

# The Vacuum.Net Theory: Foundational Paper

Eighth Edition — Leiden, 17 October 2026

J. Konstapel, Constable Research, Leiden

© 2026 J. Konstapel, Constable Research, Leiden. All rights reserved. Quotation with attribution is permitted; reproduction of the whole requires written permission.

Version 8.1

## Revision note (Edition 8)

Axioms N1–N5 are unchanged, word for word. Edition 8 writes out the derivations that earlier editions only cited. Four lines join into one chain: the ground of distinction (Spencer-Brown, Kauffman), the ternary register, nilpotent rewriting (Dirac, Rowlands) and Maxwell's quaternions. Units follow Larson's dimensions.

New in this edition:

- Part 0, the chain in one view;
- Part II, the ground of distinction (T8);
- Part III.4–III.5, the window register and the two ladders (T9–T17);
- T18, the lever of a free path;
- Part V, nilpotent rewriting (T19);
- Part VI, Maxwell's quaternions (T20–T22);
- Part VII, dimensions and conversion units (T23);
- Appendix A, the origin of the chain.

Revised: C5, with C5a, I3, P2, O1 and the note to T5 brought in line; O3 extended; O5–O7 added. The coupling exponent  $\gamma$  is a state function, as N4 requires; the fixed value  $\gamma = 2\pi$  is withdrawn. The geometric  $2\pi$  of one turn per mesh stays in  $a_0 = cH_0/2\pi$ . T5 stays proved under its own hypothesis. Planck's constant has dimension  $T^2/S$ . The proofs carry the completions of revision 6.1.

Version 8.1 writes out the three cross terms in the proof of T19 and adds Appendix B, the record of every check. No statement changes.

---

## Part 0 — The Chain in One View

---

One act, one rule, one register. The chain runs as follows. Each arrow names the part where the step is stated, with its status.

1. **Distinction.** A mark in the void; the mark that re-enters itself oscillates, and its oscillation carries  $\sqrt{-1}$  (Part II.1; C1).

2. **One strand.** Every distinction is the strand touching itself. A self-crossing has three states: over, under, none (N1).
3. **Closure.** Only what returns onto itself persists (N2). Its formal form is the eigenform  $O(A) = A$  (Part II.2).
4. **The register.** Three states are the cheapest alphabet (Part III.1); balanced ternary gives each integer one address (T7). A window of three crossings closes when its sum vanishes mod 3: 9 of 27 close, in two kinds; a ring that closes everywhere repeats every three steps; these rings are the fixed points of the rewrite (T9–T14).
5. **Depth.** Each step deeper multiplies the states by three (N5). The ladders  $\times 3$  and  $\times 2$  never meet exactly; their best near-meetings are the closure depths 1, 2, 5, 12, 41, 53, 306, ... and the comma (T15–T17).
6. **The first knot.** Three crossings are the least that cannot be undone; the trefoil is chiral (T1, T8). The rotor returns after  $4\pi$  (T2). The Hopf charge is conserved without Noether (T3).
7. **Coupling.** The stiffness of the medium is a state function of its density (T4). For a free path the lever is  $-(1 + P)/P$  (T18).
8. **Motion.** Dirac's equation in Rowlands' nilpotent form:  $Q^2 = 0$  exactly on the mass shell  $E^2 = p^2 + m^2$  (T19). Nilpotence is closure (C3).
9. **Field.** Maxwell's quaternions carry two registers in one product; the net reads them as two readings of one closure operation (Part VI.2, correspondence; T20–T22 conditionally proved).
10. **Measure.** Space and time are the two aspects of one motion; energy  $T/S$ , mass  $T^3/S^3$ ,  $h = T^2/S$  (T23).
11. **Numbers.**  $a_0 = cH_0/2\pi = 1.08 \times 10^{-10} \text{ m s}^{-2}$  (I3); the solar winding edge at 7,400 AU (P3);  $\Delta\alpha/\alpha \approx 3 \times 10^{-8}$  at G191-B2B (P1).

Steps 1–6 concern the strand alone; step 1 rests on C1, and T2 and T3 add their stated hypotheses (unit-quaternion orientation states; field maps  $S^3 \rightarrow S^2$ ). Steps 7–10 connect it to known physics, partly by proof and partly by correspondence. Step 11 gives numbers that instruments can check.

---

## Part I — Axioms

---

The theory rests on five axioms. Everything else is derived, corresponded, identified, predicted, or declared open. The five statuses are defined in Part XIII.

N1 (Strand). There is one strand. It is oriented and continuous. Its only local act is self-crossing. A self-crossing has three states: over (+1), under (−1), or none (0).

N2 (Closure). Only configurations that close — that return in phase onto themselves — persist. What does not close, does not last. Closure is the selection rule. There is no external selector.

N3 (Memory). A closed configuration constrains the next rewrite. The net carries its history as its shape.

N4 (Context). The response of the medium is a state function, not a universal constant. Universality is the empirical limiting case  $g(\chi) = g_0$  when the state variable  $\chi$  does not vary across the sample.

N5 (Scale). The net repeats its own construction across depth. One winding step deeper multiplies the count of distinguishable states by three: each site adds one three-valued crossing. Identity over scale: the same winding, read at another depth.

---

## **Part II — The Ground of Distinction (new in this edition)**

---

The axioms rest on one act: a distinction. This part states where that act comes from and which of its consequences are proved.

### **II.1 The mark and its re-entry**

Spencer-Brown's *Laws of Form* (1969) begins with one instruction: draw a distinction. A mark splits a space into an inside and an outside. Two laws govern marks. Calling twice is calling once. Crossing twice is not crossing.

Re-entry is the mark that contains itself. It cannot settle on marked or unmarked. It alternates: marked, unmarked, marked. Kauffman reads this alternation as the iterant  $[+1, -1]$  together with the time shift  $\eta$  ( $\eta^2 = 1$ ,  $[a, b]\eta = \eta[b, a]$ ), and shows that  $i = [+1, -1]\eta$  satisfies  $i^2 = -1$ . The imaginary unit is the oscillation of a self-referring distinction.

N1 states the same act in the language of the strand: every distinction is the strand touching itself (C1). A touching is a mark the strand makes on itself.

