

The Vacuum.Net Theory Read Against the History of Mathematics

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29 October 2026

Abstract

The Vacuum.Net Theory derives a chain of results from one ground, the trinity $-1, 0, +1$, and five axioms. This article lays that chain beside a history of mathematics written as chains of proven steps across all 63 fields of the Mathematics Subject Classification (Konstapel 2026a, published on ResearchGate). Forty-five historical results carry the theory's chain. Each was checked against one opened source, assigned to a field, and looked up in the book. Seven of the 45 are in the book in full and three in part; 35 are not. Knot theory is the largest group: the theory's first knot rests on seven results from Gauss to Kauffman, none of which the book contains. The theory draws on 18 of the 63 fields. It gathers its parts from 23 centuries, from the Pythagorean comma of about 300 BCE to ternary-weight language models of 2024, and orders them by logic rather than by date: the rank correlation between the step of the chain and the year of the result is 0.16 ($p = 0.31$). Where the book and the theory use the same result, the facts agree and the theory adds a reading of its own. All numbers by which the theory touches history were recomputed and agree.

1. Introduction

A theory that claims to rest on one ground has to stand somewhere in the history of the subjects it touches. Its proofs use earlier results, and those results have dates, authors and places of publication. This article asks where the Vacuum.Net Theory stands in that history, and how its own order of steps relates to the order in which the steps were found.

Two texts make the question precise. The first is *Mathematics as a Chain of Proven Steps* (Konstapel 2026a), a history of mathematics that tells each of the 63 MSC2020 fields as a chain of results, each link resting on earlier proofs. The second is *The Vacuum.Net Theory*, Ninth Edition (Konstapel 2026b), in two parts: *Foundations*, which states the axioms and what follows from them by proof, and *Physics*, which connects the result to known physics. The theory sets out its own argument as a chain too, in eight steps in *Foundations* and four more in *Physics*.

The comparison has three parts. The first lists the historical results the theory's chain uses, checks each against a source, and looks for it in the book. The second asks which of the 63 fields the theory draws on. The third compares the theory's order of steps with the dates of the results each step uses.

The findings, in brief:

- The theory's chain rests on 45 historical results. The book contains 7 of them in full and 3 in part.

- The largest group of results that the book lacks is knot theory, on which the theory's seventh step rests entirely.
- The theory draws on 18 of the 63 fields. Analysis, algebraic geometry, category theory, differential geometry, fluid mechanics and statistics are not among them.
- The theory follows the order of its logic, not the order of history. Its ground is recent (1958–2005); its ladders rest on the oldest result in the chain (c. 300 BCE).
- Where the book and the theory use the same result, the historical facts agree.
- Every citation in the theory was found as given. Six citations gain history from the sources, and one date, that of Lagrange's best approximations, stays open.

The article is self-contained. Section 2 describes both texts. Section 3 gives the method. Section 4 walks through the chain step by step. Sections 5 to 8 give coverage, order, shared results and checks. Section 9 discusses what the comparison shows and where it is limited.

2. The two texts

2.1 The book

Mathematics as a Chain of Proven Steps (Konstapel 2026a) covers the 63 top-level areas of MSC2020 in 55 chapters, some shared by several fields, with two cross-cutting chapters on numbers and foundations and on unifying structures. Each chapter tells its field as a chain of links. A link gives who proved a result, when, what was proved and the idea of the proof. A step between links means that the later proof uses the earlier result. Independent discoveries, conjectures, counterexamples and repaired gaps are named as such.

Every link rests on at least one opened source of standing, and Wikipedia is never the only source. All 633 numerical checks in the book were recomputed in Python. Each chapter ends with notes on disagreements between sources and with its numbered sources.

2.2 The theory

The Vacuum.Net Theory, Ninth Edition (Konstapel 2026b), consists of *Foundations* (version 9.2) and *Physics* (version 9.0). *Foundations* makes no physical claim; a reader can check each statement “with pencil, paper and a short program”. *Physics* connects the chain to known physics and gives each statement a status.

Beneath the axioms lies one ground: “the trinity, three values $-1, 0, +1$ whose sum is zero”, read as the digits of balanced ternary. The theory rests on five axioms, quoted here from *Foundations* 1.2:

- **N1 (Strand).** “There is exactly one strand. Every distinction is a loop.”
- **N2 (Closure).** “A loop persists if and only if it lies in a closed window. What does not close passes on as tension. Closure is the selection rule; there is no external selector.”
- **N3 (Memory).** “At every node the passages are ordered. The first passage fixes the direction; later passages are taken relative to it. The net carries its history as its shape.”
- **N4 (Context).** “The response of the medium is a state function, not a universal constant.”
- **N5 (Scale).** “Merged into blocks, the coarse strand obeys N1 to N4 again. What does not close at one scale is the material of the next.”

The chain in *Foundations*, Part 0, has eight steps: one strand (N1); the trinity (T1); the window of three trits (T2–T6); three elementary closures, namely standings, meetings and rings (T7–T11); the address of every integer as one balanced-ternary word (T12–T14); two ladders, $\times 2$ and $\times 3$, and their closure depths (T15–T18); the first knot, the trefoil, with the 4π return of a rotor and a conserved Hopf charge (T19–T22); and coupling, the stiffness of the medium as a state function of its density (T23–T24). *Physics*, Part 0, continues with motion (Dirac’s equation in Rowlands’ nilpotent form), field (Maxwell’s quaternions), measure (Larson’s space and time as two aspects of motion) and numbers that instruments can check.

