The Step Map

How the numbering of the ternary net acquires a dynamics, and how a collaboration architecture written in 2003 already contained its shape

J. Konstapel, Leiden, 18 August 2026 · second version

This paper is self-contained. It assumes no acquaintance with the Vacuum.Net programme, with balanced ternary arithmetic, or with the software architectures discussed in the second half. Every step and every number needed to follow the argument is written out here.

1. What this paper does

A previous result, theorem T7, gives every configuration of the net a unique number. That result fixes a map. It says nothing about movement across the map.

This paper supplies the movement. The rule is one line of arithmetic. From it follow three results, each proved here in full:

- A net read at depth *n* remembers exactly *n* turns and nothing older. The forgetting is exact and happens at step *n*.
- Every repeating pattern has one and only one address at which it closes. That address is a rational number, and a pattern closes if and only if its address is rational.
- Reversing every turn in a pattern moves its closing address to the mirror position. Opposite charge is the same route read backwards through zero.
- Under the natural measure on unbounded routes, the closing ones form a set of measure zero. Almost nothing closes.

The second half of the paper shows that the distinction on which all of this rests — between what repeats and what does not — was implemented in working software twice, independently, before the arithmetic existed. Once by Maarten Boasson at Hollandse Signaalapparaten from 1984 onward, once in a collaboration architecture written for Cordys in 2003. Both of those architectures are described here from the ground up.

2. The net

Take a fishing net. It has no parts brought in from outside. There is one strand, and there is what the strand does with itself.

Locally the strand does exactly one thing. It crosses itself. A crossing has three states. The strand passes over, it passes under, or the two lines do not cross at all. Write those three states as +1, −1 and 0.

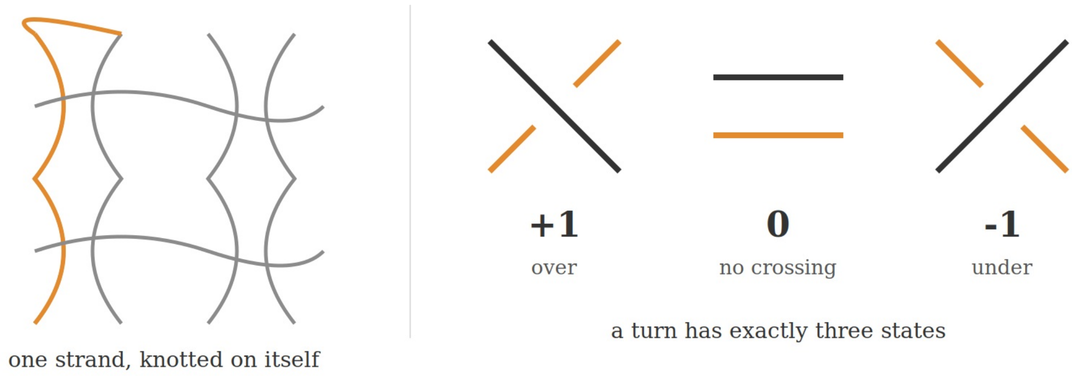


Figure 1. Left: one strand, knotted on itself. Nothing enters from outside. Right: a crossing has three states and no more. Over, under, or no crossing. This is the entire local vocabulary of the net.

The same structure has a second picture. A labyrinth is one path, folded. At every point the path turns one way, turns the other way, or runs straight on. Three states again. Net and labyrinth are two drawings of one thing. In the implementation the structure is called MAZE, which in Dutch also reads as *maas*, the smallest closed unit of a net. The doubling is deliberate.

Turn, route, depth

On the ground layer there are no things. There are turns.

A **turn** is one ternary act of the strand: +1, 0 or −1. A stretch of the maze is a sequence of turns. A sequence of n turns is a **route of depth n**.

Three states in each of n positions gives 3^n routes at depth n. This number will matter twice.

3. The number

Read the turns as digits of a number in base three, in the balanced form. The digits are −1, 0 and +1 rather than 0, 1 and 2. The turn in position i counts for its value multiplied by 3^i, with position 0 the shallowest.

Written out, deepest position first, the route (+1, −1, −1, −1, +1) reads:

$$1{\times}81 - 1{\times}27 - 1{\times}9 - 1{\times}3 + 1{\times}1 = 81 - 27 - 9 - 3 + 1 = \mathbf{43}$$

One route, one number. The theorem states that this correspondence is exact in both directions.

No two routes share a number

Suppose two different routes gave the same number. Subtract them turn by turn. Each difference lies between −2 and +2, and at least one difference is not zero.

Take the deepest position k at which they differ. That position alone contributes at least 3^k to the difference.

Everything shallower contributes at most 2 in each position. Summed over positions 0 to $k-1$ that is $2 \times (3^k - 1)/2$, which is $3^k - 1$.

That is exactly one less than the deepest difference on its own. The deepest difference can never be cancelled. The two routes cannot have the same number.

The margin is one. There is no slack in this proof and no room to widen the digit set.

Every number is a route

Any whole number can be written in ordinary base three with digits 0, 1 and 2. Wherever a 2 appears, replace it, using the identity $2 \times 3^i = 3^{i+1} - 3^i$. The 2 becomes a −1 in its own position and a carry of +1 into the next. Repeat until no 2 remains. Only −1, 0 and +1 are left.