### **II.2 The eigenform as closure**

Kauffman calls a form that reproduces itself under an operation an eigenform:

$$O(A) = A.$$

An eigenform is a fixed point of a transformation. N2 says that only configurations that return onto themselves persist. In the language of Part III, a ring closes in every window exactly when it is a fixed point of the rewrite  $R$  of its register (T14). With the window rule read as N2 on the register (III.4), N2 takes the form of the eigenform condition for  $R$ . O2 asks for the rewrite dynamics that is its own fixed point; that is the eigenform question stated for the whole net.

### **II.3 The grammar of knots**

A knot diagram changes without changing the knot by exactly three local moves (Reidemeister 1927). Kauffman calls them the grammar of knot equivalence. T1 uses this grammar: an invariant that survives the three moves separates the trefoil from the unknot.

Kauffman's bracket is such an invariant, up to one move. For an unoriented diagram it is fixed by three rules:  $\langle O \rangle = 1$ ; adding a free circle multiplies by  $d = -A^2 - A^{-2}$ ; a crossing splits into its two smoothings,  $\langle \times \rangle = A\langle \nearrow \rangle + A^{-1}\langle \searrow \rangle$ .

**T8 (Bracket of the first knot).** For the standard three-crossing diagram of the trefoil,  $\langle K \rangle = A^{-7} - A^{-3} - A^5$ . Normalised by the writhe  $w = 3$ ,  $f(K) = (-A^3)^{-w}\langle K \rangle$ , and with  $A = t^{(-1/4)}$ , the Jones polynomial is  $V(t) = t + t^3 - t^4$ . Its mirror image gives  $V(t) = t^{-1} + t^{-3} - t^{-4}$ . Status: conditionally proved (state sum over the  $2^3 = 8$  smoothings; computed).

Proof. Each of the  $2^3$  states chooses one smoothing at each crossing. A state with a A-smoothings, b B-smoothings and |s| circles contributes  $A^{(a-b) \cdot d}(|s|-1)$ . The eight states give the sum stated. Multiplying by  $(-A^3)^{-3}$  and substituting  $A = t^{(-1/4)}$  gives V. ■

The unknot has  $V = 1$ . So the trefoil is not the unknot, and it differs from its mirror image. The first knot is chiral. The three-colouring of T1 and the bracket both separate the trefoil from the unknot, and they are linked exactly: the number of Fox three-colourings equals  $3|V(e^{(i\pi/3)})|^2$  (Przytycki 1998; Lickorish and Millett 1986). For the trefoil  $V(e^{(i\pi/3)}) = i\sqrt{3}$ , which gives 9. The bracket also detects chirality, which the colouring does not. That three crossings are the least that cannot be undone is the minimality part of T1.

## II.4 Status

The mark, re-entry and the eigenform are the ground of N1 and N2. They are declared, not derived: C1 identifies the mark with the self-touching of the strand. T8 is conditionally proved. The reading of O2 as the eigenform question is a correspondence.

---

## Part III — The Ternary Ground

---

### III.1 Minimality: why three symbols

Leibniz states the design principle in the Discours de métaphysique (1686), §5-6: the best world maximizes the richness and diversity of phenomena while minimizing the ideas and laws used to build it. Comprehensibility is a consequence: a world built from few bricks can be grasped.

Chaitin translates this into a measurable criterion. A law is a compression. A theory is a program shorter than the data it produces. Comprehensibility equals compressibility. The quality of a foundation is its program length.

The principle has a numerical answer for alphabets. The cost of representing N in radix r is r times the number of digits,  $r \cdot \lceil \log_r(N+1) \rceil$ ; for large N this is  $r \cdot \log_r N = (r / \ln r) \cdot \ln N$ , so the cost per unit of  $\ln N$  tends to  $r / \ln r$ . Compute it:

$$r = 2: 2 / \ln 2 = 2.885$$

$$r = 3: 3 / \ln 3 = 2.731$$

$$r = 4: 4 / \ln 4 = 2.885$$

continuous optimum:  $r = e = 2.718$

Three is the integer closest to the optimum. Three beats two. The cheapest alphabet that expresses richness has three symbols. In the vocabulary of N1: over, under, none. The trinity is not one possible ground among many. It is the solution of Leibniz's minimization.

Note on the selector. In Leibniz the minimization is performed by God — an external builder. The net does not need one. By N2, only what closes persists. The optimum is not chosen; it is what remains. Selection by fit replaces the external optimizer. Chaitin secularizes Leibniz halfway; closure completes it.

### III.2 Symmetry: the balanced notation

Base three has two notations. Unbalanced ternary uses  $\{0, 1, 2\}$ . Balanced ternary uses  $\{-1, 0, 1\}$ , written here as  $\{T, 0, 1\}$ . The balanced form is the notation of the strand: T is counterwinding, 0 is rest, 1 is winding.

Balanced ternary closes without external conventions. Four facts, each verifiable by direct computation:

(a) No minus sign. The sign of a number is the sign of its leading nonzero trit. Negation is trit-wise mirroring: swap  $1 \leftrightarrow T$ . Example:  $8 = 10T$  ( $9 - 1$ );  $-8 = T01$  ( $-9 + 1$ ). No symbol is imported from outside the alphabet.

(b) Closed single-trit multiplication, no carry. The full table:  $1 \times 1 = 1$ ,  $1 \times T = T$ ,  $T \times T = 1$ , anything  $\times 0 = 0$ . One trit in, one trit out. The addition table has two carries in nine entries. Unbalanced ternary has three carries in nine addition entries and a carry in  $2 \times 2 = 11$ ; binary multiplication of single bits is also carry-free, but binary digits carry no sign.

(c) Truncation equals rounding (Knuth). Cut a balanced ternary expansion after position  $m$ . The discarded tail is at most  $\sum_{i>m} 3^{-i} = 3^{-(m)/2}$  in absolute value — half the last kept unit. Cutting is therefore automatically rounding to nearest. No separate rounding rule exists or is needed.

(d) Shift equals scale step. Shifting the trit string one place left multiplies by 3; one place right divides by 3. Same string, other depth. This is N5 written as notation.

