Upward in the Windings

The higher turns of the Vacuum.Net ladder

Exploration plan and first result · Version 0.2 · 20 November 2026 · J. Konstapel, Constable Research

1. What this book is about

A winding is a movement that comes back to where it began. Go once around a circle and you have made one winding. Go around twice and you have made two. A winding is not a number that is counted. It is a movement that is remembered.

“Upward in the windings” means: windings of windings. A loop on a circle is a winding of the first kind. A loop of loops sweeps over a surface, such as the skin of a ball. That is a winding of the second kind. Each step upward gives a new kind of turn. Mathematicians write $\pi_k(S^n)$ for the windings of kind k around a sphere of dimension n. The circle is S^1, the skin of a ball is S^2, and so on.

The book explores these windings rung by rung. On each rung it asks what the Vacuum.Net ladder sees, what can be proved by machine, and what it means for the whole.

2. Where we start: the net and the book that is closed

The Vacuum.Net theory begins with movement. One step is a direction: forward, back, or standing still. Such a step is a trit. Trits in a row form a stream. Read in base three, a stream is an address: a cell inside a cell inside a cell. Read in base two, the same stream computes. Two streams are the same place when they point to the same spot. A spot with two names is an edge.

The book Mathematical Theory Generated with the Vacuum.Net Theory proves this ladder in cubical Agda, a proof assistant that checks every step.[1] Its version 7.0 ends at the edge. Going from computing back to the address needs a decision that no rule of computation gives.

The same book glues the ends of the interval and calls the result Kring. It proves that Kring is a set, and therefore not the circle. A set knows where you are. It does not know how often you went around. So Kring forgets the windings. This book starts exactly there: it gives the windings back.

The idea is older than the book. In 2012 the author wrote that numbers are patterns of movement, and that reversing an operation produces new kinds of number.[2] The windings make this precise: the whole numbers return as the memory of a movement around a circle.

3. The pattern: going back produces new mathematics

Reversing an operation has repeatedly forced new mathematics. Reversing addition gave negative numbers. Reversing multiplication gave fractions. Reversing squaring gave √2 and the imaginary unit. In book 7.0, reversing the ladder (×2 back to ×3) gave not a new number but a new principle: LLPO, a decision at the edge.

The next reversal is to undo the gluing: from the circle back to the line. Forward, wrapping the line around the circle is smooth. Backward, one point on the circle has a whole row of points on the line, one for each turn. You can only go back by following a movement step by step. What appears is the number of turns. That is the first rung of this book.

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[1] J. Konstapel, Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (reference edition), 19 November 2026; files VacuumNetCircle, VacuumNetCircleFase, VacuumNetEdge, VacuumNetChoice.

[2] J. Konstapel, “About Number and Magnitude”, constable.blog, 9 January 2012.

4. The rungs

Each rung gives what is already known and machine-checked, and the question the net adds. “Known” means proved in the literature and, where stated, available in the cubical library that the book already uses.

Rung	Known (machine-checked where noted)	Question of the net
W0 · circle	The loops around the circle are the whole numbers: $\Omega S^1 \simeq \mathbb{Z}$.	Can Kring be raised from a set to the true circle, so that the turns of the shift are counted? Does the ×3 address become the universal cover, the line over the circle?
W1 · sphere	Windings of kind n around S^n are again $\mathbb{Z}$; below kind n there are none.	Does a ladder of pairs of trits close into the sphere S^2, the way one ladder closes into a circle?
W2 · Hopf	$\pi_3(S^2) \cong \mathbb{Z}$: a ball can be wrapped by a three-dimensional sphere in a way that does not unwind. The Hopf map $S^3 \to S^2$ does this.	Is the Hopf map visible as a pattern on the net, for instance as circles of trits over each point?
W3 · first twist	$\pi_4(S^3) \cong \mathbb{Z}/2$: going around twice is the same as not going around. Proved by computing the Brunerie number, −2, in cubical Agda.	Where does the 2 come from on the net? Is it related to the ×2 ladder, or is that a false resemblance?
W4 · frontier	$\pi_5(S^3) \cong \mathbb{Z}/2$ is classical. Its Brunerie number has been defined but not yet computed in cubical Agda.	Can the net make this computation shorter or clearer? This rung is real open ground.
W5 · order and chaos	Windings of spheres are irregular. Windings of the classical groups repeat with period 2 or 8 (Bott).	Where on the ladder does chaos turn into period? The 2012 text already pointed to Bott.