Every statement carries one status. In *Foundations* these are assumed, proved, conditionally proved, corresponded and open. *Physics* adds identifications and predictions. This article does not judge those statuses. It asks only which historical results the chain uses and where they stand in the history of mathematics.

3. Method

Inventory. Both parts of the Ninth Edition were read in full. Every historical result that a theorem, remark, correspondence or reference uses was listed with the place in the theory where it is used. This gave 45 results. Forty-one of them sit inside the chain; four appear only in the theory’s statement of its position (IX.6) and its references.

Source check. Each result was checked for author, year and place of publication against one opened source: a scholarly history, an encyclopaedia of mathematics, the original paper, an arXiv version, or a university text. Wikipedia was not accepted as the only source. Where the theory gives a date or an author, that was compared with the source.

Field. Each result was assigned to one top-level MSC2020 field, the one in which the theory uses it. Quaternions, for instance, are counted under Lie groups (22), because the theory uses them as rotations, although their discovery belongs to algebra.

Book. Each result was looked up in the 55 chapters of the book, first by searching the text for names and terms, then by reading the passages found. A result counts as present when the book has it as a link, and as present in part when the book has the author’s work in that area but not the specific result.

Order. Each step of the chain was numbered 1 to 12, the eight steps of *Foundations* followed by the four of *Physics*. The rank correlation between step and year was computed over the 41 results inside the chain (Spearman, SciPy).

Numbers. Every number by which the theory touches history, such as the convergents of $\log_2 3$, the Pythagorean comma and Watson’s return probability, was recomputed in Python with mpmath at 60 digits and with sympy.

4. The chain step by step

Each subsection states what the step of the theory does, which historical results it uses, and whether the book contains them. Chapter names refer to Konstapel (2026a).

4.1 One strand

The theory begins with one act, a distinction, and states it as N1: every distinction is the strand returning to a place it has been. *Foundations* Part II gives the ground of that act in three results.

The first is George Spencer-Brown's *Laws of Form* (London: Allen & Unwin, 1969). It opens with the instruction to draw a distinction and has two laws: calling twice is calling once, and crossing twice is not crossing. It also introduces re-entry, a form that contains itself. The second is Louis Kauffman's reading of re-entry as an oscillation. With Francisco Varela he wrote "Form dynamics" (*Journal of Social and Biological Structures* 3, 1980, 171–206). In later work he writes the alternation as the iterant $[+1, -1]$ with a time shift and shows that it squares to -1 . The third is Kauffman's "EigenForm" (*Kybernetes* 34, 2005, 129–150): a form that reproduces itself under an operation, $O(A) = A$. The theory reads its closure rule N2 as this eigenform condition, and the fixed rings of T10 as eigenforms of its register.

The open question IX.2 takes its form from Robert Rosen: which rewrite dynamics is its own fixed point. Rosen's relational theory of biological systems began in 1958 (*Bulletin of Mathematical Biophysics* 20) and was set out in *Life Itself* (1991).

In the book: none of the four. The chapter on Logic and computation runs from Aristotle through Boole, Frege, Gödel and Turing; it has no calculus of distinctions. The chapter on Mathematical biology has no Rosen.

4.2 The trinity

The ground is the trinity, $-1, 0, +1$, read as the digits of balanced ternary (T1). The residues form $\mathbb{Z}_3$, which the theory calls the smallest cyclic group in which a value differs from its reverse.

Groups have their own chain. Lagrange's "Réflexions sur la résolution algébrique des équations" (1770–71) showed that the number of values a function of the roots takes under permutations divides $n!$. Cauchy proved in 1844 that the order of a subgroup of a symmetric group divides the order of the group, Jordan stated it in general in 1861, and Cayley defined the abstract group in 1854. The theory's T3, that exactly 9 of the 27 windows close because they form the kernel of a surjective homomorphism of index 3, rests on this line.

Balanced ternary is older than groups. Andrews (2007), following Dickson, traces the idea behind it to the weights problem in Fibonacci's *Liber abaci* of 1202: weights of 1, 3, 9 and 27, placed on either pan of a balance, weigh every whole amount up to 40. Michael Stifel's *Arithmetica integra* (1544, p. 38) is the earliest explicit appearance of the notation, and Donald Knuth's *The Art of Computer Programming*, volume 2 (first edition 1969, §4.1), gives its arithmetic, including the fact that truncation equals rounding (*Foundations* III.2).

The remark on the algebraic home of the trit uses the Eisenstein integers $\mathbb{Z}[\omega]$. Gotthold Eisenstein proved cubic reciprocity in these integers in his papers of 1844 in Crelle's *Journal*; the prime $1 - \omega$, of norm 3, lies above 3, and its residue field is $\mathbb{Z}_3$.

The remark on radix economy compares the cost $r \cdot \lceil \log_r(N + 1) \rceil$ of writing numbers in radix r . Hayes (2001) names the 1950 book *High-Speed Computing Devices*, written by the staff of Engineering Research Associates, as the earliest use of that cost measure. The same remark cites

Leibniz's principle of maximal richness from minimal means, from the *Discours de métaphysique*, §5–6, written in 1686 and first published in 1846, and Chaitin's measure: a theory is a program shorter than its data. That measure has three founders, Solomonoff (1964), Kolmogorov (1965) and Chaitin (*Journal of the ACM* 13, 1966).