Every whole number is therefore a route. Nothing falls outside.

How many fit at each depth

At depth n there are 3^n routes. The largest is the route with +1 everywhere, which equals $(3^n - 1)/2$. The smallest is its negative. The addresses at depth n run symmetrically from $-(3^n - 1)/2$ to $+(3^n - 1)/2$, and the route of all zeros has address 0. Rest is on the map, not off it.

depth n	**routes 3^n**	**ceiling $(3^n-1)/2$**
1	3	1
2	9	4
3	27	13
4	81	40

5	243	121
6	729	364

Why three and not two

Three is not a preference. It is an optimum, and the optimum can be computed.

An alphabet of *r* signs costs, per unit of information carried, an amount proportional to *r* divided by the natural logarithm of *r*. The measure is standard in information theory and goes under the name radix economy.

base *r*	**r / ln r**
2	2.8854
3	2.7307
4	2.8854
e = 2.71828	2.71828 (minimum)

Bases 2 and 4 cost exactly the same, which is not a coincidence: 4 / ln 4 = 4 / (2 ln 2) = 2 / ln 2. The true minimum sits at *e*. Three is the only whole number adjacent to it on the cheap side, and three beats two.

The balanced form adds a second property. It closes on itself. The sign lives in the leading turn, so no minus sign is imported from outside. Negation is the mirror exchange of +1 and −1. Shifting one position is one step of scale. Truncating a number to a chosen depth gives the same answer as rounding it. There are no external conventions to supply.

4. Which series counts: forty, not forty-three

Two series occur in this programme. They agree for three terms and then part. The choice between them has to be made before any dynamics can be written, because a rule of motion must respect the size of the space it moves in.

The ternary depth ceiling is $(3^n - 1)/2$, generated by a(n) = 3a(n−1) + 1:

1, 4, 13, **40**, 121, 364, 1093, 3280, 9841, 29524, 88573

The Bronze Mean series is generated by a(n) = 3a(n−1) + a(n−2):

1, 4, 13, **43**, 142, 469, 1549, 5116, 16897, 55807, 184318

The shared prefix is an artefact

Put the two recurrences side by side. One adds 1 at every step. The other adds the term from two steps back. They give the same answer for exactly as long as the term two steps back is still 1. That is through the third term. At the fourth term the memory term has become 4, and the series separate.

There is no coincidence here to be explained, and no evidence that the two count the same thing.

The Bronze series cannot be a count of routes

At depth n there exist exactly 3^n routes. That is the whole population. Anything counted at depth n must stay below it.

depth *n*	**routes 3^n**	**Bronze term**	**ratio**
4	81	43	0.531
5	243	142	0.584
8	6,561	5,116	0.780
9	19,683	16,897	0.858
10	59,049	55,807	0.945
11	**177,147**	**184,318**	**1.041**

At depth 11 the Bronze series counts more items than there are routes to count.

The cause is the growth factor. The Bronze Mean is $(3 + \sqrt{13}) / 2 = 3.302776$. That is larger than 3. Each step gains 10.09 per cent on 3^n. The series starts far below and overtakes. The crossing between depth 10 and depth 11 is arithmetic, not approximation.

A count cannot exceed its own total. The Bronze series is therefore not a count of routes.

Where the Bronze Mean does belong

3.302776 is a ratio. It is the growth factor of a process with two-step memory, and it is a member of the metallic-mean family. It has a natural place wherever the theory speaks of proportion between successive closures, or of tiling a plane. It has no place in an answer to the question how many routes fit at depth n.

> T7 counts. The Bronze Mean scales. They are not rivals, and only one of them can appear in a rule of motion.

Everything that follows uses $(3^n - 1)/2$.

5. The step map

Adding one turn to a route does two things at once. Every turn already present is pushed one position deeper. The new turn is written in the shallowest position.

In place-value arithmetic that is multiplication by three followed by addition of the new turn.

> **The step map.** $a' = 3a + t, \quad \text{with } t \in \{-1, 0, +1\}$.

This is not a modelling decision. It is what positional notation means, written as a map. The recurrence for the depth ceiling is the same rule applied to the largest turn: $3 \times 13 + 1 = 40$.

Iterating from a starting address a_0 over turns $t_1 \dots t_p$ gives

$$a_p = 3^p \cdot a_0 + N$$

where N is the number of the turn word $t_1 \dots t_p$ read as a route of depth p. The first turn applied carries the deepest weight, 3^{p-1}.

6. Closure by depth

The step map on its own deepens without end. Closure comes from the property established in section 3: reading to a chosen depth, where truncation equals rounding.

Truncating at depth n is arithmetic modulo 3^n.

> **The closed step map.** $a' = 3a + t \pmod{3^n}$.

The address space at depth n is now finite: 3^n addresses, running from $-(3^n - 1)/2$ to $+(3^n - 1)/2$. The map sends that space into itself. Closure has become something to compute rather than something to assume.

7. Result one: depth is memory

Iterate the closed map n times from any starting address. The result is

$$a_n = 3^n \cdot a_0 + N \pmod{3^n}$$

But $3^n \cdot a_0$ is a multiple of 3^n, so it vanishes modulo 3^n. Therefore

$$a_n \equiv N \pmod{3^n}$$

The starting address is gone. After n steps the state depends on the last n turns and on nothing else.