Sources for the rungs: the circle[3]; spheres[4]; Hopf[5]; the first twist[6]; the frontier[7]; the stable stems[8]; Bott periodicity[9]; irregularity[10].

5. First result: rung W0

W0 is done in Agda.[11] It has five parts.

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[3] D. R. Licata and M. Shulman, “Calculating the Fundamental Group of the Circle in Homotopy Type Theory”, LICS 2013, 223–232. In the cubical library: $\Omega S^1 \equiv \mathbb{Z}$ (Cubical/HITs/S1/Base.agda).

[4] The Univalent Foundations Program, Homotopy Type Theory: Univalent Foundations of Mathematics, Institute for Advanced Study, 2013, chapter 8. In the cubical library: $\pi_n S^n \cong \mathbb{Z}$ (Cubical/Homotopy/Group/PinSn.agda).

[5] The Hopf fibration $S^3 \to S^2$ gives $\pi_3(S^2) \cong \mathbb{Z}$ through the long exact sequence; see HoTT book §8.5 and U. Buchholtz, lecture notes Introduction to HoTT (2019/20), lecture 14. In the cubical library: $\pi_3 S^2 \cong \mathbb{Z}$ (Cubical/Homotopy/Group/Pi3S2.agda).

[6] G. Brunerie, PhD thesis (2016), first synthetic proof that $\pi_4(S^3) \cong \mathbb{Z}/2$. A. Ljungström and A. Mörtberg, “Formalizing $\pi_4(S^3) \cong \mathbb{Z}/2\mathbb{Z}$ and Computing a Brunerie Number in Cubical Agda”, LICS 2023 (arXiv:2302.00151). In the cubical library: Cubical/Homotopy/Group/Pi4S3/.

[7] T. Jack and A. Ljungström, “Yet another homotopy group, yet another Brunerie number”, abstract, TYPES 2025: a Brunerie number for $\pi_5(S^3) \cong \mathbb{Z}/2$; normalising it in Cubical Agda is still open.

[8] H. Toda, Composition Methods in Homotopy Groups of Spheres, Annals of Mathematics Studies 49, Princeton University Press, 1962. The table of stable stems is reproduced in A. Hatcher, Algebraic Topology.

[9] R. Bott, “The stable homotopy of the classical groups”, Annals of Mathematics 70 (1959), 313–337. Period 2 for the unitary group, period 8 for the orthogonal group.

[10] R. Bott’s biographers note that for spheres the higher homotopy groups show no discernible pattern; see “The Life and Works of Raoul Bott”, arXiv:math/0201027, §13.

[11] Agda file VacuumNetWinding.agda, version 1.0 (20 November 2026), checked with Agda 2.8.0 and cubical v0.9 under --safe; judged by a separate agent.

- Kring forgets. Every map from the true circle to Kring is constant. Kring is a set, and a set cannot hold a turn (kring-vergeet).
- Steps as paths. A new type, Wind, has a point for every stream and a path for every step from a stream to its tail. A step is now a movement, not a function. A map tel sends every point to the base of the circle and every step to one turn. It counts steps; a step back counts −1.
- Periodic streams carry turns. If a stream returns to itself after k steps, the k steps close into a loop. Under tel that loop winds exactly k times. For k ≥ 1 it is not trivial (periode-winding, lus-niet-triviaal). Going round m times winds m·k times (herhaal-winding). Not yet proved: that all loops at that point are the whole numbers.
- Edges end in a cycle. Each of the two names of an edge reaches, in finitely many steps, a constant stream: L L L … or R R R …. A constant stream is its own tail, so it carries a loop that winds once (rand-naar-lus).
- On paper, judged but not in Agda: every part of Wind has at most one cycle, because every stream has exactly one tail. A part without a periodic stream is an infinitely branching tree and holds no loop. A part with a periodic stream has the shape of a circle.