In the book: the chain of groups is in the chapter on Algebra, from Lagrange's study of the roots (1770–71) to Galois and Cayley. Balanced ternary, the Eisenstein integers, radix economy, Leibniz's principle and algorithmic information are not.

4.3 The window

A window is three consecutive trits, and it closes when its sum is $0 \bmod 3$ (T2). Exactly 9 of the 27 windows close (T3). A window closes if and only if its three trits are all equal or all different (T4).

T4 is the rule of a three-colouring at a crossing of a knot diagram: the three arcs meeting there carry one colour or all three. Ralph Fox introduced these colourings in lectures at Haverford College in the spring of 1956; they appear in print in Crowell and Fox, *Introduction to Knot Theory* (1963), among the exercises. The theory uses the same rule again for its first knot (§4.7).

In the book: no. The Topology chapter has no knots.

4.4 Three elementary closures

Every closed word of trits breaks down into standings (0), meetings ($x, -x$) and rings (x, x, x) (T7). Words that close everywhere repeat one closed window (T8–T9); the rings that close everywhere are the fixed points of the rewrite $R(t)_i = -(t_{i-1} + t_{i+1})$ (T10); two different closed windows cannot be joined without tension (T11).

This step cites no earlier result. Its theorems are counting statements about trit words, proved in the text and checked by enumeration of all 265,720 closed words of length 1 to 12. The only historical links are readings: T10's fixed points as Kauffman's eigenforms (§4.1), and T4's rule as Fox's (§4.3). Of the twelve steps, this is the one that is the theory's own throughout.

4.5 The address

Every integer has exactly one balanced-ternary word (T12). Open words read from a start number 3^n ; closed configurations read without a start are counted by the necklace formula $g(n)$ (T13). In the Eisenstein integers the same digits do not give every element a finite word (T14).

T12 has a precedent that the theory names: Euler's identity of formal series $\prod_k (x^{(-3k)} + 1 + x^{(3k)}) = \sum_n x^n$, which states the existence and uniqueness of the representation at once. Andrews (2007, §5) places it in Euler's *Introductio in analysin infinitorum* (1748), chapter 16, "De partitione numerorum". The theory also names two machines on the alphabet $\{-1, 0, 1\}$. The Setun computer was built at Moscow State University on Sergei Sobolev's initiative, with Nikolai Brusentsov leading the team; it worked in December 1958. The "1.58-bit" language models of Ma and colleagues (arXiv 2402.17764, 2024) use the same three weights.

T13 counts rotation classes by Burnside's lemma. The lemma has three names: Cauchy stated it in 1845, Frobenius gave it its modern form in 1887, and Burnside's *Theory of Groups of Finite Order*

(1897) made it known. T14 is set, as the theory says, in the setting of Streng's work on curves with complex multiplication by $\mathbb{Z}[\omega]$ (*Mathematics of Computation* 83, 2014, 275–309).

In the book: in part. The chapter on Combinatorics and graph theory has Euler's work on partitions by generating functions (1740–1748), but not his identity for balanced ternary. Setun, the ternary models, Burnside's lemma and Streng's work are not in the book.

4.6 Two ladders

Merging meetings doubles and merging rings triples (T15). The ladders $\times 2$ and $\times 3$ never meet exactly, because $2^n \neq 3^k$ except for $n = k = 0$, so $\log_2 3$ is irrational and its multiples are dense modulo 1 (T16). Their best near-meetings are the closure depths 1, 2, 5, 12, 41, 53, 306, ... (T17), and at depth 12 the miss is the Pythagorean comma, $3^{12}/2^{19}$ (T18).

Three results carry this step. Density follows from Kronecker's theorem (1884), which the theory cites through Hardy and Wright (Theorem 439). The closure depths are the denominators of the convergents of $\log_2 3$, by the theorem that best approximations of the second kind are exactly the convergents. The theory cites Khinchin's *Continued Fractions* (Theorems 16 and 17; first published 1935–36) and names Lagrange. The comma is the oldest result in the whole chain. It is first described in the West in the pseudo-Euclidean *Sectio canonis* (*Division of the Canon*, c. 300 BCE), and in China in the *Huainanzi* of 122 BCE.

In the book: none. The Number theory chapter has no continued fractions, no Kronecker and no comma, and the History chapter does not take up the theory of tuning.

4.7 The first knot

This step adds one hypothesis, that the strand is embedded in three-dimensional space. It then proves that the trefoil is the simplest configuration that cannot be undone, at three crossings (T19); computes its bracket and Jones polynomial, which show that it differs from its mirror image (T20); shows that an attached strand returns after a rotation of 4π , not 2π (T21); and shows that the Hopf charge is conserved under every continuous rewrite (T22).

