

The Height of the Tower

Deriving Scale Separation, Tower Height, and the Human Level from the Vacuum.Net Base Model

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What this paper does

The Human as Throughflow Closure derived the human being from the Vacuum.Net base model, but left three things unearned. Scale separation was a chosen admissibility condition. The number of tower levels was open. The coupling of two humans into one mesh was open.

This paper closes all three, using one quantity the earlier paper introduced and then left idle: the internalised conductance η_{int} .

The price is one new explicit choice, the drift scaling. It is named, isolated, and it is the place where this construction can die.

The paper is self-contained. Every step and every number is in the text.

1. What is carried over

Only the minimum is repeated. The vacuum is one strand. Where the strand touches itself there is a contact. A winding is a cycle of segments and contacts that keeps coming back. The net is the graph of all of this; the meshes are its cycles. One scalar field χ , the tension, lives on the strand, and its total is conserved. The only dynamics is redistribution of tension toward equalisation, along the strand and across contacts.

Four results of *The Human as Throughflow Closure* are used, and nothing else.

(R1) Retention time. A winding W with capacity C_W and boundary conductances η_b has one characteristic time

$$\tau_W = C_W / \sum_b \eta_b.$$

Capacity over boundary conductance. This holds for windings that have an inside — a large internal spectral gap.

(R2) The closure lemma. Closing constituents W_i into one winding S internalises part of their boundary conductance. With $G \equiv \sum_i \sum_b \eta_{b,i} = \sum_i C_i / \tau_i$, and η_{int} the internalised part,

$$\tau_S = \sum_i C_i / (G - \eta_{\text{int}}).$$

(R3) The maintained pattern. A closure re-lays its own contacts while current runs through it. Contacts decay at rate λ . Re-lay runs at rate μJ during load, drift out of the identity pattern at rate vJ , repair at rate r during equalisation phases. Stationary:

$$f = vJ / (vJ + r), C_W^*(J) = \mu r J / [\lambda (vJ + r)].$$

(R4) The window. Closure requires $C_{W^*} \geq C_{\min}(N)$, the smallest sub-realisation that still closes every defining cycle of the identity pattern. The constitutional condition is

$$\mu r > \lambda \vee C_{\min}(N).$$

2. Scale separation is a count of boundary contacts

Let q constituents be closed into one winding. Each constituent carries b boundary contacts of comparable conductance η_b . Then $\tau_i = C_i / (b \eta_b)$, and $G = q b \eta_b$.

A closure lays c new contacts running over the W_i as wholes. Each new contact consumes one boundary contact on each of two constituents and turns both into interior. So $\eta_{\text{int}} = 2c \eta_b$.

Define the closure density $\zeta \equiv c/q$, new contacts per constituent. The minimum that closes a cycle over q constituents is a ring, $c = q$, so $\zeta \geq 1$. The maximum is $c = qb/2$, where no boundary is left.

Define the internalisation fraction

$$\phi \equiv \eta_{\text{int}} / G = 2\zeta / b.$$

Substituting into R2, with identical constituents so that $\sum_i C_i / G = \tau_i$:

$$\tau_S = \tau_i / (1 - \phi), \text{ hence } \sigma \equiv \tau_S / \tau_i = b / (b - 2\zeta).$$

Three readings follow.

$\phi = 1$ means fully sealed: no boundary, no exchange, no throughflow. Retention becomes infinite and the closure is dead. So $b > 2\zeta$ is not a technical restriction. It is the statement that a living closure keeps part of its boundary open.

$\sigma > 1$ holds for every $\phi > 0$. Scale separation is not a condition to be imposed. It follows from closing anything at all.

σ is a ratio of two whole numbers. Minimal ring closures:

b	3	4	5	6	7	8
σ at $\zeta = 1$	3	2	5/3	3/2	7/5	4/3
σ at $\zeta = 2$	—	—	5	3	7/3	2
σ at $\zeta = 3$	—	—	—	—	7	4

3. Drift, and what falls out of the constitutional condition

Choice A (drift scaling). A re-lay is pattern-preserving when the replacement contact lands in the same chord interval, bounded by the neighbouring contacts of the pattern. A pattern with more contacts has shorter intervals, so the hit probability falls with pattern size. Write the hit probability as $w = C_{\min}^{(-a)}$:

$$v = \mu (1 - C_{\min}^{-a}), a = 1 \text{ or } 2.$$

$a = 1$ corresponds to one effective degree of freedom in the landing, $a = 2$ to two independent endpoints.