Reading, not proof: turns arise exactly where a pattern returns. Cycles are periodic patterns on the net. The edges of book 7.0 end in the simplest cycle, of period 1. This is the bridge to the Guide of Humanity, which asks where a client's cycles are.

6. Relation to the Guide and to earlier work

The Guide (built separately with Claude Code) finds a client's cycles, their edges and the choice at the edge. This book gives the mathematics of those cycles. W0 shows that a cycle can be counted. W3 asks whether two rounds can cancel. W5 asks which cycles are regular and which are chaotic.

The author's earlier work already points here. The financial-windings text describes windings that never fully close, with an address and a tension.[12] Paths of Change describes every successful change as a closed path.[13] The Spiral Navigator already uses Bott periodicity and the Hopf fibration for navigation in SWARP.[14] Rungs W2 and W5 are therefore not far from practice.

7. The central question

Does the net say anything about the irregular windings, or only about the regular ones?

- If the net predicts structure in the irregular windings, that is a real test of the theory. Nobody knows a pattern there.
- If the net only reproduces the regular windings, such as the circle, the spheres and Bott periodicity, then the book shows where its reach ends. That is also a result, and the book will say so plainly.

8. How the book is made

- Each rung has three parts: Seen (the picture), Reasoned (what is proved), and Back to the whole (what it means). This is the form of book 7.0.
- Every claim carries a status: proved in Agda, known theorem with source, argument on paper, reading, or open.

[12] J. Konstapel, "Understanding Financial Markets through Vacuum.Net Theory" (The Foreseeability Boundary: Windings, Loading and the Physics Hidden inside Economics), constable.blog, 5 September 2026.
[13] J. Konstapel, "About Paths of Change (PoC)", constable.blog, 10 September 2020. Every successful change is a closed path through four world views (after W. McWhinney).
[14] J. Konstapel, "The Spiral Navigator", constable.blog, 10 March 2026. Uses Bott periodicity and the Hopf fibration for navigation in SWARP.

- The Agda work builds on the cubical library, which already contains the circle, the spheres, Hopf and $\pi_4(S^3)$. The new work is the bridge from the net to these objects.
- Every new result is checked by a separate judge agent before it enters the book.
- Each delivery has a version number in the file name and on the title page.

9. Risks

- The higher windings are hard mathematics. The proof of $\pi_4(S^3)$ took years. This book builds on that work; it does not redo it.
- Raising Kring to the true circle needs a higher inductive type. The ladder of book 7.0 lives in sets. The bridge may be harder than it looks.
- A resemblance between the net and a winding (for instance the 2 in $\mathbb{Z}/2$ and the ×2 ladder) can be a coincidence. The book will only call it a result if it is proved.

10. Next step and decision

Two steps are possible next. First, finish W0: prove that all loops at a periodic point are exactly the whole numbers, not only their image under tel. Second, start W1: a ladder of pairs of trits, and whether it closes into the sphere S^2.

Decision for the author: finish W0 first, or go on to W1.

Annotated references

J. Konstapel, “About Number and Magnitude”, constable.blog, 9 January 2012. *The author’s own text on number as movement and form. Source of the idea that reversing operations creates new numbers.*

J. Konstapel, Mathematical Theory Generated with the Vacuum.Net Theory, version 7.0 (reference edition), 19 November 2026; files VacuumNetCircle, VacuumNetCircleFase, VacuumNetEdge, VacuumNetChoice. *The closed reference edition. Proves the ladder, the edge and LLPO; shows that Kring is a set and not the circle. Starting point of this book.*