Decimal and binary in their standard signed form need two external conventions: a minus sign and a rounding rule. Both are external standards in the sense of the assumptions inventory (The Assumptions Nobody Declares, 26 July 2026). Balanced ternary needs neither. Among positional notations with radix 3 and three integer digits, it is the only one that needs neither convention: the tail bound  $\max |d| \cdot 3^{-(m)/2} \leq 3^{-(m)/2}$  forces  $|d| \leq 1$ , and three digits forming a complete residue system mod 3 within  $\{-1, 0, 1\}$  are exactly  $\{T, 0, 1\}$ . (Radix  $-3$  with the same digits is the same notation with odd positions mirrored. Balanced odd radices 5, 7, ... share both properties but have more than three symbols.)

### III.3 Theorem T7 (Unique Address) — conditional

Hypotheses. N1 (three local states per site) and N5 (scale step of factor three).

Statement. (i) Every finite net state at depth  $n$  corresponds to exactly one trit string of length  $n$ . (ii) Every nonzero integer winding sum has exactly one finite balanced ternary representation with nonzero leading trit; 0 is represented by the empty string (written 0).

Proof of (i). By N5 a net state at depth  $n$  consists of  $n$  sites, one per winding step, ordered by depth; by N1 each site is in exactly one of the states  $+1, 0, -1$ . Writing the site values in depth order gives a map from depth- $n$  states to  $\{T, 0, 1\}^n$ . It is injective, because a depth- $n$  state consists of nothing but its  $n$  site values. By N5 (one distinguishable state at depth 0, multiplied by three per step), there are  $3^n$  depth- $n$  states, as many as there are strings. So the injective map is also surjective, and the correspondence is one-to-one.

Proof of (ii). Existence. Let  $|N| \leq (3^k - 1)/2$ . Then  $M = N + (3^k - 1)/2$  lies in  $[0, 3^k - 1]$  and has a standard ternary expansion  $M = \sum_{i < k} c_i 3^i$  with  $c_i \in \{0, 1, 2\}$ . Since  $(3^k - 1)/2 = \sum_{i < k} 3^i$ ,  $N = \sum_{i < k} (c_i - 1) 3^i$  with  $c_i - 1 \in \{T, 0, 1\}$ . Delete leading zeros.

Uniqueness. Suppose two distinct finite trit strings represent the same integer. Pad the shorter with leading zeros; since both leading trits are nonzero and the strings are distinct, some  $d_i \neq 0$ . Their trit-wise difference gives  $\sum d_i 3^i = 0$  with all  $d_i \in \{-2, -1, 0, 1, 2\}$ , not all zero. Let  $k$  be the largest index with  $d_k \neq 0$ . Then  $|d_k| 3^k \geq 3^k$ . But the remaining terms satisfy  $|\sum_{i < k} d_i 3^i| \leq 2 \sum_{i < k} 3^i = 3^k - 1 < 3^k$ . The sum cannot vanish. Contradiction. ■

Corollary T7.1 (Mirror charge). The address of  $-Q$  is the address of  $Q$ , mirrored trit-wise ( $1 \leftrightarrow T$ ). The sign of the Hopf charge in T3 is a property of the string, not a label attached from outside. Winding and counterwinding are one strand, read in mirror.

Corollary T7.2 (Self-centering depth). By fact (c), the state read at finite winding depth  $m$  is automatically a nearest value at that depth (at an exact midpoint, where the expansion ends in 111... or TTT..., both neighbours are nearest and both expansions exist). Description at a given depth needs no external rounding convention.

Measurement at finite depth is well-defined from inside.

Corollary T7.3 (Capacity per depth). The maximal winding sum expressible at depth  $n$  is the all-ones string:  $(3^n - 1)/2 = 1, 4, 13, 40, 121, \dots$ . Each depth has a hard capacity; exceeding it forces the next depth. This is the quantitative content of N5.

Precedent. Euler stated the uniqueness in (ii) as an identity of formal Laurent series,  $\prod_{k \geq 0} (x^{-3k} + 1 + x^{3k}) = \sum_{n \in \mathbb{Z}} x^n$  (Andrews 2007, §1 and §5, Theorem 4; Dickson notes the result predates Euler). The notation runs on hardware: Brusentsov's Setun computer (Moscow, 1958) computed in balanced ternary. Current ternary-weight neural networks, including the 1.58-bit large-language-model line (Ma et al. 2024), use exactly the alphabet  $\{-1, 0, 1\}$ . The MAZE kernel can take it as its ground type.

Status. T7 is conditionally proved (on N1, N5). The identification of trit values with strand states is a correspondence (C6, Part VIII), not a theorem.

### III.4 The window register

**Definition (window).** A window is three consecutive crossings along the strand, with trits  $(x, y, z) \in \{T, 0, 1\}^3$ , read as elements of  $\mathbb{Z}_3$ . A window closes when its charge vanishes:  $x + y + z \equiv 0 \pmod{3}$ . A ring is a closed strand of  $n \geq 3$  crossings, read cyclically. This is the closure rule of N2 written on the register (source: "Closure Is Zero Force: The Oscillator Framework Read as the Phase Register of the Vacuum.Net Theory", PDF, and "The Relationship Between Vacuum.net.Theory (the MAZE), and Swarm-Theory", both 23 September 2026).

**T9 (Nine of twenty-seven).** Exactly 9 of the 27 windows close. They form a subgroup of  $\mathbb{Z}_3^3$  of index 3: a plane through the origin. Status: conditionally proved (on N1); computed by enumeration.

**Proof.** The sum map  $\mathbb{Z}_3^3 \rightarrow \mathbb{Z}_3$  is a surjective group homomorphism. The closing windows are its kernel, of order  $27/3 = 9$ . ■

**T10 (Two kinds of closure).** A window closes if and only if its three trits are all equal or all different. There are 3 homogeneous and 6 heterogeneous closing windows. Status: conditionally proved.

**Proof.** All equal to  $a$  gives  $3a \equiv 0$ . All different gives  $1 + 0 + (-1) = 0$ . Otherwise exactly two are equal: the sum is  $2a + b$  with  $b \neq a$ . Then  $2a + b \equiv 0$  gives  $b \equiv -2a \equiv a$ , a contradiction. ■

This is the rule of Fox three-colouring at a crossing, used in the proof of T1. The register and the first knot obey one rule.