Substitute into R4:

$$\mu r > \lambda \mu (1 - C_{\min}^{-a}) C_{\min}.$$

μ cancels. Define the repair-to-decay ratio $R \equiv r/\lambda$. Then

$$R > C_{\min} - C_{\min}^{(1-a)}.$$

For $a = 1$: $C_{\min} < 1 + R$. For $a = 2$: $C_{\min} < R + 1/C_{\min}$. To leading order, in both cases,

$$C_{\min, \max} = R.$$

The largest identity pattern a closure can hold equals its repair-to-decay ratio. The re-lay coefficient μ , which looked like a material parameter of life, does not appear in the existence condition at all. This is a consequence of Choice A, not of R1–R4 alone.

The exponent a is visible only at the smallest closures. At $C_{\min} = 3$ the two versions require $R > 2$ and $R > 2.67$. So the smallest closure needs repair running roughly three times faster than decay.

4. The rest fraction is not free

From R3, write $x \equiv vJ/r$, so $f = x/(1+x)$. Then

$$C_W^* = \mu J / [\lambda (1 + x)] = (\mu/v) \cdot (vJ/r) \cdot (r/\lambda) / (1+x) = (\mu/v) \cdot R \cdot f.$$

With $\mu/v = 1/(1 - C_{\min}^{-a})$, exactly:

$$C_W = R f / (1 - C_{\min}^{-a}).^*$$

For patterns of any size the correction factor is close to one, giving $C_W^* = R \cdot f$. Closure requires $C_W^* \geq C_{\min}$, hence

$$f \geq C_{\min} / R.$$

Rest is not recovery from exertion and it is not a schedule. It is the fraction of the pattern that repair must reach per cycle. A larger pattern on the same repair capacity must rest more.

One distinction is needed downstream. The *required* f carries no load term. The *achieved* f does: $f = vJ/(vJ + r)$ rises with throughflow, because the duty cycle sets itself. A closure prevented from reaching $J \approx 0$ has an achieved f below its requirement whatever the size of the load.

5. The growth law, derived

Each level closes q constituents with ζq new contacts. The identity pattern of the new level contains the q constituent patterns, plus the contacts that tie them:

$$C_{\min,k+1} = q \cdot C_{\min,k} + \zeta q.$$

This is the same counting that produced ϕ in Section 2. It is not a separate hypothesis.

Solve it. The fixed point is $C^* = -\zeta q/(q-1)$. Write $B \equiv \zeta q/(q-1)$ and $A \equiv C_{\min,0} + B$. With the smallest stable winding $C_{\min,0} = 3$ — the trefoil, three crossings:

$$C_{\min,k} = A q^k - B, A = 3 + B, B = \zeta q/(q-1).$$

For $\zeta = 1$:

q	A	B	C_min,0..4
2	5,000	2,000	3, 8, 18, 38, 78
3	4,500	1,500	3, 12, 39, 120, 363
10	4,111	1,111	3, 40, 410, 4110, 41110

Growth is exponential with base q . Only the prefactor shifts, from 3 to A . In levels the shift is $\log(A/3)/\log q$: 0,74 at $q = 2$; 0,37 at $q = 3$; 0,22 at $q = 5$; 0,14 at $q = 10$.

6. The height of the tower

The tower stops at the last level whose pattern still fits under the bound of Section 4:

$$A q^m - B \leq R f.$$

$$m = \lfloor \log((R f + B) / A) / \log q \rfloor.$$

With $f = 1/3$ and $\zeta = 1$:

R = r/ λ	q = 2	q = 3	q = 5	q = 10
10 ²	2	1	1	0
10 ³	6	3	2	1
10 ⁴	9	6	4	2
10 ⁶	16	10	7	4
10 ⁹	25	16	11	7
10 ¹²	35	22	15	10

Raising ζ lowers each entry by at most one.

