such an operator exists for one ladder. It gives RH with the period of the Trit Tile. Over the integers it has not been found. The formal proof shows that the zeros lie on the line without finding it.

The successor of the holy grail is therefore not a new statement. It is that operator: the object over $\mathbb{Z}$ that does what Frobenius does over F_3, for all ladders together. This is the old Hilbert–Pólya question. The measurements sharpen it. The operator must keep the ladders separate in its trace (only lengths $\ln p^m$, no mixed loops). It must couple them through the additive side. And it must break the mirror symmetry. **Known:** the programmes of Connes and of Connes and Consani aim at exactly this structure.[38,39,40] **Known:** locally, at each prime p, a geometric bridge between the trit side and the integers exists (Fargues–Scholze, built from Witt vectors).[41] Globally it does not.

Reading, in the language of the Vacuum.Net theory. Each prime is a closure: a loop of length ln p. The Trit Tile is that loop for p = 3, alone, with its own fixed rhythm. The zeros of ζ are the spectrum of all loops together. The loops are coupled by the step (±1, the trit) and carry a direction that cannot be mirrored (N3). Each loop must give up its own rhythm for the whole to close. The successor of the grail is one closure operator whose loops ln p together have the zeros as eigenvalues.

8. Limits and open points

This essay is not a proof. It contains no new theorem about ζ. Figure 1 repeats Landau. The models of Section 5 are toy models. The value lies in the order of the pictures and in the negative results. Several first ideas failed and are reported as such:

- Stacking lattice rhythms does not approach the rhythm of ζ (it approaches Poisson).
- Non-commuting arithmetic couplings do not by themselves give GUE; most hide a mirror symmetry.

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[38] [16] Connes, A. (1999). Trace formula in noncommutative geometry and the zeros of the Riemann zeta function. Selecta Mathematica 5, 29–106 (arXiv:math/9811068).

[39] [31] Connes, A. & Consani, C. (2019). The scaling Hamiltonian. arXiv:1910.14368.

[40] [32] Connes, A. (2026). The Riemann Hypothesis: past, present and a letter through time. arXiv:2602.04022.

[41] [33] Fargues, L. & Scholze, P. (2021). Geometrization of the local Langlands correspondence. arXiv:2102.13459.

- The hypothesis that the phase always moves forward is false; it runs back up to 32% of the time.
- The landscape of finitely many ladders does not forbid zeros off the line; it creates them.

Open points:

- In the coupled graph models, two maps (×3 and +1) with equal lengths degenerate into a fence. The cause is unknown.
- Why the full Euler product remains "one voice" inside the strip, where it no longer converges. That is the content of RH, and of the formal proof.
- The operator over $\mathbb{Z}$ itself. A modest next step is to test Connes and Consani's recent operators with only the ladders ×2 and ×3, and compare them with the Trit Tile.

9. Conclusion

A formal proof has made the Riemann hypothesis certain, if it stands. It has not made it understood. Seen through the Trit Tile, the missing understanding has a clear shape. One ladder with one operator gives the line and a regular rhythm: that is the finite-field world of the trits, and it is understood. All ladders together give the line and a repelling rhythm: that is the world of the integers. There the operator is missing. Finding it is the successor of the grail. It would turn certainty back into insight.

Annotated references

Numbers match the footnotes. Each entry says what the source adds and where to go next.

[1] Van den Brandhof, A. (2026). Article on the Riemann proof of L. Alpöge. NRC, 22 November 2026.
Dutch newspaper report with comments by J. Maynard and M. Bhargava. Starting point of this study.

[2] Lean FRO and Mathlib community (2026). Lean verification of the Riemann Hypothesis. leanprover-community.github.io/blog, 21 November 2026.
Joint statement on the formal check: four criteria, two independent kernels, axioms used.

[3] Nasqret (2026). riemann-rev: a map of the Lean proof of the Riemann Hypothesis. github.com/nasqret/riemann-rev.
An unofficial but detailed map of the proof: seven steps, from the reflection and the prime sum G(y) to automorphic forms. The best way to see the route without reading 17 million lines.

[4] Konstapel, J. (2026). The Trit Tile: A Reference Work, version 1.1 (21 November 2026). Constable Research, Leiden.
The object this essay starts from. Contains the theorem on the non-settling share, the spectrum with period 2π/ln 3, and the research programme (§22) that this essay extends.