> **Depth *n* is a memory of exactly *n* turns.** Everything older is unrecoverable at that depth.

This is not a tendency and not an approximation. It is exact, and it happens at step n, neither earlier nor later.

The mechanism is a loss of information at a fixed rate. Multiplying by three modulo 3^n can only reach multiples of three, which is one third of the addresses, and it reaches each of them from three different sources. The map is three-to-one. One turn of history is discarded per step.

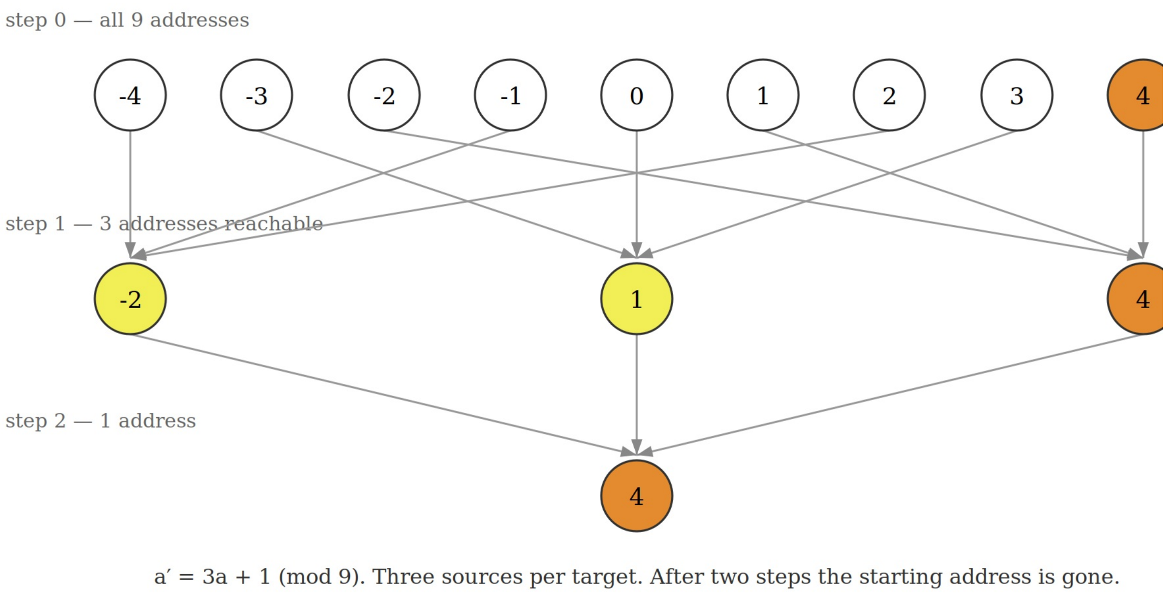


Figure 2. The closed step map at depth 2, with the turn +1 repeated. Modulus 9, addresses −4 to +4. After one step only three addresses remain reachable. After two steps every starting address has arrived at 4, which is the depth-2 ceiling. Each target has exactly three sources: the map is three-to-one, and that is why anything settles at all. A reversible map

would carry the whole past forever and nothing could close.

8. Result two: every repeating pattern closes, at exactly one address

Let a turn word of length p repeat indefinitely. Closure means the address is unchanged after p steps:

$$\mathrm{a} \equiv 3^p \cdot \mathrm{a} + \mathrm{N} \pmod{3^n}$$

$$\mathrm{a} \cdot (3^p - 1) \equiv -\mathrm{N} \pmod{3^n}$$

Now $3^p - 1$ leaves remainder 2 on division by 3. It therefore shares no factor with 3^n. It is invertible modulo 3^n, and the congruence has exactly one solution.

> **Every repeating turn word has one and only one closing address at depth *n*:**
>
> $$\mathrm{a} \equiv -\mathrm{N} \cdot (3^p - 1)^{-1} \pmod{3^n}$$
>
> No choice, no degeneracy, no missing case.

This congruence is the definition of the closing address. Everything below is a way of reading it.

The special case: period equal to depth

Take p equal to n. Then 3^n is zero modulo 3^n, so $3^p - 1$ reduces to -1, and the congruence collapses to

$$\mathrm{a} \equiv \mathrm{N} \pmod{3^n}$$

A pattern whose period equals the depth closes on its own number.

The general case: write out whole periods

For a shorter period the pattern must be written out to a length that is *a whole multiple of the period* and at least n, and only then truncated.

Let the word be repeated m times, with mp at least n. The number of the extended word is

$$N_{mp} = N \cdot (3^{mp} - 1) / (3^p - 1)$$

because the repetition is a geometric sum. Modulo 3^n the term 3^{mp} vanishes, so $3^{mp} - 1$ reduces to -1, and

$$N_{mp} \equiv -N \cdot (3^p - 1)^{-1} \equiv a \pmod{3^n}$$

> **Write the repeating pattern out to a whole number of periods, at least as deep as you are reading, and truncate. That number is the address at which it closes.**

The condition on whole periods is not decoration. Cutting the repetition mid-period gives the wrong answer, and can give the wrong sign.

A worked warning

Take the pattern (+1, −1), so $p = 2$ and N = 3 − 1 = 2. Read at depth 3, modulus 27.

