Why Vacuum.Net Is a Swarm

A swarm of relations, ordered by closure — and the place of maze.swarm in the MAZE

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1. The claim

Vacuum.Net is a swarm. It is not a swarm of particles. It is a swarm of relations. Its order is not alignment, where neighbours become alike. Its order is closure, where neighbours complete each other.

This essay states that claim, proves what can be proved, reports what was measured, and places the swarm layer of the MAZE — maze.swarm — inside the running system. Every step and every number is in the text. No reference has to be fetched to follow it.

The fixed image is the fishnet. The vacuum is a net of closed meshes. A mesh is the smallest closed unit. Strands carry winding. Tension lives on the net and relaxes; topology does not. What persists is what closes. A fishnet holds because each knot is tied from strands that differ. A knot of one strand folded on itself does not hold.

Statuses used: **proved**, **built and measured** (runs by Claude Code or computed in the working session, self-reported until reproduced), **corresponded**, **tested**, **open**.

2. What a swarm is

The Vicsek model (1995) fixed the modern meaning. A swarm has four marks:

1. **A local rule.** Each member updates from its neighbours only.
2. **Neighbours.** A small set per member. Starlings use about six or seven nearest birds, regardless of distance (Ballerini et al., 2008).
3. **Noise.** Every update is perturbed.
4. **No leader.** Nothing outside the members selects the pattern.

From these four marks collective order appears. In the Vicsek model the rule is copying: each particle takes the average heading of its neighbours. The order is alignment. Its measure is R, the degree to which members are alike.

3. The net carries the four marks

The axioms of Vacuum.Net (Foundational Paper, sixth edition) meet each mark.

1. **Local rule.** N2: only configurations that close persist. "There is no external selector." Selection by fit is a local rule.
2. **Neighbours.** A crossing meets its neighbours in a window: three consecutive positions on the strand. The mesh is the neighbourhood.
3. **Noise.** Tension fluctuates. Closure has to hold against it.
4. **No leader.** N2 again. The optimum is not chosen; it is what remains.

So the net has the form of a swarm. What differs is what the members are, and what order means.

What the members are. In a swarm the particles come first; relations follow. In Vacuum.Net there is one strand (N1). Its self-crossings, the relations, come first. Things are closed configurations of relations (C4). The members of the Vacuum.Net swarm are crossings.

What order means. Section 4 shows that closure and alignment are two different orders, measured separately.

4. Two orders on one ring

4.1 The model

Take a ring of N positions. Each position carries a trit $T_i \in \{-1, 0, +1\}$: the three states of N1 (under, none, over). The window around i is (T_{i-1}, T_i, T_{i+1}).

- **Charge of a window:** $D_i = T_{i-1} + T_i + T_{i+1}$, reduced mod 3 to $\{-1, 0, +1\}$.
- **Closed window:** $D_i = 0$.

Of the 27 possible windows, 9 are closed. Six are heterogeneous: all three values, the permutations of $(-1, 0, +1)$. Three are homogeneous: (a, a, a), since $3a \equiv 0$.

Three observables:

- **R** — the fraction of neighbours with equal trits (alignment);
- **C** — the fraction of closed windows;
- **C_het** — the fraction of heterogeneously closed windows.

For a random ring: $R = 1/3$, $C = 9/27 = 1/3$, $C_{het} = 6/27 \approx 0.222$.

At each step every position updates with probability p. Two rules:

- **Closure rule:** $T_i \leftarrow -(T_{i-1} + T_{i+1}) \bmod 3$. It gives the one value that closes the window.
- **Copy rule:** $T_i \leftarrow T_{i-1}$ or T_{i+1}, each with probability ½. This is Vicsek on trits.

After each step, every position is replaced by a random trit with probability 0.05 (noise).