**T11 (The trit cube).** Read  $\{T, 0, 1\}^3$  as a cube. The number of cells with  $k$  non-zero coordinates is  $C(3, k) \cdot 2^k$ : 1, 6, 12, 8. The closing cells are the centre, the six cells with two non-zero coordinates of opposite sign, and  $(1, 1, 1)$  and  $(T, T, T)$ . Status: conditionally proved; computed.

**Proof.** Choose the  $k$  non-zero axes and their signs. One non-zero coordinate gives  $\pm 1$ . Two give  $\pm 2$  when the signs agree and 0 when they differ. Three give  $\pm 3 \equiv 0$  when all signs agree, and  $\pm 1$  otherwise. The total is  $1 + 0 + 6 + 2 = 9$ , as in T9. ■

**T12 (Closure probability).** Let the three trits be independent with law  $(p_0, p_+, p_-)$ . Put  $\hat{p}(z) = p_0 + p_+z + p_-z^2$  and  $\omega = e^{(2\pi i/3)}$ . A window closes with probability  $(1/3) \cdot [1 + \hat{p}(\omega)^3 + \hat{p}(\omega^2)^3]$ . The uniform law gives  $1/3$ . Status: conditionally proved.

**Proof.** The indicator of  $s \equiv 0 \pmod{3}$  is  $(1/3) \cdot \sum_k \omega^{ks}$ . Take expectations over independent entries. ■

**T13 (A ring that closes everywhere).** If every window of a ring of  $n \geq 3$  trits closes, then  $t_{i+3} = t_i$  for all  $i$ . If 3 does not divide  $n$ , the ring is constant. If 3 divides  $n$ , it repeats one closing window: three constant rings and six of minimal period 3. The number of such rings, counted with labelled positions, is 9 when 3 divides  $n$ , and 3 otherwise. Status: conditionally proved; computed for  $n = 3$  to 13.

Proof. Subtract the closure conditions at  $i$  and  $i + 1$ :  $t_{i+3} - t_i \equiv 0$ . So 3 is a period, and so is  $n$ ; hence  $\gcd(3, n)$  is a period. If 3 divides  $n$ , choose  $t_0$  and  $t_1$  freely and put  $t_2 = -(t_0 + t_1)$ ; every window is a cyclic shift of  $(t_0, t_1, t_2)$  and closes: 9 rings. Otherwise the ring is constant,  $t = a$ , and  $3a \equiv 0$  always: 3 rings. ■

**T14 (The rewrite of the register).** Let  $R$  act on a ring by  $R(t)_i = -(t_{i-1} + t_{i+1})$ . A ring is a fixed point of  $R$  if and only if every window closes. Status: conditionally proved.

Proof.  $R(t)_i = t_i$  means  $t_{i-1} + t_i + t_{i+1} \equiv 0$ . ■

Since  $2 \equiv -1 \pmod{3}$ ,  $R(t)_i = 2^{-1}(t_{i-1} + t_{i+1})$ : each trit becomes the mean of its neighbours in  $\mathbb{Z}_3$ . The fixed points are the eigenforms of the register (Part II.2). They are fixed, not attracting: for  $n = 3$  to 10, the rings that are not fixed end in cycles, except 24 rings for  $n = 4$  and 24 for  $n = 8$ . Computed.

### III.5 Two ladders and their closure depths

The net carries two ladders:  $\times 3$  for addresses (N5) and  $\times 2$  for dimensions ("The Comma in the Net", 2 September 2026). They meet where  $3^k$  comes close to  $2^n$ .

**T15 (No exact meeting).**  $2^n \neq 3^k$  for all positive integers  $n, k$ . So  $\log_2 3$  is irrational, and the points  $n + k \cdot \log_3 2$  ( $n \in \mathbb{Z}, k \geq 1$ ) lie dense on the line; equivalently, the fractional parts of  $k \cdot \log_3 2$  are dense in  $[0, 1)$ . Status: conditionally proved.

Proof.  $2^n$  is even and  $3^k$  is odd. An irrational step generates a dense set (Kronecker; Hardy and Wright, Theorem 439). ■

Definition (closure depth). For  $k \geq 1$  let  $\|k \cdot \log_2 3\|$  be the distance from  $k \cdot \log_2 3$  to the nearest integer  $n$ . A closure depth is a  $k$  with  $\|k \cdot \log_2 3\| < \|j \cdot \log_2 3\|$  for every  $1 \leq j < k$ . It counts powers of 3.

**T16 (The closure depths).** The closure depths are the denominators of the convergents of  $\log_2 3 = [1; 1, 1, 2, 2, 3, 1, 5, 2, 23, \dots]$ : 1, 2, 5, 12, 41, 53, 306, 665, 15601, ... Status: conditionally proved (Lagrange's theorem on best approximations of the second kind; Khinchin, Chapter II, §6); computed for  $k \leq 10^6$ .

Proof. A best approximation of the second kind of  $\theta$  is a fraction  $n/k$  with  $|k\theta - n| < |j\theta - m|$  for all  $0 < j < k$  and all  $m$ . By Lagrange's theorem (Khinchin, Theorems 16 and 17) these are exactly the convergents  $p_m/q_m$ , with the single exception of  $p_0/q_0 = a_0$  when  $a_1 = 1$ , as here. For  $k \geq 2$  the definition of a closure depth is this condition, with  $n$  the nearest integer to  $k\theta$ .  $k = 1$  is a closure depth vacuously; it is the denominator of  $p_1/q_1 = 2/1$ . So the closure depths are  $q_1, q_2, q_3, \dots = 1, 2, 5, 12, \dots$  ■

**T17 (The comma).** At depth 12 the two ladders miss by  $3^{12}/2^{19} = 531441/524288 = 1.013643$ , the Pythagorean comma (23.46 cents). The miss never vanishes (T15). Status: conditionally proved; computed. The residues  $|2^n/3^k - 1|$  at the depths  $k = 2, 5, 12, 41, 53, 306$  are 11.1 %, 5.35 %, 1.35 %, 1.15 %, 0.21 % and 0.10 %.