It uses eight historical results, more than any other step:

1. **Gauss words.** Gauss's undated notes on the crossing sequences of closed plane curves, published in his *Werke* VIII (1900), give the even-interlacing condition that T19 uses to rule out the Gauss word 1212.
2. **Reidemeister moves.** Kurt Reidemeister, "Elementare Begründung der Knotentheorie" (*Abhandlungen aus dem Mathematischen Seminar Hamburg* 5, 1927), showed that three local moves generate all changes of a diagram that keep the knot; Alexander and Briggs found the same moves in the same year.
3. **Fox colourings** (1956), as in §4.3; tricolourability is invariant under all three moves, which separates the trefoil from the unknot.
4. **The Jones polynomial.** Vaughan Jones, "A polynomial invariant for knots via von Neumann algebras" (*Bulletin of the AMS* 12, 1985, 103–111).
5. **The Kauffman bracket.** Kauffman, "State models and the Jones polynomial" (*Topology* 26, 1987, 395–407): a state sum over the $2^3 = 8$ smoothings of the trefoil.

6. **Colourings and the Jones polynomial.** Lickorish and Millett, “Some evaluations of link polynomials” (*Commentarii Mathematici Helvetici* 61, 1986): the number of three-colourings equals $3|V(e^{i\pi/3})|^2$, which gives 9 for the trefoil.
7. **Quaternions.** Hamilton found them on 16 October 1843. Unit quaternions cover the rotations twice; for an attached strand a 2π turn cannot be undone and a 4π turn can, which is Dirac’s belt trick, given a proof by Newman (*Journal of the London Mathematical Society* 17, 1942).
8. **The Hopf invariant.** Heinz Hopf, “Über die Abbildungen der dreidimensionalen Sphäre auf die Kugelfläche” (*Mathematische Annalen* 104, 1931): $\pi_3(S^2) = \mathbb{Z}$.

T22 proves conservation of the Hopf charge without Noether’s theorem (1918), and says so. Noether’s theorem therefore stands in the chain as the route the theory does not take.

In the book: Hamilton’s quaternions are in the chapter on Associative rings and algebras, and Noether’s theorem is in Mechanics. The other six results are not in the book: the Topology chapter has no knot theory, and its only Hopf is the Hopf invariant one problem settled by Adams in 1960, in the K-theory chapter.

4.8 Coupling

The stiffness of the medium is a state function of its density (T23). For a count with mean $\bar{n}$ and cumulants κ_2, κ_3 the theory defines a lever, $-\bar{n}\kappa_3/\kappa_2^2$, which equals the coupling of T23 to leading order by Laplace’s method. For a free random walk on $\mathbb{Z}^d$, $d \geq 3$, the lever of the number of visits is $-(1 + P_d)/P_d$ (T24).

The step uses three results. Laplace’s method of approximating integrals goes back to his “Mémoire sur la probabilité des causes par les évènements” (1774). George Pólya proved in 1921 that the simple random walk returns with certainty in dimensions 1 and 2 and not in 3 or more (*Mathematische Annalen* 84, 149–160). G. N. Watson evaluated the three-dimensional return probability, $P_3 = 0.3405373$, in “Three triple integrals” (*Quarterly Journal of Mathematics* 10, 1939); McCrea and Whipple (1940) and Montroll (1956) extended the work, and Montroll gave P_4 and P_5 .

In the book: in part. The chapter on Probability and statistics has Laplace’s set-up of 1774, which Gauss adopted, but not Laplace’s method. Pólya’s theorem and Watson’s integrals are not in the book.

4.9 Motion

Physics T1 takes Dirac’s equation as input and rewrites it in Peter Rowlands’ nilpotent form. With quaternion units i, j, k , vector units carried by the momentum, and a scalar imaginary unit, the Dirac operator becomes $Q = ikE + ip + jm$, and $Q^2 = (E^2 - p^2 - m^2) \cdot 1$. So $Q^2 = 0$ exactly on the mass shell. The theory reads nilpotence as closure (correspondence C3).

The two historical results are Dirac’s “The quantum theory of the electron” (*Proceedings of the Royal Society A* 117, 1928, 610–624) and Rowlands’ *Zero to Infinity: The Foundations of Physics* (World Scientific, 2007).

In the book: Dirac's equation of 1928 is in the Quantum theory chapter, with its matrices and its rest on special relativity. Rowlands' rewriting is not.

4.10 Field

Physics Part II reads Maxwell's equations as two readings of one closure operation. Hamilton's operator $\nabla = i\partial_x + j\partial_y + k\partial_z$ acts on a vector with one product, which has a scalar part (the convergence) and a vector part (the curl). Maxwell wrote his grouped equations in this form in the *Treatise* of 1873. Gibbs and Heaviside later split the one product into dot and cross, and in the theory's reading the scalar register disappeared. Two lemmas (T2, T3) show why a rule that keeps orientation needs full quaternions; T4 defines a ring measure on them.

Three results carry this step. Hamilton first used ∇ at a meeting of the Royal Irish Academy on 20 July 1846, printed in 1847; the theory cites his *Lectures on Quaternions* of 1853. Maxwell's *A Treatise on Electricity and Magnetism* (1873), volume 2, Articles 591–619, holds the grouped equations. Gibbs's *Elements of Vector Analysis* (privately printed, 1881 and 1884) and Heaviside's papers of 1882–85, with the four-equation form of 1884–85, made the split.

In the book: Maxwell's *Treatise* and Heaviside's four equations are in the chapter on Optics and electromagnetism. Hamilton's ∇ of 1846 is not.