By the congruence: $a \cdot (9 - 1) \equiv -2$, so $8a \equiv -2 \pmod{27}$. The inverse of 8 modulo 27 is 17, since 8 × 17 = 136 = 5 × 27 + 1. Therefore $a \equiv 17 \times (-2) = -34 \equiv -7 \pmod{27}$. The closing address is **−7**.

By whole periods: repeat twice to depth 4, giving (+1, −1, +1, −1) with number 27 − 9 + 3 − 1 = 20. And 20 − 27 = −7. Same answer.

Cutting at exactly depth 3 would give (+1, −1, +1) with number 9 − 3 + 1 = 7. That is not the closing address, and it carries the opposite sign. Period 2 does not divide depth 3, and the shortcut of reading exactly n turns is valid only when the period divides the depth.

Closing addresses are rational

Nothing in the congruence depends on the depth except the modulus. Written without a modulus, the solution is

$$a = N / (1 - 3^p)$$

The denominator $3^p - 1$ is never divisible by 3, so this fraction is a well-defined three-adic integer: it can be read at any depth by reducing modulo 3^n. Conversely, every rational number whose denominator is not divisible by 3 has a repeating expansion in balanced ternary, and is therefore the closing address of exactly one repeating pattern.

A pattern closes if and only if its address is rational.

The addresses computed earlier are the truncations of small fractions.

pattern	closing address	depth 2	depth 3	depth 5
(+1)	−1/2	4	13	121
(−1)	+1/2	−4	−13	−121
(0)	0	0	0	0
(+1, −1)	−1/4	2	−7	−61
(+1, +1, −1)	−11/26	2	11	65

The depth ceiling of section 3 now has a name. The all-plus pattern closes at −1/2, and $(3^n - 1)/2$ is simply that fraction read at depth n.

One corollary

At depth n there are 3^n words of length n and 3^n addresses, and by the special case above the correspondence between them is the identity. Every address at depth n is the closure of exactly one pattern of length n. Everything has its number, and at any fixed depth every number has its cycle.

That last clause carries a restriction which section 11 takes up: at a fixed depth, closure is universal. The distinction between what closes and what does not exists only in the unbounded reading.

9. Result three: opposite charge is the mirror address

Negate every turn in a word. Each place keeps its weight and each digit flips sign, so N becomes −N.

Substituting −N into the congruence of section 8 gives −a. The closing address flips with the word.

The mirror word closes at the mirror address.

The minus sign of physics is not a label attached from outside. It is the same route read in the mirror. In T7 this followed from the notation. Here it follows from the dynamics as well, which is a stronger statement: it is preserved by motion, not merely by naming.

10. Worked examples

Depth 2, modulus 9, addresses −4 to +4

Pattern (+1), written out to depth 2 as (+1, +1). N = 3 + 1 = 4. Closing address **4**. Check directly: 3 × 4 + 1 = 13, and 13 − 9 = 4. Fixed. This is the depth-2 ceiling, and it is the address every start reaches in Figure 2.

Pattern (−1), written out as (−1, −1). N = −4. Closing address **−4**. The mirror of the previous line.

Pattern (0, 0). N = 0. Closing address **0**. Rest has an address like everything else.

Pattern (+1, −1). N = 3 − 1 = 2. Closing address **2**. This one is not a fixed point but a cycle of two. From 2, applying +1: 3 × 2 + 1 = 7, and 7 − 9 = −2. From −2, applying −1: 3 × (−2) − 1 = −7, and −7 + 9 = 2. The orbit is 2 → −2 → 2. A winding that alternates rather than rests, symmetric about zero.

Depth 3, modulus 27, addresses −13 to +13

Pattern (+1, +1, −1). N = 9 + 3 − 1 = 11. Closing address **11**.

The orbit, step by step. From 11 with +1: 3 × 11 + 1 = 34, and 34 − 27 = 7. From 7 with +1: 3 × 7 + 1 = 22, and 22 − 27 = −5. From −5 with −1: 3 × (−5) − 1 = −16, and −16 + 27 = 11. Closed in three steps, as the period requires.

Pattern (+1), written out as (+1, +1, +1). N = 9 + 3 + 1 = 13, the depth-3 ceiling, as expected.

11. Why anything persists

The results so far are arithmetic. This section draws a distinction from them, and the last step of the section is an interpretation rather than a theorem. The two are marked apart.

The arithmetic part

Read without a depth limit, a turn word is either repeating or it is not.

A **repeating** word has exactly one closing address, and that address is rational. Started anywhere, the state arrives at its orbit within n steps at depth n, and stays there. It maintains itself and needs nothing from outside.

A **non-repeating** word has no closing address. Its three-adic value is irrational. The state never settles.

How many are there of each? The repeating words are exactly the rationals with denominator prime to 3. That set is countable. The unbounded routes carry a natural measure, in which each turn is drawn independently and uniformly from the three states; this is the Haar measure on the three-adic integers. A countable set has measure zero under it.

> **Almost no route closes.** Closure is exceptional in the strict sense: the closing routes have measure zero among all routes.

The restriction that matters

This statement belongs to the unbounded reading and not to any fixed depth. At depth n the situation is the reverse. Every word of length n has period n, which divides n, so every one of the 3^n addresses closes.