The tower is finite in every case, and short. A repair-to-decay ratio of a million supports four to sixteen levels, depending on how many constituents each closure gathers.

Two properties of the shape. The height depends on R only through its logarithm, so it is insensitive to the exact value: a factor of a thousand in repair capacity buys three to ten extra levels. And the

height falls as q rises. A closure that gathers many constituents at once climbs faster and stops sooner.

7. The consistency check

Two series were built from different countings. Pattern size grows as $C_{\min,k} = A q^k - B$. Retention time grows as $\tau_k = \tau_0 \sigma^k$. Eliminate k . Because $q^k = (C_{\min,k} + B)/A$, the relation is exact in the shifted variable:

$$\tau = \tau_0 \cdot [(C_{\min} + B) / A]^{\log \sigma / \log q}.$$

The exponent:

b, ζ, q	σ	exponent
3, 1, 3	3	1,000
4, 1, 2	2	1,000
4, 1, 4	2	0,500
3, 1, 10	3	0,477
5, 1, 5	5/3	0,317

One case gives exactly one. When $b = q = 3$ and $\zeta = 1$, retention time is proportional to pattern size plus a constant of 1,5. That is the case in which three is the only number in the model: three crossings in the smallest winding, three constituents per closure, three boundary contacts per constituent, one ring to close them. There

$$\tau_m / \tau_0 = (C_{\min,m} + 1,5) / 4,5.$$

With $m = 10$ this gives $\sigma^m \approx 5,9 \times 10^4$ and $C_{\min} \approx 2,66 \times 10^5$.

8. The human level, as an interval

The human is the last level that still clears its own bound. That statement is conditional, and the conditions are named here rather than in the ledger alone. It holds given q , given the growth law of Section 5, given Choice A, and given the two inputs below.

Input 1. The rest fraction f is empirical. This paper uses $f = 1/3$.

Input 2. The identification of the equalisation phase with sleep. An equalisation phase is defined by $J \approx 0$. Whether sleep is that phase is a separate claim, and it is what allows f to be read off a daily cycle at all. If rest is distributed differently — many short phases rather than one long one — f changes and every number below scales with it.

From $C_{\min,m} \leq R f < C_{\min,m+1}$, with $f = 1/3$:

$$3 C_{\min,m} \leq R < 3 C_{\min,m+1}.$$

R is bracketed within a factor of roughly q, not fixed. The table gives the lower bound:

q	m	C_min, human	R ≥	R <	τ_m/τ_0 at σ = 3
3	10	2,66 × 10 ⁵	7,97 × 10 ⁵	2,39 × 10 ⁶	5,9 × 10 ⁴
10	5	4,11 × 10 ⁵	1,23 × 10 ⁶	1,23 × 10 ⁷	2,4 × 10 ²
3	16	1,94 × 10 ⁸	5,81 × 10 ⁸	1,74 × 10 ⁹	4,3 × 10 ⁷

What the construction fixes: pattern size and retention time are locked by Section 7, and rest fraction and pattern size by Section 4. What remains free is the count — b, ζ, q, a — and R within its bracket.

Those are countable quantities, not fitted constants. Three model choices have been replaced by four things one could go and count, plus two named inputs.

9. Two closures

A second closure at the same depth is not a second object. Two closures can be constituents of one closure, by the same operator. Three things follow.

The internalisation fraction is a state, not a constant. For a pair the counting of Section 2 is not fixed by construction, because the number of shared contacts varies in time. Write it directly:

$$\phi_{ij}(t) = \eta_{int,ij}(t) / (G_i + G_j), \tau_{ij}(t) = (C_i + C_j) / (G_i + G_j - \eta_{int,ij}(t)).$$

Interaction is the time course of $\eta_{int,ij}$. Part of what was boundary for each becomes interior for the pair. The measurable quantity is not one number but a curve.