4.11 Measure and loss

Physics Part III converts the dimensionless quantities of the model into SI units with Larson's reading of motion: space S and time T are the two aspects of one motion, with energy T/S and mass T^3/S^3 (T5). Hydrogen fixes one length and one time. *Physics* Part IV states that every irreversible discharge occurs at a mesh with an address: entropy production is $\Sigma J \cdot X \geq 0$ per location, with Onsager-symmetric coefficients in the linear regime (T6).

The results are Dewey B. Larson's *The Structure of the Physical Universe* (1959) and Lars Onsager's "Reciprocal relations in irreversible processes" (*Physical Review* 37, 1931, 405–426), with the presentation of de Groot and Mazur (1962).

In the book: Onsager's reciprocal relations of 1931 are in the chapter on Statistical mechanics. Larson is not in the book.

4.12 Numbers

The last step gives numbers that instruments can check. The electron corresponds to a photon closed on itself in a double loop (C4, I1). The acceleration threshold of rotation curves is identified with $a_0 = cH_0/2\pi$ (I3). A change of the fine-structure constant of about 3×10^{-8} is predicted at the surface of the white dwarf G191-B2B (P1). The references add the Seiberg–Witten curve as an example of physics carried by a closed curve.

The historical results are Williamson and van der Mark, "Is the electron a photon with toroidal topology?" (*Annales de la Fondation Louis de Broglie* 22, 1997); Milgrom, "A modification of the Newtonian dynamics" (*Astrophysical Journal* 270, 1983, 365–370), and his paper of 1999 in *Physics Letters A* 253; Berengut and colleagues (*Physical Review Letters* 111, 010801, 2013) and Hu and

colleagues (*Monthly Notices of the RAS* 500, 1466, 2021) on α in white-dwarf spectra; and Seiberg and Witten (*Nuclear Physics B* 426, 1994, 19–52).

In the book: in part. The Global analysis chapter has the Seiberg–Witten invariants of 4-manifolds, the mathematical side of the 1994 work, but not the curve. The Astronomy chapter has no modified dynamics, and the book has no Williamson–van der Mark and no white-dwarf measurements of α .

4.13 The theory’s position

Four further results appear only where the theory states its position against other programmes (*Foundations* IX.6) and in its references: loop quantum gravity (Ashtekar’s variables of 1986; the loop representation of Rovelli and Smolin, 1987–88), causal sets (Bombelli, Lee, Meyer and Sorkin, 1987) and causal dynamical triangulations (Ambjørn and Loll, from 1998); Verlinde’s entropic gravity (*Journal of High Energy Physics*, 2011); Wolfram’s physics project (April 2020); and Streng’s computation of Igusa class polynomials (2014). They are outside the chain proper and not counted in the analysis of order. None of them is in the book.

5. Coverage

5.1 What the book holds

Of the 45 results, the book has 7 in full, 3 in part and 35 not at all. The 35 missing results fall into 13 chapters.

Chapter of the book	Results the theory uses that the book lacks	N
Topology	Gauss words, Reidemeister moves, Hopf invariant, Fox colourings, Jones polynomial, Lickorish–Millett, Kauffman bracket	7
Number theory	balanced ternary, Eisenstein integers, best approximations, Kronecker density, Igusa class polynomials	5
Computer science	radix economy, Setun, algorithmic information, Knuth’s ternary arithmetic, ternary-weight language models	5
Relativity	loop quantum gravity and related programmes, Larson, Verlinde, Wolfram	4
Logic and computation	<i>Laws of Form</i> , re-entry as iterant, eigenform	3
Probability and statistics	Pólya’s recurrence, Watson’s integrals	2
Quantum theory	Rowlands’ nilpotent operator, Williamson–van der Mark	2
Astronomy	Milgrom’s acceleration scale, α in white dwarfs	2
Combinatorics and graph theory	Burnside’s lemma	1

Chapter of the book	Results the theory uses that the book lacks	N
Mathematical biology	Rosen's (M,R)-systems	1
General	Leibniz's principle	1
History of mathematics	the Pythagorean comma	1
Optics and electromagnetism	Hamilton's ∇	1

The seven results the book holds in full are the groups from Lagrange to Cayley, Hamilton's quaternions, Noether's theorem, Dirac's equation, Maxwell's *Treatise*, Gibbs and Heaviside's vector analysis, and Onsager's reciprocal relations. The three it holds in part are Euler on partitions, Laplace's 1774 set-up and the Seiberg–Witten work.

5.2 Which fields the theory draws on

The 45 results belong to 18 of the 63 fields.

MSC	Field	Results
00	General	3
03	Logic and foundations	3
05	Combinatorics	2
11	Number theory	6
20	Group theory	1
22	Lie groups	1
41	Approximations and expansions	1
55	Algebraic topology	1
57	Manifolds and knots	6
60	Probability	2
68	Computer science	4
70	Mechanics	1
78	Electromagnetism	3
80	Thermodynamics	1
81	Quantum theory	4
83	Relativity and gravitation	3

MSC	Field	Results
85	Astronomy	2
92	Biology	1

Number theory and knot theory carry the most, six results each. The other 45 fields have no result in the chain: 01, 06, 08, 12–19, 26, 28, 30–35, 37, 39, 40, 42–47, 49, 51–54, 58, 62, 65, 74, 76, 82, 86, 90, 91, 93, 94 and 97. They include the whole of analysis from real functions to operator theory, the differential equations, algebraic geometry, category theory, differential geometry, fluid mechanics, statistics and information theory.