Closure is therefore not a property a route has at a depth. It is a property of the route across depths: whether the same pattern still closes when the reading is taken deeper. A route that closes at depth n and fails at depth $n+1$ was never a thing; it was a coincidence of the resolution.

That is a sharper criterion than the one this programme has used so far, and it is checkable: a pattern is real to the extent that its closing address is stable under deepening. The rational addresses are exactly the stable ones.

The interpretation

The programme reads a self-maintaining closed orbit as what ordinary language calls a thing. That reading is not proved by anything above. What is proved is that a repeating input has a unique periodic state, that its address is rational, and that such states are exceptional.

The step from there to *thing* is an interpretation, and it should stay visible as one. It is made here because it does work elsewhere in the programme, not because the arithmetic forces it.

What the arithmetic does say, without interpretation, is that nothing performs the selection. There is no selector and no cause. What persists is what the map leaves standing.

The rest of this paper concerns that distinction, because it was built in software twice before it was proved.

12. The collaboration architecture of 2003

The following architecture was written for Cordys and published under the title *Moving Up and Moving Down*. Its subject was the connection between small specialised human networks and large-scale collaborative technology. It is described here in enough detail to be read cold.

One figure at three depths

Its opening move is a single diagram, drawn three times with different labels.

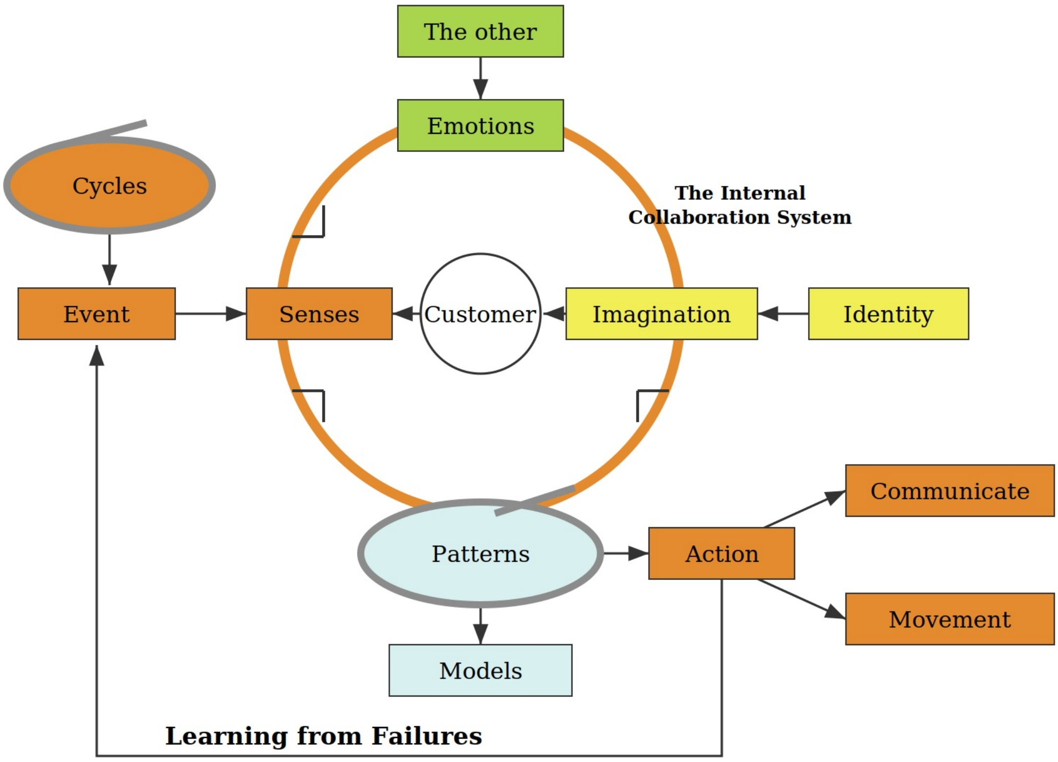


Figure 3. The human cognitive system, redrawn from the 2003 original. An event enters through the senses. The imagination supplies the interior image. The emotions set priority. Action goes out. The two ovals hold what recurs: cycles on the way in, patterns on the way out. The lower line closes the figure by returning failure to the entry point.

Four components carry the figure: senses, emotions, patterns, imagination. Around them the same shape is redrawn twice more. Once for a specialised human network, where the ring positions become roles. Once for the tool layer, where they become sensor, simulator, comparator, mover, communicator, advisor and master.

The roles are derived, not listed. Taking the four components two at a time gives six pairs and therefore six roles: craftsman from senses and patterns, entrepreneur from senses and emotions, politician from emotions and patterns, creator from imagination and patterns, motivator from emotions and imagination, inventor from senses and imagination. Two roles can work together when they share a component. That is a closed derivation from a single principle, and it is the same style of derivation as sections 3 and 8 above.

Three points are worth marking, because they recur in the arithmetic.

First, there is no causal arrow in the figure. The senses do not cause the imagination. They are positions on one ring around one centre. The arrows are flows and the ring closes.

Second, closure is the criterion for survival. The network diagram carries two poles at the top, sustaining the network and killing it, with trust against the fear of loss. What returns persists; what does not return falls away.

Third, the same figure at another depth is the whole method. The title says it: moving down to the scale of the network, moving up by connecting networks.