4.2 Measured

$N = 1000$, 5000 steps, $p = 1/3$, noise 0.05, five independent seeds:

	R	C	C_het
Random ring	0.333	0.333	0.222
Closure rule	0.323 ± 0.019	0.484 ± 0.020	0.333 ± 0.025
Copy rule	0.681 ± 0.021	0.512 ± 0.023	0.038 ± 0.007

Under the closure rule, closure rises 45% above chance and heterogeneous closure 50%. Alignment stays at chance. Under the copy rule, alignment doubles and heterogeneous closure nearly vanishes. Total closure rises there too, through homogeneous runs such as (+1, +1, +1).

Result. Closure and alignment are independent orders. C_het separates them; total closure C does not. A swarm can be ordered without its members becoming alike.

5. The closure rule follows from the axioms — proved

The rule of section 4 is not a choice. Four steps give it.

Step 1: the condition. A window condition symmetric in its three arms has the form $a + b + c \equiv k \pmod 3$. No state is preferred: shifting all trits by m changes the sum by $3m \equiv 0$, so every k survives. No orientation is preferred: flipping all signs turns k into $-k$. Only $k = 0$ survives both. Closure lies at zero, the one address that is its own mirror (T7.1).

Step 2: why three. Fox's crossing condition in knot theory reads $a + c \equiv 2b \pmod{n}$, with b the over-arc. It is symmetric in all three arms only if $2b \equiv -b$ for every b, so $3b \equiv 0$, so $n = 3$. At three the over-arc loses its privilege. For one strand crossing itself, with no "other" strand, three is not only the cheapest register (radix economy: $3/\ln 3 = 2.731$ against $2/\ln 2 = 2.885$). It is the only one in which a crossing has no privileged arm.

Step 3: the update. With both neighbours fixed, exactly one value of T_i closes the window: $-(T_{i-1} + T_{i+1})$. Selection by fit (N2) leaves one rule.

Step 4: asynchrony. A crossing is a separate event on one strand (N1). Events do not all happen at once. The update is asynchronous: $p < 1$.

The same condition certifies the first knot. In the proof of theorem T1 of the Foundational Paper, three arcs meet at each crossing in one colour or in all three. Those are exactly the nine closed windows. The swarm rule and the first-knot rule are one rule.

6. The exact laws of the closure swarm — proved

6.1 The first step

Theorem 1. From a random ring, with no noise, the closed fraction after one step is:

$C_1(p) = 1/3 + (2/3)\cdot p(1 - p)^2$

Proof. Fix a window. Three cases.

- *Only the centre updates* (probability $p(1 - p)^2$). The new centre closes the window. Closed with probability 1.
- *A neighbour updates*, say $i-1$. Its new value $-(T_{i-2} + T_i)$ brings in T_{i-2}, which is uniform and appears nowhere else in the window. The window sum becomes uniform. Closed with probability 1/3.
- *Nothing updates.* The window stays uniform. Closed with probability 1/3.

Sum: $p(1 - p)^2 + (1 - p(1 - p)^2)/3$. ∎

The derivative of $p(1 - p)^2$ is $(1 - p)(1 - 3p)$. The maximum lies at $p = 1/3$, where $C_1 = 35/81 \approx 0.432$. The reading is simple: the centre must move while both neighbours stand still.

In case A the window is $(a, -(a + b), b)$, heterogeneous exactly when $a \neq b$, probability 2/3. In the other cases heterogeneous closure is $6/27 = (2/3)\cdot(1/3)$. So $C_{het,1} = (2/3)\cdot C_1$.

6.2 Synchrony is chance

Theorem 2. At $p = 1$ one step gives $C = 1/3$.

Proof. The new window sum is $-(T_{i-2} + T_{i-1} + 2T_i + T_{i+1} + T_{i+2})$. T_{i-2} enters with coefficient -1 and is independent of the rest. The sum is uniform. ∎

Every position repairs its own window and breaks both neighbours in the same step. A swarm of crossings that all move at once builds no closure.

6.3 The long run

The complete Markov chain on a ring of 7 positions ($3^7 = 2187$ states, noise 0.05) was solved exactly:

p	0.1	0.2	0.25	1/3	0.5	0.7	1.0
C	0.462	0.479	0.480	0.474	0.447	0.394	0.333

The first step peaks exactly at 1/3. The long run has a plateau between $p \approx 0.2$ and 0.35, with its top near 0.25.