Between the closure depths, nothing closes better. *Reading*: this is the arithmetic content of "between the rungs, nothing is stable".

## Part IV — Theorems (conditional)

---

Each theorem states its hypotheses and scope. None claims more than its conditions deliver.

### T1 (First knot)

Hypotheses. Closed embedding ( $S^1$  or fixed endpoints); crossings countable.

Statement. The trefoil is the simplest configuration that cannot be undone, at three crossings.

Proof. Distinctness. Color the arcs of a diagram with three colors such that at every crossing the three meeting arcs carry either one color or all three. The crossingless unknot diagram has one arc and so admits only single-color colorings; by the invariance below, no diagram of the unknot admits a coloring with more than one color. The standard trefoil diagram admits a coloring using all three colors: assign each of its three arcs a different color; at each crossing all three colors meet, satisfying the rule. Tricolorability is invariant under all three Reidemeister moves (checked case by case on the finitely many local pictures). Hence the trefoil is not the unknot. Minimality. Diagrams with 0, 1, or 2 crossings are finitely many up to planar isotopy: the only planar Gauss words are  $\emptyset$ , 11, 1122 and 1221 (the word 1212 violates Gauss's even-interlacing condition). In each of them some crossing is a kink removable by a Reidemeister I move, and repeating this reduces the diagram to the unknot. For fixed endpoints, the endpoints lie on the boundary of the ambient ball (or at infinity); closing the arc along the boundary identifies these arc classes with knot classes, so the closed case covers it. (Endpoints fixed at interior points would not suffice: every such arc can be undone.) So no nontrivial knot exists below three crossings. At three crossings, the only planar Gauss word without a kink is 123123, the trefoil shadow. Of its eight crossing choices, the two alternating ones are the right- and left-handed trefoils. Each of the six non-alternating ones has a bigon whose one edge passes over at both ends; a Reidemeister II move removes that bigon and leaves one crossing, hence the unknot. So every nontrivial three-crossing knot is a trefoil. ■

Remark. The proof itself is ternary: three arcs, three colors, three crossings. The first stable form is certified by a three-coloring. The trinity appears in the first proof of the first knot.

### T2 (Double cover)

Hypotheses. Orientation states form the unit quaternions; a strand is attached (has ends or environment).

Statement. A strand returns to its initial state after a rotation of  $4\pi$ , not  $2\pi$ .

Proof. Unit quaternions  $q = a + bi + cj + dk$  with  $i^2 = j^2 = k^2 = -1$  represent rotations by  $v \rightarrow q v q^*$  on pure-imaginary quaternions  $v$ . Both  $q$  and  $-q$  give the same rotation: the representation is a double cover. A continuous rotation by  $2\pi$  carries  $q$  to  $-q$ ; only  $4\pi$  returns  $q$  to  $q$ . For a free frame the sign is unobservable. For an attached strand the

configuration records the homotopy class of the rotation path in  $SO(3)$  relative to the attachment.  $\pi_1(SO(3)) = \mathbb{Z}/2$ : the  $2\pi$  loop is the nontrivial class (its lift starting at  $q$  ends at  $-q \neq q$ ), the  $4\pi$  loop is trivial. So the  $2\pi$ -rotated attached configuration cannot be deformed back to the initial one with the attachment fixed, while the  $4\pi$ -rotated one can (Dirac belt trick). The sign is thereby observable. ■ (C2 identifies  $\text{spin-}\frac{1}{2}$  as this rotation label.)

### T3 (Charge conservation)

Hypotheses. Field configurations map into the target space of closed windings; the strand is continuous (N1); no boundary is crossed.

Statement. The Hopf charge  $Q \in \pi_3(S^2) = \mathbb{Z}$  is conserved under all continuous rewrites.

Proof.  $Q$  is an integer-valued homotopy invariant. Here a field configuration is a map  $S^3 \rightarrow S^2$  (space compactified by a fixed value at infinity, which is what "no boundary is crossed" secures), and a continuous rewrite is a homotopy of such maps. Since  $S^2$  is simply connected, free and based homotopy classes agree:  $[S^3, S^2] = \pi_3(S^2) = \mathbb{Z}$ , the integer being the Hopf invariant (the linking number of the preimages of two regular values; for the Hopf map these are two fibres). A continuous deformation cannot change an integer continuously.  $Q$  can change only through discontinuity: cutting the strand, leaving the target space, or crossing a boundary. All three are excluded by hypothesis. Conservation follows from continuity of the family of field maps — strand continuity (N1) together with the two other hypotheses — and no symmetry argument (Noether) is invoked. ■ By T7.1, the addresses of  $Q$  and  $-Q$  are trit-wise mirrors of each other.

### T4 (Constitutive equation)

Hypotheses. The energy density of the medium is  $e(n) = (g_2/2) n^2 + (g_3/6) n^3$ , with  $g_3 \neq 0$ . A reference density  $\bar{n} \neq 0$  is chosen such that  $\bar{g} \equiv g_2 + g_3 \bar{n} \neq 0$ . Scope: one explicit realization of N4.

Statement.  $g_{\text{eff}} = \bar{g} (1 + \gamma\delta)$ , with  $\bar{g} \equiv g_2 + g_3 \bar{n}$ ,  $\gamma \equiv g_3 \bar{n} / \bar{g}$ , and  $\delta \equiv (n - \bar{n})/\bar{n}$ .

Proof. Two derivatives:  $g_{\text{eff}} = d^2e/dn^2 = g_2 + g_3 n$ . Insert  $n = \bar{n}(1 + \delta)$ :  $g_{\text{eff}} = (g_2 + g_3 \bar{n}) + g_3 \bar{n} \delta = \bar{g} (1 + \gamma\delta)$ . ■ The stiffness of the medium is a state function of its density. This is N4 made explicit.

### T5 (Sign theorem)

Hypotheses. T4 holds;  $\bar{g} > 0$  at the reference state (positive net stiffness); at the reference state  $\gamma = 2\pi$  (a hypothesis; this edition no longer asserts it, see C5 and the note to T5).

Statement.  $g_2 < 0$ , and in the stiffness at reference density,  $g_{\text{eff}}(\bar{n}) = g_2 + g_3 \bar{n}$ , the quadratic contribution  $g_2$  cancels 84.1% of the cubic contribution  $g_3 \bar{n}$ .