D. R. Licata and M. Shulman, “Calculating the Fundamental Group of the Circle in Homotopy Type Theory”, LICS 2013, 223–232. In the cubical library: $\Omega S^1 \equiv \mathbb{Z}$ (Cubical/HITs/S1/Base.agda). *First machine-checked proof that the loops around the circle are the whole numbers. Basis for rung W0.*

The Univalent Foundations Program, Homotopy Type Theory: Univalent Foundations of Mathematics, Institute for Advanced Study, 2013, chapter 8. In the cubical library: $\pi_n S^n \cong \mathbb{Z}$ (Cubical/Homotopy/Group/PinSn.agda). *Standard introduction to homotopy type theory. Chapter 8 treats loops, spheres, Freudenthal and Hopf. Basis for W1 and W2.*

The Hopf fibration $S^3 \to S^2$ gives $\pi_3(S^2) \cong \mathbb{Z}$ through the long exact sequence; see HoTT book §8.5 and U. Buchholtz, lecture notes Introduction to HoTT (2019/20), lecture 14. In the cubical library: $\pi_3 S^2 \cong \mathbb{Z}$ (Cubical/Homotopy/Group/Pi3S2.agda). *Clear lecture notes on $\pi_3(S^2)$ via the Hopf fibration and the long exact sequence.*

G. Brunerie, PhD thesis (2016), first synthetic proof that $\pi_4(S^3) \cong \mathbb{Z}/2$. A. Ljungström and A. Mörtberg, “Formalizing $\pi_4(S^3) \cong \mathbb{Z}/2\mathbb{Z}$ and Computing a Brunerie Number in Cubical Agda”, LICS 2023 (arXiv:2302.00151). In the cubical library: Cubical/Homotopy/Group/Pi4S3/. *The milestone: $\pi_4(S^3) \cong \mathbb{Z}/2$ proved by computation in cubical Agda. Basis for W3.*

T. Jack and A. Ljungström, “Yet another homotopy group, yet another Brunerie number”, abstract, TYPES 2025: a Brunerie number for $\pi_5(S^3) \cong \mathbb{Z}/2$; normalising it in Cubical Agda is still open. *The current frontier: a Brunerie number for $\pi_5(S^3)$ that has not yet been computed. Basis for W4.*

H. Toda, Composition Methods in Homotopy Groups of Spheres, Annals of Mathematics Studies 49, Princeton University Press, 1962. The table of stable stems is reproduced in A. Hatcher, Algebraic Topology. *Classical computation of the homotopy groups of spheres. Reference for the irregular pattern.*

R. Bott, “The stable homotopy of the classical groups”, Annals of Mathematics 70 (1959), 313–337. Period 2 for the unitary group, period 8 for the orthogonal group. *The periodicity of the classical groups. Basis for W5.*

J. Konstapel, “Understanding Financial Markets through Vacuum.Net Theory” (The Foreseeability Boundary: Windings, Loading and the Physics Hidden inside Economics), constable.blog, 5 September 2026. *The author’s windings in financial markets: address, tension, loading, discharge, foreseeability boundary.*

J. Konstapel, “About Paths of Change (PoC)”, constable.blog, 10 September 2020. Every successful change is a closed path through four world views (after W. McWhinney). *Paths of Change: change as a closed path. A cycle in the Guide’s sense.*

J. Konstapel, “The Spiral Navigator”, constable.blog, 10 March 2026. Uses Bott periodicity and the Hopf fibration for navigation in SWARP. *The author’s navigation design for SWARP, already built on Bott periodicity and the Hopf fibration.*

Agda file VacuumNetWinding.agda, version 1.0 (20 November 2026), checked with Agda 2.8.0 and cubical v0.9 under --safe; judged by a separate agent. *The proofs behind section 5.*

R. Bott’s biographers note that for spheres the higher homotopy groups show no discernible pattern; see “The Life and Works of Raoul Bott”, arXiv:math/0201027, §13. *Survey of Bott’s life and work; states the contrast between irregular spheres and periodic classical groups.*