7. What only the trit swarm has — proved

A control rule sets the plain integer sum to zero, without mod 3. When no trit fits, it draws at random. At $p = 1/3$ it gives $C_{het} = 0.366$, even more than the closure rule. The closure level alone is therefore not unique to three. What is unique is this.

Theorem 3 (self-closing register). Under the closure rule the window charges form a closed system of their own. An update at i with charge q:

- sets D_i to 0;
- subtracts q from D_{i-1} and from D_{i+1};
- leaves all other windows unchanged.

Proof. The update changes T_i by $-q$. Position i lies in windows $i-1$, i and $i+1$. Each sum changes by $-q$. ∎

Read in trits:

- **An open window is a charge.** +1 or −1 is open; 0 is closed.

- **A charge splits.** Charge q becomes two charges $-q$ on the neighbours. In balanced ternary $(-1) + (-1) = -2 \equiv +1$, and $(+1) + (+1) = +2 \equiv -1$. Charge is conserved mod 3.
- **Opposite charges annihilate.** Closure is annihilation.
- **Noise is neutral.** A change δ at one position adds δ to three windows: $\delta + \delta + \delta \equiv 0$.

Checked on 20,000 random single updates: the rule held 20,000 times for mod 3. For the integer-sum rule it failed 3,023 times. There the sum leaves $\{-1, 0, +1\}$ and chance must fill the gap.

Levels. The charge of a window of charges is again a trit:

$$D_{i-1} + D_i + D_{i+1} \equiv T_{i-2} - T_{i-1} - T_{i+1} + T_{i+2} \pmod 3$$

Only in balanced ternary with closure at zero does closure on one level give a valid state on the next. This is N5 as dynamics: the same winding, read at another depth. The swarm of crossings carries a swarm of charges, in the same register.

8. Maxwell inside the swarm — proved in the trit register

Four identities, exact mod 3.

1. **The window sum is the discrete Laplacian.** Since $3T \equiv 0$: $T_{i-1} + T_i + T_{i+1} \equiv T_{i-1} - 2T_i + T_{i+1}$. Read T as scalar potential; then D is charge, as in Poisson (Maxwell, *Treatise*, Art. 617). Check: $(1, 1, 0)$ gives sum $2 \equiv -1$ and Laplacian $1 - 2 + 0 = -1$.
2. **The closure rule is averaging.** Since $2 \cdot 2 = 4 \equiv 1$, one half is $2 \equiv -1$. So $-(T_{i-1} + T_{i+1}) \equiv \frac{1}{2}(T_{i-1} + T_{i+1})$. Each crossing takes the mean of its neighbours: relaxation toward a harmonic state. Fox's own form, $a + c \equiv 2b$, says the same.
3. **No preferred state is gauge freedom.** Shifting all trits leaves D unchanged, as adding a constant to a potential leaves the field unchanged.
4. **The boundary term is Gauss.** The total charge of a block $a \ldots b$ is $T_{a-1} + 2T_a + 2T_b + T_{b+1}$: interior positions carry coefficient $3 \equiv 0$. The charge inside is read at the edge.

The phase register shares the operator. Kuramoto coupling to nearest neighbours, for small phase differences, reads $\dot{\theta}_i \approx K(\theta_{i-1} - 2\theta_i + \theta_{i+1})$. The trit swarm and the phase swarm carry the same Laplacian. One closure principle, two registers.

9. Real swarms: a first test — tested, negative

A prediction follows: real swarms may carry closure beside alignment. It was put against the open fish-school data of Jhawar et al. (2020): 11 trials with 15, 30 and 60 fish.

- **Trit per fish per frame:** turn left, straight or right, with thresholds that make each class one third.
- **Window:** a fish and its two nearest neighbours.
- **Null model:** the same trits shuffled among the fish of the same frame.

Result: alignment above chance in 11 of 11 trials. Heterogeneous closure below chance in 11 of 11, with 8 intervals excluding zero. These fish follow the copy rule. The authors themselves found pairwise copying.