Proof. From  $\gamma = g_3 \bar{n} / \bar{g}$  and  $\bar{g} = g_2 + g_3 \bar{n}$ :  $g_2 = \bar{g} (1 - \gamma)$ . With  $\gamma = 2\pi > 1$ ,  $g_2 = \bar{g} (1 - 2\pi) < 0$  for  $\bar{g} > 0$ . The cancellation fraction is  $|g_2| / (g_3 \bar{n}) = (2\pi - 1)/2\pi = 1 - 1/2\pi =$

$0.8408 \rightarrow 84.1\%$ . The residual stiffness is  $\bar{g} = g_3 \bar{n} / 2\pi$ . ■ Under this hypothesis the vacuum sits near criticality: two large contributions, nearly balanced, one small net stiffness. This is the formal version of the weather picture.

Note to T5 (this edition). T5 remains proved under its hypothesis  $\gamma = 2\pi$ . This edition no longer asserts that hypothesis (C5, Part VIII). By N4  $\gamma$  is a state function. For  $\gamma < 1$  the same algebra gives  $g_2 = \bar{g}(1 - \gamma) > 0$ , and the cancellation fraction of T5 does not apply.

## **T6 (Loss has an address)**

Hypotheses. Local equilibrium holds per mesh (Gibbs relation); local balance equations hold; the second law holds for every sub-volume; fluxes  $J$  and forces  $X$  are defined per mesh; channels may couple.

Statement. Entropy production is  $\sigma = \sum J \cdot X \geq 0$  for the total, per location. In the linear regime  $J = LX$  with Onsager-symmetric  $L$ , the individual terms are sign-definite in the channels that diagonalize  $L$ ; in coupled channels a single  $J_i X_i$  can be negative.  $\sigma = 0$  does not force every  $X$  to zero: in the linear regime it forces  $J = LX = 0$ , with  $X$  free in the kernel of  $L$ ; outside the linear regime it constrains only the total.

Proof sketch. Under local equilibrium, the Gibbs relation and the balance equations give  $\sigma = \sum J \cdot X$  (de Groot-Mazur). The second law applied to every sub-volume, however small, gives  $\sigma \geq 0$  pointwise. Positivity binds the sum, not the parts. For  $J = LX$  with symmetric  $L$ ,  $\sigma = X^T L X$ . Since  $\sigma \geq 0$  for every  $X$ ,  $L$  is positive semidefinite. In an orthonormal eigenbasis of  $L$ ,  $J'_k = \lambda_k X'_k$ , the pairing is preserved ( $\sum J'_k X'_k = \sum J_i X_i$ ), and  $\sigma = \sum \lambda_k X'^2_k$  with  $\lambda_k \geq 0$ ; each term  $J'_k X'_k = \lambda_k X'^2_k \geq 0$ . ■

Consequence: every irreversible discharge occurs at a mesh with an address. Loss is bookkeeping on the net, not a leak out of it.

## **T7 (Unique Address)**

Stated and proved in Part III.3.

## **T18 (Lever of a free path) — new in this edition**

Definition (lever of a count). For an integer-valued count  $V$  with mean  $\bar{n}$  and cumulants  $\kappa_2 > 0$ ,  $\kappa_3$ , the lever is  $\gamma(V) = -\bar{n} \cdot \kappa_3 / \kappa_2^2$ . For a smooth closing energy whose fluctuations  $x = n - \bar{n}$  have weight  $\exp(-N \cdot f(x))$  near the minimum, with  $f(x) = (a/2)x^2 + (b/6)x^3$ ,  $a = e''(\bar{n}) > 0$  and  $b = e'''(\bar{n})$ , the lever equals  $\bar{n} \cdot b/a = \bar{n} \cdot e'''(\bar{n})/e''(\bar{n}) = \gamma$  of T4 to leading order in  $1/N$  (Laplace's method). The lever is not shift-invariant, so the count must be stated exactly.

Hypotheses. The strand is a free path: the simple random walk on  $\mathbb{Z}^d$ ,  $d \geq 3$ , with return probability  $P_d < 1$ . A knot counts the visits  $V$  to its place, the first included.

Statement.  $V$  is geometric,  $P(V = k) = (1 - P_d) \cdot P_d^{k-1}$ , and its lever is  $\gamma = -(1 + P_d)/P_d$ . With  $P_3 = 0.3405373$ ,  $P_4 = 0.1932017$  and  $P_5 = 0.1351786$  (Watson 1939; OEIS A086232, A086233),  $\gamma = -3.9365$ ,  $-6.1759$  and  $-8.3976$ .

Proof. By the strong Markov property each visit is followed by another with probability  $P_d$ , independently. With  $q = 1 - P_d$ :  $\bar{n} = 1/q$ ,  $\kappa_2 = P_d/q^2$ ,  $\kappa_3 = P_d(1 + P_d)/q^3$ . Then  $-\bar{n} \cdot \kappa_3 / \kappa_2^2 = -(1 + P_d)/P_d$ . ■

A simulation in  $d = 3$  ( $10^6$  walks of  $10^4$  steps) gives  $-3.939 \pm 0.013$ , against the exact  $-3.961$  for walks of that length. The lever depends on the freedom of the path, through  $P_d$ . This is N4 at work: the coupling is a state function. That the lever of the free path equals  $\gamma$  of the medium is not proved here (O1).

---

## Part V — Nilpotent Rewriting: from Dirac to Closure (new in this edition)

---

### V.1 Input: Dirac's equation

Dirac (1928):  $(\gamma^\mu p_\mu - m)\psi = 0$ , in natural units with metric  $(+, -, -, -)$ .

Multiplying by  $(\gamma^\nu p_\nu + m)$  gives  $(E^2 - p^2 - m^2)\psi = 0$ , because  $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu}$ . Dirac's equation enters this paper as given input.