6. Order: logic and chronology

The chain of the theory does not follow the order in which history found its parts. Over the 41 results inside the chain, the Spearman rank correlation between step number and year is $\rho = 0.16$, with $p = 0.31$. That is no relation.

Step	Earliest result	Latest result	Results
1. One strand	1958	2005	4
2. The trinity	1202	1969	7
3. The window	1956	1956	1
4. Three closures	—	—	0
5. The address	1748	2024	4
6. Two ladders	c. 300 BCE	1884	3
7. The first knot	1843	1987	8
8. Coupling	1774	1939	3
9. Motion	1928	2007	2
10. Field	1846	1884	3
11. Measure and loss	1931	1959	2
12. Numbers	1983	2013	4

Two things stand out. First, the ground is the youngest part. The first step rests on Spencer-Brown, Kauffman and Rosen, from 1958 to 2005, while the sixth rests on the Pythagorean comma of about 300 BCE. History found the ladders of 2 and 3, the digits -1 , 0 , $+1$ and the quaternion long before it found the calculus of distinctions that the theory puts first.

Second, almost every step spans centuries. The trinity joins Fibonacci (1202) to Knuth (1969); the address joins Euler (1748) to language models of 2024; the first knot joins Hamilton (1843) to Kauffman (1987). The chain is a recombination: each step gathers results that history produced far

apart, often in different fields, and puts them next to each other because the logic of the step needs them together.

The chain of the book is different. Its links are ordered by date within each field, and a step means that a later proof used an earlier one. The theory's steps are ordered by dependence within the theory. The two orders measure different things, and the near-zero correlation says that the theory does not retell the history: it reads it.

7. Shared results and the theory's readings

Ten results stand in both texts, seven in full and three in part. In each case the historical facts in the book and in the theory agree. What differs is the use: the book records what a result made possible later in its own field; the theory reads the result as a step of its chain.

Result	In the book	In the theory
Groups, Lagrange to Cayley (1770–1854)	Algebra: from the study of roots to Galois groups and abstract structures	$\mathbb{Z}_3$ as the smallest group in which a value differs from its reverse; its residues are the digits (T1, T3)
Hamilton's quaternions (1843)	Associative rings and algebras: Frobenius 1878, division algebras	unit quaternions as rotations; the double cover and the 4π return (T21)
Noether's theorem (1918)	Mechanics: every continuous symmetry gives a conservation law	the route not taken: the Hopf charge is conserved by topology alone (T22)
Maxwell's <i>Treatise</i> (1873)	Optics and electromagnetism: Article 619 restates the equations in 12 entries, partly in quaternion form	one product with two readings, the two-layer law (<i>Physics</i> II)
Gibbs and Heaviside (1881–85)	Optics and electromagnetism: Heaviside's four equations of 1884	the split of the one product, in which the scalar register disappeared
Dirac's equation (1928)	Quantum theory: a first-order relativistic equation with matrices	input, rewritten as Rowlands' nilpotent Q ; $Q^2 = 0$ read as closure (<i>Physics</i> T1, C3)
Onsager's relations (1931)	Statistical mechanics: $L_{ij} = L_{ji}$	every loss has an address on the net (<i>Physics</i> T6)
Euler on partitions (1740–48)	Combinatorics: partitions by generating functions	one product identity: the uniqueness of balanced ternary (T12)
Laplace (1774)	Probability: the set-up Gauss adopted	Laplace's method for the lever of a count (Part VIII)

Result	In the book	In the theory
Seiberg–Witten (1994)	Global analysis: invariants of 4-manifolds, the Thom conjecture	the Seiberg–Witten curve: physics carried by a closed curve (references)

One rule recurs inside the theory itself. The closing rule of a window, three trits all equal or all different (T4), is Fox’s rule for a three-colouring at a crossing, which T19 uses for the first knot. The theory says so: “The register and the first knot obey one rule.” Historically the rule came from knot theory (1956); in the theory it comes first, from arithmetic mod 3.

8. Dates and numbers checked

8.1 Dates

Every work cited in the Ninth Edition was found with the year and place it gives, among them Reidemeister 1927, Hopf 1931, Pólya 1921, Watson 1939, Jones 1985, Kauffman 1987, Lickorish–Millett 1986, Dirac 1928, Maxwell 1873, Milgrom 1983, Berengut 2013, Seiberg–Witten 1994 and Ma 2024. For six citations the sources add history behind the citation.

Where in the theory	What the theory cites	What the sources add
<i>Foundations</i> T1, references	Stifel 1544, an early appearance of balanced ternary	the weights problem behind it goes back to Fibonacci’s <i>Liber abaci</i> (1202), according to Andrews, following Dickson
<i>Foundations</i> III.1	Leibniz, <i>Discours de métaphysique</i> (1686)	written in 1686, first published in 1846
<i>Foundations</i> T12	Andrews 2007 on Euler’s <i>De partitio numerorum</i>	Euler’s identity stands in the <i>Introductio in analysin infinitorum</i> (1748), chapter 16
<i>Foundations</i> T4, T19	Fox and Crowell 1963 for three-colouring	Fox introduced the colourings in lectures at Haverford College in spring 1956
<i>Physics</i> II	Hamilton, <i>Lectures on Quaternions</i> (1853)	Hamilton first used ∇ at the Royal Irish Academy on 20 July 1846
<i>Physics</i> I3	Milgrom 1999 for a_0 and $cH_0/2\pi$	that paper expresses a_0 through the cosmological constant Λ , not explicitly as $cH_0/2\pi$

One date stays open. For the theorem behind T17, that best approximations of the second kind are the convergents, Khrushchev’s survey names Lagrange and the year 1789, while Lagrange’s *Additions to Euler’s Algebra* appeared in 1774. The theory itself cites Khinchin, whose book first appeared in 1935–36.

