Two Ends of One Medium

Neutron Stars, Black Holes, and the Constitutive Vacuum

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1. The discomfort

Modern astrophysics is remarkably successful at observing compact objects and remarkably uncomfortable at explaining them. Consider the open ledger.

We do not know the equation of state of matter above nuclear density. Sixty years after the first pulsar, the interior of a neutron star is still a family of competing curves (Özel & Freire 2016). We do not know why the heaviest neutron stars and the lightest black holes are separated by a mass gap — and we are startled when an object like GW190814's 2.6 solar-mass companion lands inside it (Abbott et al. 2020). We do not know what happens at the center of a black hole. The singularity theorems tell us general relativity predicts its own breakdown there (Penrose 1965). We do not know where the information goes; fifty years after Hawking (1975), the paradox is still generating firewalls, fuzzballs, and conference series rather than a resolution. And now JWST finds billion-solar-mass black holes fully grown a few hundred million years after the Big Bang, faster than accretion models comfortably allow (Bogdán et al. 2024; Maiolino et al. 2024).

Each of these puzzles is treated as its own crisis, with its own literature. We propose that they are one puzzle. They all arise from the same assumption: that the vacuum is nothing, and that its parameters are universal constants rather than properties of a medium.

Drop that assumption and the picture simplifies dramatically. This essay presents the simple version, and then shows that laboratory physics has already tested its core mechanism.

2. The vacuum as a constitutive medium

Our framework treats the vacuum as a physical condensate with a constitutive equation — a stress-strain relation, exactly as a materials scientist would write one (Konstapel 2026a, 2026b). Three statements carry everything that follows.

First: the vacuum has a state. Its local condition is measured by a single dimensionless variable χ, the ratio of local acceleration to the cosmic background scale, $\chi = a/(cH)$. "Constants" such as the effective coupling are functions of this state, not universal numbers. This is the context principle: the law you measure is the response of the medium you are standing in.

Second: the response is nonlinear. The effective stiffness of the medium obeys $g\_eff = \bar{g}(1 + \gamma\delta)$, where δ is the local compression and γ is the leading nonlinearity. This is the acoustoelastic form familiar from nonlinear elasticity (Thurston & Brugger 1964): stressed media stiffen. We derive γ from the topology of the underlying network and find it is of order 2π. That single number already pays rent: it predicts the Milgrom acceleration scale $a_0 = cH_0/2\pi$, turning a forty-year-old numerical coincidence (Milgrom 1983) into the phase winding of one closed loop.

Third: matter is frozen topology. Particles are stable knots in the medium — topologically protected excitations, not foreign objects placed into empty space. Mass is stored tension. This is not a private fantasy of ours; it is the working picture of superfluid vacuum theory, developed by Volovik (2003) directly from his laboratory work on superfluid helium-3, where knotted and topological excitations are routinely created and measured.

With these three statements, neutron stars and black holes stop being two separate mysteries. They become the two ends of one constitutive curve.

3. Neutron stars: the medium at maximum packing

A neutron star, in this picture, is the knot network compressed against its packing limit.

The consequences are immediate. The "unknown equation of state" above nuclear density is not unknown — it is the upper branch of the same constitutive equation $g_{eff} = \bar{g}(1 + \gamma\delta)$ whose lower branch, at small χ, produces flat galaxy rotation curves. One curve, two regimes. The astronomer measuring the radius of PSR J0740+6620 with NICER (Miller et al. 2021; Riley et al. 2021) and the astronomer fitting the radial acceleration relation of a dwarf galaxy are probing the same material property at opposite ends of its range.

This reading is testable against data that already exist. GW170817 gave us the tidal deformability of neutron-star matter (Abbott et al. 2017). NICER gives radii. Radio timing gives maximum masses above two solar masses (Cromartie et al. 2020). A constitutive equation with one nonlinearity parameter is far more constrained than the current zoo of phenomenological equations of state. Either one γ fits the tidal, radius, and mass data together with the galactic data, or the framework fails. That is a feature. Most proposals in this field cannot fail this cheaply.

The Tolman-Oppenheimer-Volkoff mass limit (Oppenheimer & Volkoff 1939) also changes character. It is not an arbitrary threshold where gravity "wins." It is the saturation point of the medium — the compression beyond which the condensate cannot store further tension in knot form. And the question "what happens beyond saturation?" now has a materials-science answer.