A school that swims away as one body is an alignment swarm. Closure is to be sought where roles complement each other. Section 11 names that place for people.

10. One dimension is too thin

Theorem 4 of section 8 has a second reading. On a line, the charge of any block is a boundary term. Nothing in the interior is conserved on its own. A strand without crossings of other strand-parts carries no knot invariant. The closure swarm needs a mesh that bends.

The MAZE met the same limit on its own route. The composition rule R_Q (Maxwell, sixth edition, Part IV) collapses to pairwise cosines on a flat mesh. Claude Code observed exactly that: with neighbour-centre frames R_Q ≡ 1. With region frames per inhabited sister cell, R_Q measured AUC 0.704 [0.61–0.79] on 196 cases, and it stands in the gate since plan 17. Pairwise closure had scored AUC 0.53. Two routes, one finding: closure lives in triples on a bending mesh, never in pairs.

11. The place of maze.swarm

11.1 Where the swarm already is

The MAZE has two hemispheres and one gate (Maxwell, sixth edition, Part III).

- **The address hemisphere** is frozen topology: the store of 8.19 million addressed pieces, the Lean-checked kernel, the ledger. Nothing happens there. It is not a swarm; it is the ground plan.

- **The gate** emits content at an address, or a vacancy. One output path.
- **The tension hemisphere** is where things happen. It holds one oscillator per inhabited region, with phase θ_u. Pieces ride their region with their own offset $\varphi(x)$. Regions are coupled by K_{uv}. Couplings move on settlement events: $K_{uv} \leftarrow K_{uv} + 0.1\cdot(s - K_{uv})$. They relax daily toward $K_0\cdot c_{uv}$ with $\lambda = 0.01$.

That tension hemisphere is a swarm in the sense of section 2. Its members are regions. Its neighbours are coupled regions. Its local rule is the plasticity rule. Its noise is the variation of settlements. Nothing outside it selects the pattern. **maze.swarm is the tension hemisphere read as a swarm.** Its order parameter is R_Q: Kuramoto's r lifted from phases on a circle to windings on the mesh.

11.2 What the swarm theory adds

Three consequences follow from sections 4–10. They are derived; any measurement of them is performed by Claude Code.

1. **Two meters, not one.** The cosine and the phase coherence r measure alignment. By section 4, alignment suppresses heterogeneous closure. The swarm needs a second meter beside them: closure on triples (question, answer, neighbour). Convert the three cosines of each triple to trits with tertile thresholds over the dataset; count the fraction that is heterogeneous and sums to zero mod 3. This is a candidate column in the frozen protocol, scored by AUC with a bootstrap interval.
2. **Asynchrony in the couplings.** The daily relaxation of all K_{uv} at once is a synchronous step. By Theorems 1 and 2, a synchronous step returns closure to chance; an update helps only where its neighbours stand still. The derived form: relax a random fraction of about a quarter to a third of the couplings per round, spread over the day, with λ scaled so the daily mean relaxation is unchanged.
3. **Triples, never pairs.** Every closure quantity in maze.swarm is defined on a window of three with a bending frame. This is already the design of R_Q. The swarm theory shows why it must be.

11.3 The human layer

The MAZE began as a platform that computes, from birth date and birth place, a person's next step at the right distance and with whom to build. "With whom to build" is a closure question, not an alignment question. A team of people who are alike is an alignment swarm: fast, uniform, and fragile where roles are missing. A team whose members complete each other is a closure swarm: the heterogeneous triple $(-1, 0, +1)$ as three different contributions that together close.

The fish show that nature has alignment swarms. The theory shows that closure swarms are a different order. Where people choose partners, colleagues or a village to live in, the question is which order they seek. maze.swarm can hold both meters for people too: how alike a group is, and how well it closes.