### V.2 Rowlands' rewriting

Rowlands replaces the four gamma matrices by two commuting sets of units. The quaternion units  $i, j, k$  square to  $-1$  and anticommute. The vector units square to  $+1$  and anticommute; they are carried by the momentum vector  $\mathbf{p}$ . Each unit of one set commutes with each unit of the other. A third ingredient is the scalar imaginary unit  $i$  ( $i^2 = -1$ ), which commutes with all six units; it is the  $i$  in  $ikE$  and in the operator, while the  $i$  of  $i\mathbf{p}$  is the quaternion unit. The Dirac operator becomes the nilpotent

$$Q = ikE + i\mathbf{p} + jm,$$

and the equation reads  $(\mp k\partial_t \mp i\nabla + jm)(\pm ikE \pm i\mathbf{p} + jm)e^{(-i(Et - \mathbf{p}\cdot\mathbf{r}))} = 0$ .

**T19 (Nilpotence is the mass shell).**  $Q^2 = (E^2 - p^2 - m^2) \cdot 1$ . Hence  $Q^2 = 0$  exactly when  $E^2 = p^2 + m^2$ . Status: conditionally proved (hypothesis: Dirac's equation in Rowlands' form); computed in a  $4 \times 4$  representation (quaternions  $\otimes$  Pauli matrices), on and off the shell.

Proof.  $(ikE)^2 = -k^2E^2 = E^2$ .  $(i\mathbf{p})^2 = i^2\mathbf{p}^2 = -p^2$ , because the vector units square to  $+1$  and anticommute, so  $\mathbf{p}^2 = p^2$ .  $(jm)^2 = -m^2$ . There are three cross terms, each an anticommutator:

- $ikE$  with  $i\mathbf{p}$ :  $iE(ki + ik)\mathbf{p} = 0$ , since  $ki = -ik$ ;
- $ikE$  with  $jm$ :  $iEm(kj + jk) = 0$ , since  $kj = -jk$ ;
- $i\mathbf{p}$  with  $jm$ :  $m(ij + ji)\mathbf{p} = 0$ , since  $ji = -ij$ .

In each line  $\mathbf{p}$  and the scalar  $i$  commute with  $i, j, k$  and can be moved out; what remains is an anticommutator of two different quaternion units, which is zero. So  $Q^2 = E^2 - p^2 - m^2$ . ■

Computed: in the  $4 \times 4$  representation (quaternion units  $-i\sigma \otimes 1$ , vector units  $1 \otimes \sigma$ ) each of the three anticommutators is the zero matrix, and  $Q^2 = (E^2 - p^2 - m^2) \cdot 1$  to  $10^{-14}$ .

### V.3 The link to the net

The three units of  $Q$  are the three floors  $E, p, m$  of the crossing alphabet, with signature  $(+, -, -)$ .  $N2$  applied to itself adds nothing:  $\Psi^2 = 0$ .  $C3$  identifies this closure with Rowlands' nilpotence. The chain is:

Dirac (input)  $\rightarrow$  nilpotent form (Rowlands)  $\rightarrow Q^2 = 0 \leftrightarrow$  closure ( $C3$ )  $\rightarrow E^2 = p^2 + m^2$  (T19).

Status. T19 is conditionally proved (hypothesis: Dirac's equation in Rowlands' form). The identification of nilpotence with closure is a correspondence ( $C3$ ). A derivation of Dirac's equation from  $N1$ - $N5$  without input is open ( $O6$ ).

## Part VI — Maxwell's Quaternions: Two Registers (new in this edition)

### VI.1 The record

Hamilton's operator  $\nabla = i\partial_x + j\partial_y + k\partial_z$  acts on a vector with one product. The product has two parts: a scalar part, the convergence, and a vector part, the curl. Maxwell wrote his grouped equations of 1873 (Treatise, Arts. 591-619) in this form:  $\mathfrak{B} = \nabla \nabla \mathfrak{A}$  ( $\mathfrak{B} = \nabla \times \mathfrak{A}$ ),  $\mathfrak{e} = S \nabla \mathfrak{D}$  ( $\mathfrak{J}$ ) as printed, read today as  $\rho = \nabla \cdot \mathfrak{D}$  (with Hamilton's sign convention  $S \nabla \mathfrak{D} = -\nabla \cdot \mathfrak{D}$ , the convergence; the printed form carries Maxwell's sign),  $\mathfrak{C} = \mathfrak{R} + \mathfrak{D}$  ( $\mathfrak{H}$ ) (the total current, with the displacement term; in Maxwell  $\mathfrak{J}$  denotes the intensity of magnetization), and the constitutive moduli  $K, C, \mu$ , with  $c^2 = 1/(K\mu)$  (Art. 784 ff.). Gibbs (Elements of Vector Analysis, 1881-84) and Heaviside (1882-85) split the one product into dot and cross; Heaviside's four-equation vector form dates from 1884-85. The scalar register disappeared. In this paper's reading,  $K, C$  and  $\mu$  were then treated as fixed constants; in Maxwell (Art. 618) they are coefficients of the medium.

### VI.2 The rewriting in net terms (correspondence)

| Maxwell                                                      | Net                                                              |
|--------------------------------------------------------------|------------------------------------------------------------------|
| one product, two readings ( $S \nabla$ and $\nabla \nabla$ ) | two readings of one closure operation: the two-layer law         |
| potential $\mathfrak{A}$ first, fields as readings of it     | the winding state of the mesh is the object; fields are readings |
| free charge $\mathfrak{e} = S \nabla \mathfrak{D}$           | the scalar residue of non-closure, written as a trit $+1/0/-1$   |
| displacement term                                            | the closure-completing term: the total current always closes     |

| Maxwell                 | Net                                                        |
|-------------------------|------------------------------------------------------------|
| moduli $K, C, \mu$      | state functions $g(\chi)$ of the local vacuum (N4)         |
| $c^2 = 1/(K\mu)$        | $c$ as a state function, not a postulate                   |
| non-commutative product | order is orientation (N3); a full return takes $4\pi$ (T2) |

Status: correspondence. Maxwell's equations are translated into the net, not derived from N1-N5 (O5).

### VI.3 The composition rule (conditionally proved)

A passage question  $\rightarrow$  neighbour  $\rightarrow$  answer is read with quaternions. Two lemmas show why the scalar register is needed.

**T20 (The 4c identity).** For unit vectors  $q, a$  with  $c = \cos(q, a)$ :  $|q + a|^2 - |q - a|^2 = 4c$ . Status: conditionally proved.