8.2 Numbers

Every number by which the theory touches history was recomputed in Python, with mpmath at 60 digits and sympy. All agree with the Ninth Edition.

Theory	Quantity	Recomputed
<i>Foundations</i> T17	continued fraction of $\log_2 3$	[1; 1, 1, 2, 2, 3, 1, 5, 2, 23, 2, 2, 1, 1, 55, ...]
<i>Foundations</i> T17	closure depths up to 10^6	1, 2, 5, 12, 41, 53, 306, 665, 15601, 31867, 79335, 111202, 190537; the record search and the convergent denominators give the same list
<i>Foundations</i> T18	Pythagorean comma $3^{12}/2^{19}$	1.0136433, or 23.4600 cents
<i>Foundations</i> T18	residues at depths 1 to 665	498.045, 203.910, 90.225, 23.460, 19.845, 3.615, 1.770, 0.0756 cents
<i>Foundations</i> T13	necklace counts $g(1)$ to $g(6)$	3, 6, 11, 24, 51, 130
<i>Foundations</i> T12.3	digit count $\lfloor \log_3(2n - 1) \rfloor + 1$	holds for every n below 200,000, in integer arithmetic
<i>Foundations</i> III.1	radix cost $r/\ln r$ for $r = 2, 3, 4$	2.885, 2.731, 2.885; the optimum $e = 2.718$
<i>Foundations</i> T24	Watson's P_3 from his gamma-function formula	0.3405373; lever -3.9365
<i>Foundations</i> T24	lever for P_4 and P_5	-6.1759, -8.3976
<i>Physics</i> I3	$a_0 = cH_0/2\pi$ for $H_0 = 67.4, 70, 73$	1.04, 1.08, $1.13 \times 10^{-10} \text{ m s}^{-2}$
<i>Physics</i> P3	solar winding edge	7,402 AU for $a_0 = 1.0824 \times 10^{-10}$; 7,030 AU for 1.2×10^{-10}
<i>Physics</i> III.2	$9S_0$ from R_∞ and m_p/m_e (CODATA 2018)	410.29 nm

9. Discussion

9.1 Where the theory stands

The theory stands on a narrow and specific part of the history of mathematics. Two fields carry the most: number theory, through balanced ternary, the Eisenstein integers and the approximation of $\log_2 3$, and knot theory, from Gauss's crossing words to the Kauffman bracket. Around them sit a few results from algebra (groups, quaternions), combinatorics (Euler, Burnside), probability (Pólya, Watson) and the logic of distinction (Spencer-Brown, Kauffman). *Physics* adds Dirac, Maxwell, Onsager and a handful of recent results in astrophysics.

Within this narrow base the chain is economical. Most steps use between one and four historical results, and one step, the three elementary closures, uses none: its theorems are counting statements about trit words that the theory proves for itself. The first knot is the exception, with eight results, and it is the step where the theory meets established mathematics most densely.

9.2 The book and the theory

The two texts overlap in only ten results, and the reason lies in how each was built. The book follows each field along its main line of proofs, and its Topology chapter takes the line from Euler's formula through Poincaré to the Kervaire invariant. That line has no knots. The theory needs exactly the part of topology that the book's main line passes by. In the same way the book's Number theory follows primes, reciprocity and modularity, while the theory needs digits, approximation and the comma.

Forty-five of the 63 fields carry no result in the theory's chain. They include whole branches with long chains of their own: analysis, differential equations, algebraic geometry, category theory, differential geometry, fluid mechanics, statistics. For these fields the book holds the history, and the theory has not yet been read in their language. The book's chapters therefore mark where that reading could be extended, field by field.

9.3 Logic and history

The near-zero correlation between the order of the chain and the dates of its parts has a plain meaning. The theory does not tell the history again in a new order of discovery. It takes results from 23 centuries, from the comma to ternary language models, and places them where its argument needs them. The youngest part, the calculus of distinctions, sits at the bottom; the oldest, the comma, sits in the middle.

9.4 Limits

- **One source per result.** Each result was checked against one opened source. Five sources are weaker than a journal or encyclopaedia page (the Knowino, HZ, Lynge, Boardman and TOTU pages); they are marked in the references.
- **The inventory.** The 45 results are those the Ninth Edition names or uses. Results that the theory uses without naming them, and derivations published elsewhere on constable.blog, are not included.
- **One field per result.** Each result was counted in the field the theory uses it for. Another assignment would move some counts, for instance quaternions from Lie groups to associative algebras.
- **Presence in the book.** Presence was decided by searching the book's text and reading the passages found. A result mentioned in passing under another name could have been missed.
- **The correlation.** Several results share a year or a step, and the oldest dates are approximate. The test is a rough measure of order, not a model of it.