4. Black holes: a phase transition, not a singularity

Beyond saturation, a real medium does not compress to a point of infinite density. It changes phase. We propose that this is what a black hole is: a region where the vacuum condensate has undergone a phase transition to its saturated state.

The horizon, in this reading, is a critical surface — the locus where the medium's signal speed collapses. This is not a metaphor. It is precisely how acoustic horizons work in laboratory condensates. Unruh (1981) showed that sound in a flowing fluid obeys the same wave equation as light near a black hole; where the flow exceeds the local sound speed, an acoustic horizon forms. The mathematics is identical, term for term (Barceló, Liberati & Visser 2011).

Here the evidence becomes unusually strong for a foundational proposal, because the key phenomenon has been *measured*. Steinhauer (2016) observed spontaneous Hawking radiation — entangled phonon pairs — at the acoustic horizon of a Bose-Einstein condensate. The follow-up measured the full thermal spectrum at the predicted temperature (Muñoz de Nova et al. 2019). Hawking radiation is real, and it demonstrably does not require a singularity, an event horizon in the strict causal sense, or any quantum gravity. It requires a medium with a critical surface. That is the only laboratory in the universe where Hawking's mechanism has ever been observed, and it is a condensate.

In our framework the curvature of spacetime is a refractive-index profile of the medium — a reading with a long pedigree (Gordon 1923; de Felice 1971). A black hole is then simply the region where the medium goes optically opaque: the refractive index diverges at the critical surface, and the interior is a saturated condensate core at finite density.

Astronomy has been circling this object for two decades under other names. Mazur and Mottola (2004) proposed the gravastar: a condensate interior matched to an exterior Schwarzschild geometry, indistinguishable from a black hole at current resolutions. Chapline (2005) proposed the dark energy star on similar grounds. What these proposals lacked was a reason — why should the vacuum condense there, of all places? The constitutive equation supplies the reason: saturation of the medium at the end of the neutron-star branch. The gravastar stops being an exotic alternative and becomes the generic strong-field state of the vacuum.

5. What dissolves

Once the black hole is a phase, not a singularity, the crisis ledger of Section 1 shortens line by line.

The singularity. Gone by construction. The singularity theorems assume the medium responds classically and linearly all the way down. A saturating constitutive equation violates that assumption in exactly the way real materials do. Infinite density was never a prediction about nature; it was a statement about the breakdown of a linear extrapolation.

The information paradox. Nothing falls out of the universe. Infalling structure is absorbed into the saturated core and its critical surface, and the correlations measured in the BEC experiments (Steinhauer 2016) show how a horizon-forming medium keeps its books: the radiation is entangled with interior modes that still exist. The paradox was manufactured by insisting the interior is vacuum-plus-singularity.

The mass gap. A phase transition has a width. Between the heaviest stable knot-packing (neutron star) and the lightest fully saturated core (black hole) lies a transition region where objects are marginal. The observed gap between roughly 2.2 and 5 solar masses, and the occasional object inside it (Abbott et al. 2020), is what a first-order-like transition looks like in a mass function. In the standard picture the gap is an accident of supernova engines; here it is a material property, and its edges encode γ.

Too-early giants. If black-hole formation is a phase transition of the medium rather than the end point of stellar collapse plus slow accretion, then the early universe — hot, dense, with the vacuum state χ far from today's — can nucleate saturated regions directly. Massive black holes shortly after the Big Bang (Bogdán et al. 2024) stop being an embarrassment of growth rates and become an expected feature of a medium quenched through its transition.

6. What it predicts

A promotion is worthless without exposure to refutation. The framework makes at least four predictions that astronomers can act on with existing or planned instruments.

Echoes. If the horizon is a critical surface of finite depth rather than a mathematical one-way membrane, the ringdown of a merger should show late-time echoes — pulses partially reflected from the surface. This search is already running on LIGO-Virgo data (Abedi, Dykaar & Afshordi 2017; Cardoso & Pani 2019). The claims are contested; the framework says: keep looking, and the echo delay measures the depth of the transition layer.

Strong-field spectroscopy. The context principle says the fine-structure constant is a function of the vacuum state, $\alpha(\chi)$, with $\Delta\alpha/\alpha = \beta\cdot\Delta\chi$. Our benchmark at the white dwarf G191-B2B predicts a shift of order 10^{-8} (Konstapel 2026a), consistent with current bounds (Berengut et al. 2013). At a neutron-star surface, χ is orders of magnitude larger. Metal lines in neutron-star atmospheres — or in the photospheres of accreting systems — should show α shifts far above the white-dwarf level, with a specific scaling in surface gravity. No competing framework predicts this correlation.