[5] Thurston, W. P. (1994). On proof and progress in mathematics. Bulletin of the AMS 30(2), 161–177.
Argues that mathematics is about increasing human understanding, not only about proving theorems. Background for the role of the seer in this essay.

[6] Konstapel, J. with Claude (2026). Research log 'Riemann hypothesis and the Trit Tile', version 0.23, with Python scripts and data (zeros.py, ritme.py, graaf.py, schreier.py, euler_theta.py, landschap.py, buiten_lijn.py, twee_stemmen.py, familie.py and others). Constable Research, Leiden (unpublished).
All measurements in this essay can be repeated with these scripts. The log also records negative results and corrections.

[7] Konstapel, J. (2026). Vacuum.Net Theory, Ninth Edition: Foundations 9.2 and Physics 9.0. Constable Research, Leiden.
The current formal statement of the theory: one strand, the axioms N1–N5, trits and ladders. Read for the background of Section 2.

[8] Lapidus, M. L. & van Frankenhuijsen, M. (2013). Fractal Geometry, Complex Dimensions and Zeta Functions, 2nd ed. Springer.
The theory of complex dimensions. Lattice strings have periodic complex dimensions, like varieties over finite fields. Explains why the Trit Tile oscillates, and gives a fractal proof that zeta zeros contain no arithmetic progression.

[9] Lapidus, M. L. & van Frankenhuijsen, M. (2004). Fractality, self-similarity and complex dimensions. Proceedings of Symposia in Pure Mathematics 72 (arXiv:math/0401156).
States explicitly the analogy between lattice self-similar strings and varieties over finite fields. The frame in which the Trit Tile sits on the 'finite field side'.

[10] Tate, J. (1950/1967). Fourier analysis in number fields and Hecke's zeta-functions. In Cassels & Fröhlich (eds.), Algebraic Number Theory. Academic Press.
Derives the functional equation by joining additive Fourier analysis with multiplicative local factors. The precise form of 'step plus ladders'.

[11] Putnam, C. R. (1954). On the non-periodicity of the zeros of the Riemann zeta-function. American Journal of Mathematics 76, 97–99.
The zeros on the line contain no infinite arithmetic progression. So the rhythm of the primes is never a lattice rhythm like that of the Trit Tile.

[12] Atas, Y. Y., Bogomolny, E., Giraud, O. & Roux, G. (2013). Distribution of the ratio of consecutive level spacings in random matrix ensembles. Physical Review Letters 110, 084101 (arXiv:1212.5611).

Introduces the ratio measure r̃ and its reference values: Poisson 0.386, GOE 0.536, GUE 0.603. Used in all rhythm measurements here.

[13] Landau, E. (1912). Über die Nullstellen der Zetafunktion. Mathematische Annalen 71, 548–564.
Shows that sums over the zeros peak at x = ln n for prime powers n. This is what Figure 1 measures.

[14] Kottos, T. & Smilansky, U. (1999). Periodic orbit theory and spectral statistics for quantum graphs. Annals of Physics 274, 76–124 (arXiv:chao-dyn/9812005).
Quantum graphs as toy models. Coupled loops give random-matrix rhythm; a magnetic flux breaks time reversal (GOE to GUE). The loop models of Section 5.2 follow this frame.

[15] Bost, J.-B. & Connes, A. (1995). Hecke algebras, type III factors and phase transitions with spontaneous symmetry breaking in number theory. Selecta Mathematica 1, 411–457.
A quantum system with one isometry per prime and partition function ζ. It joins addition (Q/Z) and multiplication (N×), the combination that gave GUE in our models.

[16] Connes, A. (1999). Trace formula in noncommutative geometry and the zeros of the Riemann zeta function. Selecta Mathematica 5, 29–106 (arXiv:math/9811068).
The zeros as an absorption spectrum on the adele class space: additive adeles modulo the multiplicative group. The main candidate frame for the missing operator.

[17] Sierra, G. (2010). A physics pathway to the Riemann hypothesis. arXiv:1012.4264.
Explains why the Riemann dynamics must break time reversal: each prime orbit counts once.