The machine

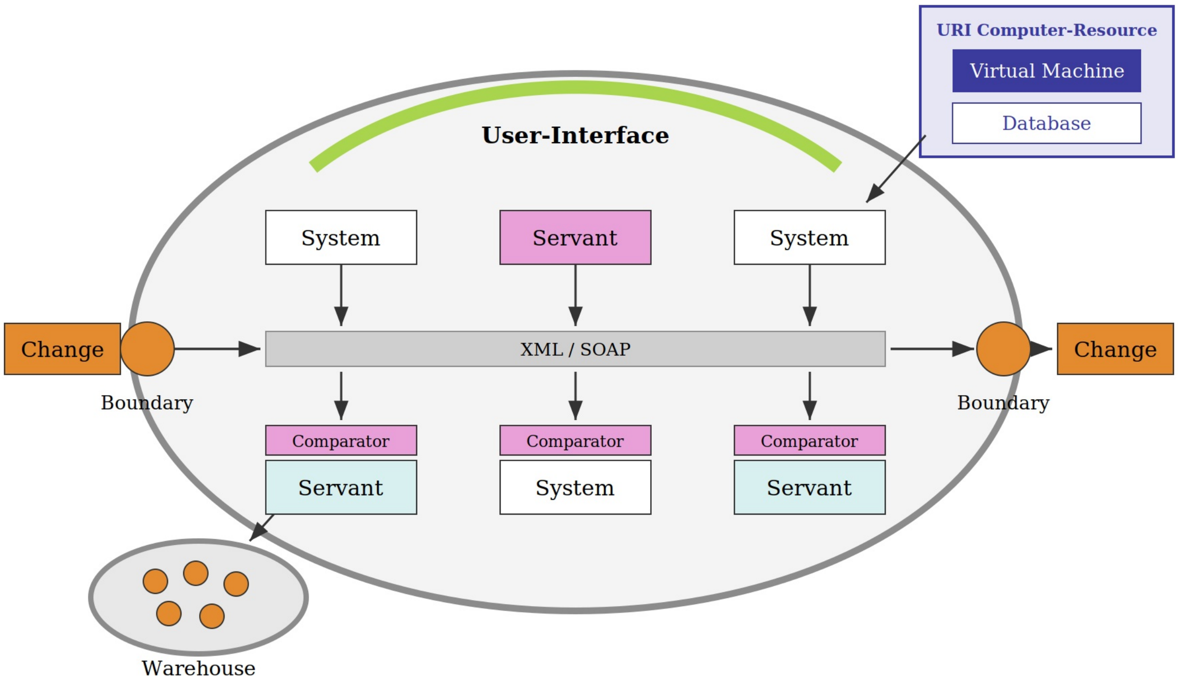


Figure 4. The collaboration system, redrawn from the 2003 original. A system is anything with a boundary. Change enters through the boundary and is transformed into an internal format. Servants take the repeating work. Comparators take the exceptions. The warehouse stores reusable sensors and comparators. The appliance is a computing resource identified by an address.

The definitions are terse and worth having exactly.

A **system** is anything with a boundary. It contains sub-systems and servants. The **boundary** protects and sustains the identity of the system. It filters what comes in, converts it to an internal format, and converts what goes out into a format the world outside can handle.

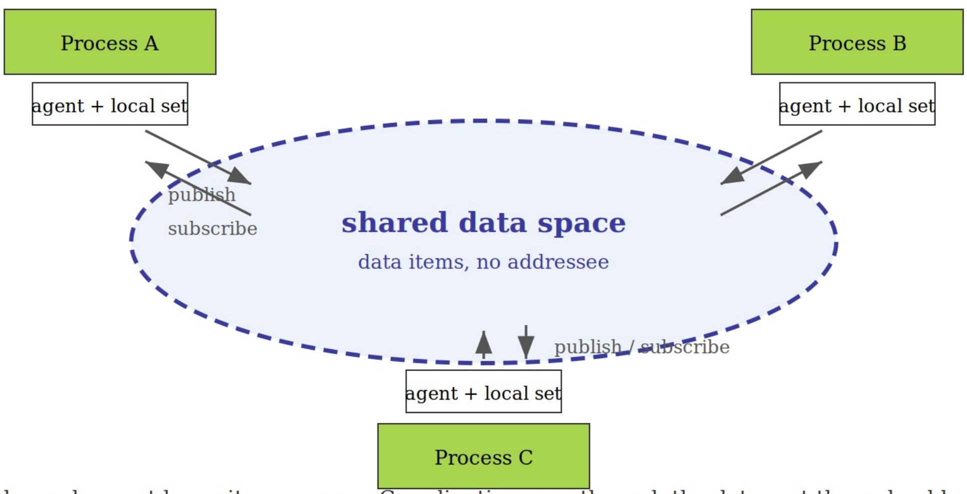
A **servant** takes over repetitive patterns. Three requirements are placed on it. It must be invisible, so that it does not interrupt. It must be undividable, so that it cannot be broken into parts. It must be replaceable without destroying the system that contains it. The example given is a central heating system: a person sets a temperature and everything else is hidden.

A **comparator** acts only on the exception. The design rule attached to it is a proportion. A system needs just enough comparators to articulate its expected behaviour. With too few it is rigid, and a legacy system is the example. With too many it is over-articulated, and has grown past its own function. The working figure given is the familiar eighty-twenty split: a fifth of the cases take four fifths of the code.