One γ across twelve orders of magnitude. The same nonlinearity must fit the SPARC rotation-curve data, the radial acceleration relation and its external-field effect (Chae et al. 2020), the neutron-star mass-radius-deformability data, and the mass-gap edges. This closure test is executable on public data today. Multi-parameter equations of state cannot lose it; a one-parameter constitutive equation can.

The photon ring. The Event Horizon Telescope images of M87* and Sgr A* (EHT Collaboration 2019, 2022) are consistent with general relativity but do not yet resolve the near-horizon structure. A saturated core with a critical surface predicts subtle deviations in the photon subring structure — the target of the next-generation EHT. A surface, however compact, is not a horizon, and interferometry will eventually tell the difference.

7. The company we keep

We close with a point about method, because astronomers are rightly wary of vacuum theories written from armchairs.

The condensate picture of the vacuum was not invented to solve the black-hole problem. It was assembled from the far end of physics: from thermodynamic derivations of the Einstein equations (Jacobson 1995), from four decades of analogue-gravity work culminating in measured Hawking radiation (Barceló et al. 2011; Steinhauer 2016), and above all from Volovik's helium-3 program — the one research line in this field led by someone who held the medium in his hands. Every exotic structure we invoke — critical surfaces, saturating stiffness, topological excitations, phase nucleation — exists on a laboratory bench somewhere.

The proposal, then, is not that astronomers adopt a speculation. It is that they recognize that the objects they observe every night are behaving exactly like the media that condensed-matter physicists measure every day — and that one constitutive equation, with one topological nonlinearity, connects a dwarf galaxy's rotation curve to a neutron star's radius to the silence at the center of a black hole.

The vacuum is not nothing. It is the material. Black holes and neutron stars are its two ends, and both are within reach of measurement.

Annotated References

Abbott, B. P., et al. (LIGO/Virgo) (2017). "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral." ***Phys. Rev. Lett.*** **119, 161101.** First binary neutron-star merger. Its tidal-deformability bound is one of the three data pins any constitutive equation of dense matter must pass.

Abbott, R., et al. (LIGO/Virgo) (2020). "GW190814: Gravitational Waves from the Coalescence of a 23 M ⊙ Black Hole with a 2.6 M ⊙ Compact Object." ***Astrophys. J. Lett.*** **896, L44.** The 2.6 M ⊙ companion sits inside the classical mass gap. Read here as an object caught in the phase-transition region between knot-packing and saturation.

Abedi, J., Dykaar, H., & Afshordi, N. (2017). "Echoes from the Abyss: Tentative Evidence for Planck-Scale Structure at Black Hole Horizons." ***Phys. Rev. D*** **96, 082004.** The first systematic search for post-merger echoes in LIGO data. Contested, but exactly the observable a critical-surface horizon predicts.

Barceló, C., Liberati, S., & Visser, M. (2011). "Analogue Gravity." ***Living Reviews in Relativity*** **14, 3.** The standard review of the field showing that horizons, Hawking radiation, and effective metrics arise generically in moving media. The mathematical backbone of Section 4.

Berengut, J. C., et al. (2013). "Limits on the Dependence of the Fine-Structure Constant on Gravitational Potential from White-Dwarf Spectra." ***Phys. Rev. Lett.*** **111, 010801.** Current observational bound on α variation in a strong potential, using G191-B2B — the benchmark object of our own $\alpha(\chi)$ prediction.

Bogdán, Á., et al. (2024). "Evidence for Heavy-Seed Origin of Early Supermassive Black Holes from an X-ray-Detected Quasar at z ≈ 10." ***Nature Astronomy*** **8, 126.** UHZ1: a massive black hole far too early for comfortable accretion growth. Here read as direct nucleation of the saturated phase in the early medium.

Cardoso, V., & Pani, P. (2019). "Testing the Nature of Dark Compact Objects: A Status Report." ***Living Reviews in Relativity*** **22, 4.** Authoritative survey of horizonless compact objects (gravastars, boson stars, echoes) and how observations can discriminate them from classical black holes.

