Why Silence Is a Vibration

Rest, tone and the edge between them

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Abstract

We think of silence as the place where sound ends. This essay argues that it is not. Silence is a vibration whose size has shrunk to zero. The argument starts from one assumption: movement comes first, and everything we see or hear is a picture of it. From that assumption, rest turns out to be an oscillation like any other. A model from dynamics, the Hopf bifurcation, shows how a tone grows out of rest and shrinks back into it without a break. Between rest and tone lies an edge. A sound that stays on that edge never quite repeats. It almost returns, at moments that follow a fixed series. That series is known to tuning theory. As an order of time and form, it seems not to have been used. A bell, and the artwork Sino, serve as the test case.

1. What a bell tells us

Strike a bell and listen. At first you hear many tones at once. The high ones fade quickly. A low tone stays. Bell founders call it the hum. It lies an octave below the main tone of the bell, and it lingers long after the others are gone.

Listen to the end. There is no moment at which the hum stops. It thins. It grows weaker and weaker, until you can no longer tell it from the room. The vibration does not switch off. It fades towards zero.

John Cage made the same discovery in another way. In 1951 he sat in an anechoic chamber at Harvard, a room built to absorb all sound. He expected to hear nothing. He heard two tones, one high and one low. He was told they were his nervous system and his blood. His conclusion was that silence does not exist. There is always something that vibrates.

These are observations. This essay asks for more: a reason. Why should silence be a form of vibration and not its absence? The reason, we will argue, lies in what comes first.

2. Movement comes first

Most of our thinking starts with things. First there is a bell, then it moves. First there is a string, then it vibrates. Rest comes first; movement is added.

The Vacuum.Net theory reverses this order. In the beginning there is movement. Nobody was there at the start, but without movement nothing happens. So the movement never began and it never stops. It goes round. The simplest picture is a circle: one point and one path that leaves the point and comes back to it.

Stand at one place on that circle and look. You can see three things. The movement can go back. It can stay. It can go out. We write these three directions as T (back), 0 (stay) and 1 (out). They are not three numbers. They are three snapshots of one movement.

This matters for silence. In this picture, staying is not the absence of a direction. It is one of the three directions. It is the direction in which the movement does not leave its place.

3. An oscillation is a repeated block

Write a movement as a row of directions, one after the other. For example: 1, T, 1, T, 1, T, and so on. Out, back, out, back. This row repeats a short block, the pair "1 T", for ever. We call any such row an oscillation. A tone is the clearest example: a pattern that repeats.

Each oscillation has a place on the circle. There is a simple rule for it. Read the block as a number in base three, with T worth −1. Call it W. If the block has n directions, the oscillation sits at the fraction $W/(3^n - 1)$ of a full turn. Two examples make this concrete:

block 1 → $1/(3 - 1) = 1/2$ of a turn

block 1 T → $(3 - 1)/(9 - 1) = 2/8 = 1/4$ of a turn

The rule is the same one that turns 0.333… into 1/3 in ordinary decimals. Repeating digits always give a fraction.

Now take the block that contains only 0. Stay, stay, stay, for ever. By the same rule its place is $0/(3 - 1) = 0$. It is an oscillation like the others. It has a block, a length and a place on the circle. Its block is "stay". Its place is the starting point.

This is the first half of the answer. Rest is not outside the family of oscillations. It is a member of that family. It is the oscillation that repeats nothing but staying. This statement has been checked by a proof assistant, a program that accepts a proof only when every step is correct [1].

4. When does a movement close?

A second way leads to the same place. Think of a swinging movement, described step by step. The next position depends on the last two, by the rule

$$\text{next} = c \times \text{now} - \text{before}$$

The number c sets the character of the swing. Start with before = 0 and now = 1 and see what happens:

c = −1 : 0, 1, −1, 0, 1, −1, … returns after 3 steps

c = 0 : 0, 1, 0, −1, 0, 1, … returns after 4 steps

c = 1 : 0, 1, 1, 0, −1, −1, 0, … returns after 6 steps

c = 2 : 0, 1, 2, 3, 4, 5, … never returns

For c = −1, 0 and 1 the movement closes. It is a cycle. For c = 2 it does not close. It drifts away in a straight line. These are the only whole numbers for which the swing closes from every start. Crystals obey the same law. Apart from the half turn and the full turn, they allow only rotations of order 3, 4 and 6. That is a classical fact of crystallography.