Late binding is the implementation. Nothing is fixed in advance. A node resolves an address, connects to what it finds, and only then loads the interface that tells it what it can do. Identity is determined at the moment of connection, by what fits.

13. Splice, 1984

The second implementation is older and more radical. Splice was developed by Maarten Boasson and colleagues at Hollandse Signaalapparaten, later Thales Nederland, from the mid-nineteen-eighties. The name stands for a subscription paradigm for the logical interconnection of concurrent engines. It was built for naval command and control, and the same architecture was later applied to process control and air traffic management.



A producer does not know its consumer. Coordination runs through the data, not through addressing.

Figure 5. The coordination model of Splice. Processes publish data items into a logically

shared data space and subscribe to the kinds of item they need. Each process has an agent holding a local replica; the shared space is distributed rather than central. A producer does not know who consumes what it writes.

The decisive property is what Splice removes. In an ordinary distributed system a sender addresses a receiver. In Splice there is no addressee. A process writes a data item into the shared space and subscribes to the kinds of item it needs. Coordination runs through the data itself. Who produced an item, and whether anyone will consume it, are not part of the transaction.

Boasson and de Jong gave the model a formal semantics as a transition system, in two stages: first the shared data space with its basic operations, then the refinement in which the data is distributed and replicated over a network by a publish-and-subscribe protocol. The line of descent from Splice runs to the Data Distribution Service standard and to the publish-subscribe middleware now used in robotics.

For the present purpose one point matters. A system with no addressee cannot be described by arrows between senders and receivers. What determines whether a data item has any effect is whether it matches a subscription. That is selection by fit, and it is the same relation as closure in section 8.

14. The correspondence

The two architectures and the arithmetic are the same structure at three depths.

collaboration architecture, 2003	**Splice, 1984**	**the maze, 2026**
event	data item published	turn
servant: takes the repeating work	a subscription that matches	repeating word, one closing address
comparator: takes the exception	an item nobody subscribes to	non-repeating word, no closure
boundary of a system	the agent's local set	the mesh, read at depth *n*
late binding: look up, connect, then load	no addressee; selection by fit	the address is the pattern's own number, computed at closure
the system does not keep its own history	local replicas, no global log	depth *n* holds exactly *n* turns

One qualification belongs with that table. It is a correspondence between three levels of description, not a mathematical equivalence. No claim is made that the 1984 or 2003 architecture is a model of the arithmetic in the technical sense, and none of the results in sections 7 to 9 depends on the comparison. What the table records is that the same distinction was drawn three times, twice in working systems and once in a proof, and that the three drawings line up. That is a historical and structural observation. It is offered as one.

Two lines of that table deserve to be read slowly.

Late binding and the closing address. In the 2003 architecture a node does not know what it is until it connects. In section 8 a pattern does not have an address until it closes, and the address it acquires is its own number. These are the same statement. Identity is not carried in advance; it is what the connection resolves to.

Forgetting. The 2003 text observes that a system with too many comparators has grown past its function, and that layers of unremoved history are the central disease of large software. Section 7 says why forgetting is not a defect. The map is three-to-one. Losing one turn of history per step is precisely what allows anything to settle. A system that kept everything could never close.

What each side supplies the other

The 2003 architecture had the dynamics and no arithmetic. It described how state moves through a net with no centre, and nothing in it could be computed. The 2026 theorem had the arithmetic and no dynamics. It numbered every state and said nothing about movement.

The step map joins them. What the architecture called a servant is a repeating word with a unique closing address, and it can now be calculated rather than designed. What it called a comparator is a word that does not close.

15. What remains open

The proportion of closure. Section 11 shows that the closing routes have measure zero among all routes, so the naive question has a degenerate answer: the fraction is nought. The real question is therefore a bounded one. Restrict to patterns of period at most k and read at depth n. What fraction of the address space is then occupied by closing orbits, as a function of k and n, and does any natural pairing of k and n reproduce a proportion near four to one? The 2003 architecture answered empirically with the eighty-twenty rule and a warning against both rigidity and over-articulation. The maze should answer by derivation. This is the sharpest open question in the paper, and the one where the older material is still ahead.

Coupled readings. The step map governs one reading of one strand. Two windings that share turns require a rule for what happens where the readings meet. Splice is the relevant precedent, because it coordinates without addressing.

Continuous depth. The map is defined at whole depths. Truncation equals rounding, so intermediate depths are readable, but the behaviour between two depths is not yet written.

Numbering histories. Gödel numbered proofs as well as formulas. Section 7 shows that a history longer than n turns cannot be recovered at depth n. Which depth a given history requires, and whether a history is codeable there as a single number, follows the same arithmetic and has not been carried out.

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16. Annotated references

Konstapel, J. (2026). *The Vacuum.Net Theory: Foundational Paper*, sixth edition. Leiden.

Axioms N1 to N5, theorem T7 with its corollaries, and the open problem O2 that the present paper answers for the single-strand case.

Why read? Sections 2 and 3 above restate T7 with its proofs, so this paper can be followed without it. Read the Foundational Paper for the axioms the theorem rests on, and for the corollary that truncation equals rounding, which is what makes closure possible in section 6.

Konstapel, J. (2026). *Everything Has Its Number in the Maze*. Leiden.