Chae, K.-H., et al. (2020). "Testing the Strong Equivalence Principle: Detection of the External Field Effect in Rotationally Supported Galaxies." ***Astrophys. J.*** **904, 51.** Evidence that galactic dynamics respond to environment — data selecting acceleration, not potential, as the state variable χ.

Chapline, G. (2005). "Dark Energy Stars." ***Proc. 22nd Texas Symposium.*** Independent proposal that black-hole interiors are condensed vacuum phases. Supplies the object; our constitutive equation supplies the reason it forms.

Cromartie, H. T., et al. (2020). "Relativistic Shapiro Delay Measurements of an Extremely Massive Millisecond Pulsar." ***Nature Astronomy*** **4, 72.** J0740+6620 at ≈ 2.1 M ⊙ : the anchor for the maximum-mass pin, read here as the saturation point of the medium.

*EHT Collaboration (2019, 2022). "First M87 Event Horizon Telescope Results" (ApJL 875, L1) and "First Sagittarius A Results" (ApJL 930, L12).** The horizon-scale images. Consistent with GR so far; the photon-subring structure is where a critical surface and a mathematical horizon will eventually part company.

Hawking, S. W. (1975). "Particle Creation by Black Holes." ***Commun. Math. Phys.*** **43, 199.** The origin of black-hole radiation and, with it, the information paradox. The mechanism survives in a medium; the paradox does not.

Jacobson, T. (1995). "Thermodynamics of Spacetime: The Einstein Equation of State." ***Phys. Rev. Lett.*** **75, 1260.** Derives the Einstein equations as a thermodynamic equation of state — the first strong hint that gravity is the macroscopic behavior of a medium.

Konstapel, H. (2026a). "The Missing Weight and the Medium." ResearchGate. The companion paper: dark matter as a collective mode of the vacuum medium, with the constitutive curve $g(\chi)$ and the $a_0 = cH_0/2\pi$ prediction.

Konstapel, H. (2026b). "The Constitutive Equation of the Vacuum Condensate." ResearchGate. The technical derivation: acoustoelastic stiffening $g\_eff = \bar{g}(1 + \gamma\delta)$, the Bogoliubov machinery, and the closure test on public SPARC data extended here to compact objects.

Mazur, P. O., & Mottola, E. (2004). "Gravitational Vacuum Condensate Stars." ***PNAS*** **101, 9545.** The gravastar: a condensate interior with no horizon and no singularity, observationally near-degenerate with a black hole. The generic strong-field object of our framework.

Milgrom, M. (1983). "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis." ***Astrophys. J.*** **270, 365.** Identified the acceleration scale $a_0 \approx cH_0/2\pi$ empirically. Forty years later, the 2π obtains a derivation as the phase winding of the medium's topology.

Miller, M. C., et al. (2021) and Riley, T. E., et al. (2021). NICER radius measurements of PSR J0740+6620. ***Astrophys. J. Lett.*** **918, L28 and L27.** The radius pin: simultaneous mass and radius for the most massive well-measured pulsar, directly constraining the upper branch of the constitutive curve.

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Özel, F., & Freire, P. (2016). "Masses, Radii, and the Equation of State of Neutron Stars." ***Annu. Rev. Astron. Astrophys.*** **54, 401.** The standard review documenting how unconstrained the dense-matter equation of state remains — the gap the constitutive equation fills with one parameter.

Penrose, R. (1965). "Gravitational Collapse and Space-Time Singularities." ***Phys. Rev. Lett.*** **14, 57.** The singularity theorem. Its conclusion is only as strong as its premise of a linearly responding medium — the premise a saturating vacuum violates.

Steinhauer, J. (2016). "Observation of Quantum Hawking Radiation and its Entanglement in an Analogue Black Hole." ***Nature Physics*** **12, 959.** The landmark experiment: spontaneous, entangled Hawking phonons at a BEC horizon. Hawking's mechanism observed — in a condensate, without a singularity.

Thurston, R. N., & Brugger, K. (1964). "Third-Order Elastic Constants and the Velocity of Small Amplitude Elastic Waves in Homogeneously Stressed Media." ***Phys. Rev.*** **133, A1604.** The classical acoustoelastic result: stressed media stiffen. The mathematical template of our constitutive equation, imported from the materials bench.

Volovik, G. E. (2003). ***The Universe in a Helium Droplet.*** **Oxford University Press.** The master text of the condensate vacuum, written by an experimentalist from his own superfluid helium-3 laboratory: emergent metrics, horizons, topological matter — all measured, not postulated.