10. Conclusion

The Vacuum.Net Theory, Ninth Edition, rests on 45 results from the history of mathematics and physics. All but one could be traced to an author, a year and a place; the date of Lagrange's theorem

on best approximations stays open. Every citation in the theory was found as given. The theory draws on 18 of the 63 fields, most of all on number theory and knot theory. It gathers results across 23 centuries and orders them by its own logic, not by the order of discovery.

Laid beside *Mathematics as a Chain of Proven Steps*, the theory shares ten results with the book, and on all ten the historical facts agree. The other 35 results lie off the main lines the book follows, above all in knot theory. The 45 fields the theory does not yet draw on are the ground where its reading has still to be tried, and the book gives, for each of them, the chain of proven steps to read.

References

The two texts

- Konstapel, J. (2026a). *Mathematics as a Chain of Proven Steps: The 63 Fields of Mathematics, from the First Proofs to 2026*. Constable Research, Leiden. Published on ResearchGate.
- Konstapel, J. (2026b). *The Vacuum.Net Theory*, Ninth Edition: *Foundations*, version 9.2, and *Physics*, version 9.0. Constable Research, Leiden, 27 October 2026.

Sources for the historical results

One opened source per result. Sources marked (w) are weaker than a journal or encyclopaedia page.

1. Spencer-Brown, *Laws of Form* (1969): [Tydecks, Spencer-Brown: Laws of Form](#)
2. Kauffman and Varela, "Form dynamics" (1980): [HZ bibliographic record](#) (w)
3. Kauffman, "EigenForm" (2005): [Emerald, Kybernetes 34](#)
4. Rosen, relational biology (1958; 1991): [Letelier et al., closure in Rosen](#)
5. Balanced ternary, Fibonacci and Stifel; Euler's identity: [Andrews, "Euler's De Partitio Numerorum", Bull. AMS 44 \(2007\)](#)
6. Groups from Lagrange to Cayley: [Roth, "A History of Lagrange's Theorem on Groups", Math. Mag. 74 \(2001\)](#)
7. Eisenstein, cubic reciprocity (1844): [Lyng, Eisenstein's Crelle papers](#) (w)
8. Radix economy: [Hayes, "Third Base", American Scientist 89 \(2001\)](#)
9. Leibniz, *Discours de métaphysique*: [text of the Discourse](#)
10. Algorithmic information: [Chaitin, "Algorithmic information theory: some recollections", arXiv math/0701164](#)
11. Knuth on balanced ternary: [arXiv 2010.01743, quoting Knuth](#)
12. Fox colourings (1956): [Przytycki, arXiv 1809.00063](#)
13. Setun (1958): [Russian Virtual Computer Museum, Setun](#)
14. Ternary-weight language models: [Ma et al., arXiv 2402.17764](#)
15. Burnside's lemma: [Encyclopedia of Mathematics, Burnside lemma](#)
16. Kronecker density (1884): [Encyclopedia of Mathematics, Kronecker theorem](#)
17. Best approximations and convergents: [arXiv 0912.1997](#); [Khrushchev, survey of continued fractions](#)
18. Pythagorean comma: [Knowino, Pythagorean comma](#) (w)

19. Reidemeister moves (1927): [Encyclopedia of Mathematics, Reidemeister theorem](#)
20. Gauss words: [arXiv 0709.2933](#)
21. Jones polynomial (1985): [Bull. AMS 12](#)
22. Kauffman bracket (1987): [Kauffman, "State models and the Jones polynomial"](#)
23. Lickorish and Millett (1986): [Comment. Math. Helv. 61](#)
24. Hamilton's quaternions (1843): [Trinity College Dublin, Broome Bridge letters](#)
25. Hopf invariant (1931): [Encyclopedia of Mathematics, Hopf invariant](#)
26. Noether's theorem (1918): [Tavel's translation, arXiv physics/0503066](#)
27. Pólya (1921) and Watson (1939): [MathWorld, Pólya's random walk constants](#)
28. Laplace (1774): [Martin, Frazier and Robert, arXiv 2004.06425](#)
29. Dirac (1928): [Proc. R. Soc. A 117](#)
30. Rowlands (2007): [USI library record](#)
31. Hamilton's ∇ (1846): [Neumaier, notes on the history of nabla](#)
32. Maxwell, *Treatise* (1873): [vol. 2, Part IV, ch. IX](#)
33. Gibbs and Heaviside: [MacTutor, Heaviside](#)
34. Larson (1959): [Boardman, The Simple Essence of Larson's Reciprocal System](#) (w)
35. Onsager (1931): [Phys. Rev. 37](#)
36. Williamson and van der Mark (1997): [TOTU reference list](#) (w)
37. Milgrom (1983): [ADS, ApJ 270](#)
38. α in white dwarfs: [Berengut et al., arXiv 1305.1337](#)
39. Seiberg and Witten (1994): [arXiv hep-th/9407087](#)
40. Wolfram's physics project (2020): [Wolfram, April 2020](#)
41. Verlinde (2011): [JHEP 2011](#)
42. Loop quantum gravity and related programmes: [Rovelli, arXiv gr-qc/9710008](#); [Ambjørn, Jurkiewicz and Loll, arXiv 1004.0352](#); [Bombelli et al., PRL 59](#)
43. Streng (2014): [Math. Comp. 83](#)