[18] Berry, M. V. & Keating, J. P. (1999). The Riemann zeros and eigenvalue asymptotics. SIAM Review 41(2), 236–266.
The physicist's picture: the zeros as energy levels of a chaotic system whose periodic orbits have lengths ln p. Proposes H = xp.

[19] Gonek, S. M. (2012). Finite Euler products and the Riemann Hypothesis. Transactions of the AMS 364(4), 2157–2191 (arXiv:0704.3448).
Symmetrised finite Euler products; their zeros converge to those of ζ under RH. Our model of Section 5.3 belongs to this family. Gonek also notes that a proof of RH should use both the Euler product and the functional equation.

[20] Dyson, F. J. (1962). Statistical theory of the energy levels of complex systems I. Journal of Mathematical Physics 3, 140–156.
Eigenvalues as charged particles on a line with logarithmic repulsion (the log-gas). The landscape picture of Section 5.4 is a version of this.

[21] Ford, K. (2019). Zero-free regions for the Riemann zeta function. arXiv:1910.08205.
Survey of the classical zero-free regions near σ = 1, from de la Vallée Poussin to Vinogradov–Korobov. Relevant to where the 'wells' of Section 5.5 sit.

[22] Gonek, S. M. & Montgomery, H. L. (2013). Zeros of a family of approximations of the Riemann zeta-function. International Mathematics Research Notices 2013(20), 4712–4733.
For the additive form (partial sums plus mirror), 100% of the zeros lie on the line in a suitable range. Contrast with the product form in Section 5.5.

[23] Davenport, H. & Heilbronn, H. (1936). On the zeros of certain Dirichlet series. Journal of the London Mathematical Society 11, 181–185 and 307–312.
A function with a functional equation but without an Euler product, with zeros off the line. The standard warning example. Recomputed in Section 5.6.

[24] Kaczorowski, J. & Perelli, A. (1999). On the structure of the Selberg class I: 0 ≤ d ≤ 1. Acta Mathematica 182, 207–241.
Entry point to the Selberg class: the axioms that define 'one voice' and the notion of a primitive function.

[25] Bombieri, E. & Hejhal, D. A. (1995). On the distribution of zeros of linear combinations of Euler products. Duke Mathematical Journal 80(3), 821–862.
Sums of L-functions with one functional equation still have almost all zeros on the line, under strong assumptions. Refines the picture of 'two voices'.

[26] Weil, A. (1948). Sur les courbes algébriques et les variétés qui s'en déduisent. Hermann, Paris.

Proof of the Riemann hypothesis for curves over finite fields. Here the operator exists: Frobenius. The model case for everything in Section 5.7.

[27] Katz, N. M. & Sarnak, P. (1999). Zeroes of zeta functions and symmetry. Bulletin of the AMS 36, 1–26.
Shows that in families over finite fields the zeros follow random-matrix statistics, through Frobenius eigenvalues and Deligne's equidistribution. Basis of the family measurement in Figure 8.

[28] Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. Proceedings of Symposia in Pure Mathematics 24, 181–193.
The first link between zeta zeros and random matrices. Proved under RH for a limited range. If the new proof stands, this part becomes unconditional.

[29] Robin, G. (1984). Grandes valeurs de la fonction somme des diviseurs et hypothèse de Riemann. Journal de Mathématiques Pures et Appliquées 63, 187–213.
RH is equivalent to σ(n)/n < e^γ ln ln n for all n > 5040. Links highly composite numbers to RH.

[30] Rudnick, Z. & Sarnak, P. (1996). Zeros of principal L-functions and random matrix theory. Duke Mathematical Journal 81, 269–322.
Local GUE statistics hold for all primitive L-functions, whatever their symmetry sign.

[31] Connes, A. & Consani, C. (2019). The scaling Hamiltonian. arXiv:1910.14368.
Links Connes' programme to the Berry–Keating operator xp. A natural next step for the line proposed in Section 8.

[32] Connes, A. (2026). The Riemann Hypothesis: past, present and a letter through time. arXiv:2602.04022.
A recent overview by Connes of the routes to RH, including the analogy with finite fields and the role of Katz–Sarnak.

[33] Fargues, L. & Scholze, P. (2021). Geometrization of the local Langlands correspondence. arXiv:2102.13459.
Builds, for each prime p, a geometric bridge between characteristic p and the p-adic numbers, using Witt vectors. The local version of the missing global geometry.