Now let c take any value between −2 and 2. Then the swing is a rotation by a fixed angle, and c = 2 cos(angle). As c comes close to 2, the angle becomes small. The swing turns more and more slowly. Its period grows without limit. At c = 2 the turning stops and the drift begins.

So there is an edge. Below it the movement turns. At it, the turning slows to nothing. Beyond it, the movement runs away. A slow swing just below the edge is still a cycle. It is the same kind of thing as a fast one. It is only longer.

5. How a tone is born out of rest

Dynamics has a precise model for the moment when a cycle appears. It is called the Hopf bifurcation, after Eberhard Hopf. Let r be the size of a vibration and let μ be a setting that we can turn, like the bow pressure on a violin. In the simplest case the size changes as

$$\text{change of } r \;=\; \mu \cdot r - r^3$$

If μ is below zero, every vibration dies out. The size goes to zero and stays there. That is rest. If μ is above zero, the vibration grows until it reaches the size $\sqrt{\mu}$, and stays there. That is a steady tone.

Now turn μ down slowly towards zero. The tone does not stop at one moment. Its size $\sqrt{\mu}$ shrinks smoothly. At μ = 0 the size is zero. The cycle has not been switched off. It has shrunk into a point. That point is the state of rest.

This is the second half of the answer. Seen from dynamics, rest is the tone at size zero. Seen from the circle of Section 3, rest is the oscillation that only stays. The two descriptions meet. Rest and tone are not two states with a wall between them. They are one state at different strengths.

The author has used the Hopf bifurcation before, to describe creativity [6]. In that reading, new forms appear near the critical point, where a small push changes the whole pattern. A system far from the edge either rests or repeats. A system near the edge invents.

6. Two ladders that never meet

What does a sound do if it stays near the edge? To answer this we need two simple movements: doubling and tripling.

Doubling a frequency gives the octave. Tripling a frequency gives the twelfth, an octave plus a fifth. These are the two purest intervals in music. Call them the ×2 ladder and the ×3 ladder.

The two ladders never meet. No number of octaves equals any number of twelfths. The proof takes one line. A power of 2 is even. A power of 3 is odd. An even number is never equal to an odd one, except in the trivial case of 1 = 1, with no steps at all.

Musicians know the consequence. Stack twelve fifths and you overshoot seven octaves by a small amount. That amount is the Pythagorean comma. It is 23.46 cents, about a quarter of a semitone. One cent is a hundredth of an equal-tempered semitone.

Now picture the octave as a circle. One turn is one octave. A step of a twelfth moves you round by a fixed amount: $\log_2 3$ turns, about 1.585. Because the ladders never meet, this amount is not a fraction. So you never land exactly where you started. Step after step, you visit new places on the circle. In the end you come as close as you like to every point. This is Kronecker’s theorem.

7. The rhythm of near-returns

You never return exactly. But some steps bring you closer to the start than all steps before them. These are the near-returns. They come after a fixed number of steps:

1, 2, 5, 12, 41, 53, 306, 665, …

Between two of these numbers, no step comes closer. The numbers come from the continued fraction of $\log_2 3$, a standard way to find the best fractions for a given number. The best fractions are 2/1, 3/2, 8/5, 19/12, 65/41, 84/53, 485/306 and 1054/665. The denominators are the near-returns.

Some of these numbers are familiar. Twelve is the number of keys in an octave of a piano. Fifty-three is a tuning that the music theorist Nicholas Mercator described in the seventeenth century, and that later tuning theory took up. In tuning, these numbers are used to divide the octave.

Here they mean something else. They are moments in time. A sound that drifts near the edge almost returns after 1 step, then after 2, then after 5, then after 12, and so on. The returns come further apart. Each return is closer than the last. This is a rhythm with its own law. It neither repeats like a beat, nor scatters like noise.

An earlier survey by the author searched the literature for such a ladder of near-returns used as an ordering principle [7]. It found locking at simple ratios in systems with two frequencies. It did not find the ladder itself used to order time or form.

8. What does not fit, goes on

There is one more piece. When two movements do not fit, something is left over. The comma is such a remainder. In ordinary practice a remainder is a nuisance: an error, a noise, a slight mistuning.