The pair has its own window. Nothing guarantees that two closures form one. The condition of Section 4 applies unchanged at the higher level:

$$R_{ij} f_{ij} \geq C_{min,ij}.$$

By the growth law with $q = 2$, $C_{min,ij} = 2 C_{min} + 2\zeta$. So the pair must maintain twice a person's pattern plus its tie contacts. Since a person satisfies $C_{min} = R f$ with $f = 1/3$, and $f_{ij} \leq 1$:

$$R_{ij} \geq 2 C_{min} + 2\zeta \approx (2/3) R.$$

A pair closes only if its joint repair-to-decay ratio reaches about two thirds of an individual's. Most pairings do not meet that. This predicts that closure between people is the exception, not the rule, and that where it holds it is costly — f_{ij} must be large.

Ordering. With ϕ_{pair} small, τ_{pair} exceeds the members' retention time while q_{pair} stays below theirs. The pair is more slowly held and more weakly held at once.

What is not supplied here: which concrete interaction corresponds to which value of $\eta_{int,ij}$. The model gives the quantity and the condition. The mapping from attention, speech, joint action or entrainment onto $\eta_{int,ij}$ is measurement work, not derivation.

10. What this does to the tests

T4 — tempo ratios. Adjacent levels stand in the ratio $\sigma = b/(b - 2\zeta)$, with b and ζ whole numbers and $2\zeta < b$. No rational value is forbidden: a measured 2,7 is reachable, requiring $b = 54$ and $\zeta = 17$. The content of the test is that a measured σ *determines* b and ζ , and those must agree with an independent count of open boundary channels. A σ implying $b = 54$ in a closure whose open channels can be counted at 7 is the falsification.

T5 — coupling as a curve. Measure $\phi_{ij}(t) = 1 - (C_i + C_j)/[(G_i + G_j) \tau_{ij}(t)]$ through an interaction, rather than one value at its end. The predicted signature is a rise during interaction and a decay after it, with the closure condition of Section 9 separating pairs that hold from pairs that do not.

T6 — tower height. The number of separable tempo levels in one individual is finite and equals m of Section 6. Counting levels, and independently measuring R from repair and decay rates, must give the same m within the bracket of Section 8. Levels found beyond that count falsify the constitutional bound of Section 3.

T7 — rest scales with pattern. From Section 4, $f \geq C_{\min}/R$. Within one individual, deeper closures require larger rest fractions in proportion to pattern size. Measured rest fractions independent of pattern size falsify Choice A.

T8 — growth law. From Section 5, pattern size at adjacent levels satisfies $C_{\min,k+1} = q C_{\min,k} + \zeta q$, with the same q and ζ that set σ in T4. So T4 and T8 must return consistent whole numbers. Two levels whose tempo ratio implies one q and whose pattern ratio implies another falsify the counting of Sections 2 and 5 together.

T10 — distribution of load. From Section 4: at equal integrated throughflow, outcomes differ according to how often and how long J reaches zero, because repair runs only there. Falsified if only the integral predicts.

Failure map: T3, T7, T10 $\rightarrow$ Choice A. T4, T8 $\rightarrow$ the contact counting. T5 $\rightarrow$ the application of the window above the individual.

11. Status ledger

Base. Definitions and postulate of the Vacuum.Net base model; R1–R4 from *The Human as Throughflow Closure*.

Derived. $\phi = 2\zeta/b$ and $\sigma = b/(b - 2\zeta)$. The cancellation of the re-lay coefficient and $C_{\min,\max} = R$, given Choice A. $C_W^* = Rf$ and $f \geq C_{\min}/R$. The growth law and its closed form. The tower height and its interval. The scaling law, exact in the shifted variable. The pair window.

Chosen. Choice A, the drift scaling $v = \mu(1 - C_{\min}^{-a})$. Comparable boundary conductance across constituents at one level. The same q and ζ at every level.

Empirical input. $f = 1/3$, and the identification of the equalisation phase with sleep, as stated in Section 8.

Removed from the earlier construction. Scale separation as a chosen admissibility condition. It follows from $\phi > 0$.

Open. The values of b , ζ and q for an actual closure. The exponent a , decidable only at the smallest closures. Whether the trefoil count $C_{\min,0} = 3$ propagates exactly. The form of N for an actual human. The mapping from concrete interaction onto $\eta_{\text{int},ij}$. And an operationalisation of T1–T10 precise enough that two independent researchers reach the same falsification criteria.

Annotated References

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