The essay version of T7, with the lineage running from Ecclesiastes through Stifel, Leibniz and Gödel.

Why read? It states the open problem in the form answered here, and it gives the historical argument that the numbering claim is far older than its proof.

Konstapel, J. (2003). *Moving Up and Moving Down: ideas and opinions about the connection between small scale specialized human networks and large scale collaborative IT-technology*. Written for Cordys, Leiden.

The architecture of sections 12 and 14. Systems, boundaries, servants, comparators, warehouses and appliances; the derivation of six network roles from four cognitive components; and the diagnosis of accumulated software layers that runs through chapter 4.

Why read? It is the same structure at another depth, written twenty-three years before the arithmetic. Chapter 8 carries the architecture. Chapters 5 and 6 give the human-scale reading of the same figure, including the derivation of the six roles.

Reading advice: chapter 8 alone if time is short. Note that the header of the published version dates the text to 22 April 1993; the content places it in 2003, since it discusses Web Services, the commercialisation of Linux, consumer broadband and the World Wide Web Consortium as an established body.

Boasson, M. and de Jong, E. (1997). "A software architecture for distributed control systems and its transition system semantics". Hollandse Signaalapparaten B.V.

Splice presented in two stages: the shared data space with its basic operations and a transition-system semantics, then the refinement in which data is distributed and replicated across a network by a publish-and-subscribe protocol.

Why read? This is the architecture of section 13 in the authors' own hands, and it is the most radical of the three: it removes the addressee entirely. If you read only one item from this list besides the Foundational Paper, read this one.

Boasson, M. (1998). "Software architecture for distributed reactive systems". ***Lecture Notes in Computer Science*****, SOFSEM.**

The design problem behind Splice. Large embedded systems face continual interaction with a changing environment under hard requirements on timing, robustness and maintainability, and coordination models are offered as the way to separate function from the rest of the design.

Why read? For the motivation rather than the mechanism. It states plainly why a centre had to be removed, which is the argument section 11 arrives at from arithmetic.

Stifel, M. (1544). ***Arithmetica Integra*****. Nuremberg.**

The first printing of balanced ternary notation.

Why read? To see that the alphabet is five centuries older than the theory built on it, and was not designed for it.

Leibniz, G. W. (1686). ***Discours de métaphysique*****, sections 5 and 6.**

Maximal richness from minimal means, and the demand of the *characteristica universalis* that every concept receive its own number so that reasoning becomes calculation.

Why read? It is the design requirement the ternary ground happens to satisfy. The difference is that Leibniz needs a minimiser standing outside the world, and the net does not: the optimum is not chosen, it is what remains.

Chaitin, G. (2005). ***Meta Math! The Quest for Omega*****; and (2006). "The Limits of Reason",** ***Scientific American*** **294(3), 74–81.**

Leibniz read as algorithmic information theory. A law is a compression, and comprehensibility is compressibility.

Why read? It supplies the cost measure used in section 3. Reading advice: the Scientific American article carries the argument on its own; the book if the incompleteness side is wanted.

Gödel, K. (1931). "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I". *Monatshefte für Mathematik und Physik* 38, 173–198.

Unique numbering of formulas and of proofs.

Why read? It is the precedent for numbering routes, and for the open question in section 15. Gödel numbered the process and not only the state, which is exactly the step still missing here.

Knuth, D. E. (1997). *The Art of Computer Programming*, volume 2: Seminumerical Algorithms, third edition, section 4.1.

The arithmetic of balanced ternary, including the property that truncation gives the same result as rounding.

Why read? Closure in section 6 rests on that property. Reading advice: section 4.1 only, and cite by section rather than by page, since the pagination differs between editions.

Hayes, B. (2001). "Third Base". *American Scientist* 89(6), 490–494.

A readable survey of ternary number systems, the radix-economy argument, and the Setun computer built in Moscow from 1958.

Why read? The shortest route into the cost argument of section 3, and a reminder that a working ternary machine existed.

Spinadel, V. W. de (1999). "The family of metallic means". *Visual Mathematics* 1(3).

The metallic means, of the form $(p + \sqrt{p^2 + 4}) / 2$. For $p = 3$ this is the Bronze Mean, $(3 + \sqrt{13}) / 2 = 3.302776$.

Why read? Section 4 removes the Bronze Mean from the counting of routes. This shows what it is instead: a proportion within a family of proportions, defined by a two-step recurrence, and therefore not a population.

Gouvêa, F. Q. (2020). *p-adic Numbers: An Introduction*, third edition. Springer.

Arithmetic modulo p^n, the p-adic metric, and multiplication by p as a contraction.

Why read? Section 7 is a three-adic statement. Depth as memory, and the three-to-one loss per step, are standard facts in that setting and are worth having in general form. Reading advice: chapters 1 and 3.

Note on this version. The first version of section 8 stated that a repeating pattern may be written out to exactly the depth being read. That shortcut is valid only when the period divides the depth. The general rule requires whole periods, as shown above, and the general congruence given in both versions was already correct. Tracing the correction produced the rationality criterion of section 8 and the measure statement of section 11, neither of which appeared in the first version. The correction is owed to a review by GPT.

A closed pattern is a turn that returns.
Write it out to a whole number of periods, at least as deep as you are reading, and its number is where it returns.