In the theory it is material. Think of adding in base three with the directions T, 0 and 1. One plus one is two. But two is not a direction. Two is three minus one. So one plus one is written as “1 T”: one at the next place up, and minus one here. The part that does not fit at one scale is carried to the next.

Read as sound, this says: what a tone lacks becomes the seed of the next layer. The comma is not a defect of the octave and the fifth. It is what stops their joint movement from repeating, and so it lets that movement reach everything. Silence after a climax, read this way, is not the end of the material. It holds the remainder that the next layer will carry.

The same survey looked for this passing on of a remainder from one scale to the next. It found the related idea that noise can improve detection, known as stochastic resonance. It did not find the passing on itself as a principle [7].

9. The octave belongs to doubling

Back to the bell. Its hum lies an octave below the main tone. Going an octave down means halving the frequency. On the ×2 ladder halving is trivial. You put one zero in front, and every place moves to half its value.

On the ×3 ladder halving is not possible step by step. Take one third, written in base three as 0.1000… . Its half is one sixth. But one sixth sits exactly on the border between two cells. Just

above one third, the half must start with the digit 1. Just below, it must start with 0. To decide, you would have to read the whole infinite row of zeros. No machine that reads one digit at a time can do that.

So the octave is not just one interval among others. It is the interval of the ladder that can compute. The fifth belongs to the ladder that names places exactly but cannot even halve. A bell carries both.

10. A bell as a test

A bell is a good test because its tones are not perfect. Its five lowest tones are named hum, prime, tierce, quint and nominal. In an ideal bell the hum is an octave below the prime and the nominal an octave above. The quint is a fifth above the prime. The tierce is a minor third. Real bells deviate from these ideals by several cents.

The octaves belong to doubling. The fifth belongs to tripling. The minor third does not belong to either. Its pure form, 6/5, contains the factor 5. A bell therefore holds three cases at once: what closes, what almost closes, and what does not close at all.

The artwork Sino by Mike Rijnierse and Rob Bothof uses such a bell. It stands upside down, mouth to the sky, like a singing bowl. A resonator replaces the clapper. It does not strike the bell. It brings out one tone and holds it as long as desired. The bell came from the Royal Eijsbouts foundry. The work was inspired by the Netherlands Carillon in Washington, a gift for American aid after the war. According to Rijnierse, its bells were cast in haste in the 1950s. They were never properly in tune and sounded dissonant for most of their life.

Sino builds up to one climax and then falls silent. In the terms of this essay it crosses the edge once: from rest to tone, and back to rest. A next work could stay on the edge. Hold one tone from the octave family and one from the fifth family. Let them drift. Place the moments of near-return at 1, 2, 5, 12, 41 and 53 units of time. Then let the strength fall towards zero, slowly, so that the silence still carries the phase of the last tone.

Whether listeners hear the near-returns is an open question. It can be tested.

11. How sure is this?

The essay combines three kinds of statement. They should not be confused.

Some are proved. That rest is the oscillation with only “stay” in its block, that the swing closes only for $c = -1, 0$ and 1, that the ladders never meet, and that halving is impossible digit by digit on the $\times 3$ ladder: these are checked by a proof assistant [1]. Two of them are simple enough to check by hand, as shown above.

Some are known results. The Hopf bifurcation, Kronecker’s theorem, continued fractions, the crystal rotations and the tones of a bell are textbook material [3, 4, 5, 8, 9].

Some are readings. That rest in the circle and rest at the Hopf edge are one thing, that silence carries the phase of the last tone, and that the near-returns can order a work of sound: these are interpretations. No proof forces them. They are offered to be tried.

12. Conclusion

Silence is a vibration. It is the movement that only stays. It is the tone whose size has shrunk to zero. It is the hum of a bell that thins but never stops.

This view removes the wall between rest and sound. There is one state, with a strength. Between rest and tone lies an edge. A sound that stays on that edge neither rests nor repeats. It almost returns, at moments that follow the series 1, 2, 5, 12, 41, 53. Each return is rarer and purer than the last.

Tuning theory knows these numbers as divisions of the octave. As a rhythm of time and form they are still unused. That is an invitation, not a result.

Tool disclosure. This text was made with the AI system Claude (Anthropic) under the direction of the author. The author chose the question and decides what counts as a result.

Annotated references

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