12. Status

Status	Items
Proved	the four marks met by N1/N2; closure at zero; n = 3 as the only symmetric modulus; the update rule; Theorem 1 (maximum at 1/3); Theorem 2 (synchrony is chance); Theorem 3 (self-closing register, conserved mod 3); levels; the Laplacian, averaging, gauge and Gauss identities; boundary term in 1D
Built and measured	closure and copy simulations; exact Markov chain on 2187 states; control rule; 20,000-update check; R_Q 0.704 in the gate; R_Q ≡ 1 on the flat mesh
Corresponded	members as crossings; maze.swarm as the tension hemisphere; teams as closure swarms
Tested	fish schools: alignment, not closure
Open	the closure column scored under the frozen protocol; asynchronous relaxation in the plasticity rule; the 90-day plasticity test; a two-dimensional knot test of closure under noise; closure in systems with complementary roles

Vacuum.Net is a swarm of crossings. Its order is closure. The MAZE carries that swarm in its tension hemisphere. The next MAZE gives it its second meter.

Annotated references

Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I. & Shochet, O. (1995). "Novel type of phase transition in a system of self-driven particles." ***Physical Review Letters*** **75, 1226–1229.** *Why read?* The founding swarm model and the alignment order against which closure is defined. *Reading advice:* Four pages; read whole.

Ballerini, M. et al. (2008). "Interaction ruling animal collective behavior depends on topological rather than metric distance." ***PNAS*** **105, 1232–1237.** *Why read?* Starlings use six or seven neighbours regardless of distance: the swarm as a net of relations. *Reading advice:* The results section and figures.

Cavagna, A. et al. (2010). "Scale-free correlations in starling flocks." ***PNAS*** **107, 11865–11870.** *Why read?* Correlations spanning the whole flock; a candidate dataset for a closure meter. *Reading advice:* Focus on how fluctuations are defined.

Kuramoto, Y. (1984). ***Chemical Oscillations, Waves, and Turbulence.*** **Springer.** *Why read?* The phase swarm and its order parameter r, which R_Q lifts to the mesh. *Reading advice:* Chapter 5.

Fox, R. H. (1970). "Metacyclic invariants of knots and links." ***Canadian Journal of Mathematics*** **22, 193–201.** *Why read?* The crossing condition $a + c \equiv 2b$ that becomes the closure rule at $n = 3$. *Reading advice:* The definitions suffice.

Przytycki, J. H. (1998). "3-colorings and other elementary invariants of knots." ***Banach Center Publications*** **42, 275–295.** *Why read?* Why tricolourings are a knot invariant; the basis for the two-dimensional test. *Reading advice:* Sections 1–2.

Martin, O., Odlyzko, A. M. & Wolfram, S. (1984). "Algebraic properties of cellular automata." ***Communications in Mathematical Physics*** **93, 219–258.** *Why read?* Rules linear mod a prime: the class of the closure rule. *Reading advice:* The polynomial representation.

Fatès, N. (2014). "A guided tour of asynchronous cellular automata." ***Journal of Cellular Automata*** **9, 387–416.** *Why read?* Asynchronous updating with a rate p, the setting of Theorems 1 and 2. *Reading advice:* The α-asynchronous sections.

Jhawar, J. et al. (2020). "Noise-induced schooling of fish." ***Nature Physics*** **16.** *Why read?* The open fish data of section 9 and the pairwise-copying finding. *Reading advice:* Compare with the negative closure test.

Markley, F. L. et al. (2007). "Averaging Quaternions." *Journal of Guidance, Control, and Dynamics* 30(4), 1193–1196. *Why read?* The sign-invariant form of R_Q. *Reading advice:* The accumulator-matrix derivation.

Konstapel, J. (2026). *The Vacuum.Net Theory: Foundational Paper*, sixth edition. *Why read?* Axioms N1–N5, T1 and T7: the ground of sections 3 and 5. *Reading advice:* Parts I–III.

Konstapel, J. (2026). *Clerk Maxwell and the Vacuum.Net Theory*, sixth edition. *Why read?* The two hemispheres, R_Q, the phase of a piece and the plasticity rule of section 11. *Reading advice:* Parts III–V.

Konstapel, J. (2026). *Closure Without Alignment* and *Progress Report: The MAZE Project*. *Why read?* The full proofs behind sections 4–8, and the place of this step in the MAZE sequence. *Reading advice:* Read the report first for orientation.