Proof.  $|q \pm a|^2 = 2 \pm 2c$ . ■

**T21 (Pure quaternions collapse).** With pure quaternions  $Q = (0, q)$ ,  $N = (0, n)$ ,  $A = (0, a)$  and  $|q| = 1$ , every term of the triple product reduces to pair dot products (cosines when  $n$  and  $a$  are unit) or to  $\det[q, n, a]$ :  $S(QNA) = -\det[q, n, a]$ ;  $V(QNA) = -(q \cdot n)a + (q \cdot a)n - (n \cdot a)q$ ;  $-V(QNA) \cdot q = n \cdot a$ ;  $S(QNAQ) = n \cdot a$ . Status: conditionally proved.

Proof.  $QN = (-q \cdot n, q \times n)$ . Expand with  $(u \times v) \times w = v(u \cdot w) - u(v \cdot w)$  and the Binet-Cauchy identity. ■

So a rule that keeps orientation needs full quaternions, scalar part included, and one frame per neighbour. With frames  $e_1^{(i)}, e_2^{(i)}, e_3^{(i)}$  per sister cell, each item becomes  $Q^{(i)} = (\cos(\theta/2), \sin(\theta/2) \cdot w)$ , the shortest rotation that carries the region centre of cell  $i$  to  $q$  (angle  $\theta$ , unit axis  $w$ ), with  $A^{(i)}$  likewise for  $a$ . The relative winding is  $D_i = (Q^{(i)})^* \cdot A^{(i)}$ .

**T22 (Ring measure).** For unit quaternions  $D_1, \dots, D_k$ , put  $B = (1/k)\sum D_i D_i^T$  and  $R_Q = \lambda_1(B)$ . Then  $1/4 \leq R_Q \leq 1$ , and  $R_Q$  is unchanged when any  $D_i$  is replaced by  $-D_i$ . Status: conditionally proved.

Proof. Each  $D_i D_i^T$  has trace 1, so trace  $B = 1$  and  $B$  is positive semidefinite. Its largest of four eigenvalues lies in  $[1/4, 1]$ .  $(-D)(-D)^T = DD^T$ . ■

The sign invariance is the double cover (T2) at work:  $q$  and  $-q$  give the same rotation. Worked example:  $q = (1, 0, 0)$ ,  $a = (0, 1, 0)$ , region centres  $(1, 1, 0)/\sqrt{2}$  and  $(1, 0, 1)/\sqrt{2}$ ,  $60^\circ$  apart. Then  $D_1 = (0.707, 0, 0, 0.707)$ ,  $D_2 = (0.653, -0.653, -0.271, 0.271)$ ,  $D_1 \cdot D_2 = 0.653$ , and  $R_Q = (1 + 0.653)/2 = 0.827$ . Computed.

## Part VII — Units and Dimensions

The model is dimensionless. Its native quantities are ratios: winding sums, densities relative to reference,  $\chi_a$ . There is no  $\hbar$  in the model.  $c$  is a state function of the medium, not a universal input. SI numbers are conversions performed after the fact,

for comparison with instruments. By T7.2, finite-depth readout is self-centering: the notation needs no external rounding convention. Because its quantities are dimensionless ratios, it needs no unit convention either.

## VII.1 The dimensional ladder (Larson)

Conversions follow Larson's reading of motion: space  $S$  and time  $T$  as the two aspects of one motion. Motion is  $S/T$ . Energy is  $T/S$ . Mass is  $T^3/S^3$ . Momentum is  $T^2/S^2$ .

**T23 (The bridge).** The map  $\varphi(x, y, z) = (x - 3z, y + 3z)$  sends the SI dimension  $L^x \cdot T^y \cdot M^z$  to  $S^{(x-3z)} \cdot T^{(y+3z)}$ . It sends speed to  $S/T$ , energy to  $T/S$ , momentum to  $T^2/S^2$  and mass to  $T^3/S^3$ . Energy per unit time is  $1/S$ . Mass times motion squared is energy. The exchange of  $S$  and  $T$  is an involution that maps motion to energy. Its kernel is generated by  $M \cdot L^3/T^3$ . Status: conditionally proved.

Proof.  $\varphi(1, -1, 0) = (1, -1)$ ;  $\varphi(2, -2, 1) = (-1, 1)$ ;  $\varphi(1, -1, 1) = (-2, 2)$ ;  $\varphi(0, 0, 1) = (-3, 3)$ .  $(T/S)/T = 1/S$ .  $(T^3/S^3)(S/T)^2 = T/S$ .  $x - 3z = 0$  and  $y + 3z = 0$  give  $(x, y, z) = z(3, -3, 1)$ . ■

Planck's constant.  $E = h\nu$  with  $E = T/S$  and  $\nu = 1/T$  gives  $h = T^2/S$ . Earlier texts wrote  $S^2/T$ ; that form is withdrawn.

## VII.2 The conversion units

For comparison with instruments the conversions use one length and one time, both fixed by hydrogen:  $S_0 = 1/(2R_H)$ , with  $R_H = R_\infty/(1 + m_e/m_p)$ , and  $T_0 = S_0/c$ . Inputs (CODATA 2018):  $R_\infty = 10,973,731.568160 \text{ m}^{-1}$ ,  $m_p/m_e = 1,836.15267343$ ,  $c = 299,792,458 \text{ m/s}$ ,  $h = 6.62607015 \times 10^{-34} \text{ J}\cdot\text{s}$ .

| Unit  | Definition               | Value                                                          |
|-------|--------------------------|----------------------------------------------------------------|
| $R_H$ | $R_\infty/(1 + m_e/m_p)$ | $10,967,758.340 \text{ m}^{-1}$                                |
| $S_0$ | $1/(2R_H)$               | $4.55881671 \times 10^{-8} \text{ m}$                          |
| $T_0$ | $S_0/c$                  | $1.52065757 \times 10^{-16} \text{ s}$                         |
| $E_0$ | $h/T_0 = 2hcR_H$         | $4.35737162 \times 10^{-18} \text{ J} = 27.1965745 \text{ eV}$ |

$E_0$  is the Hartree energy of hydrogen with its reduced mass. The layer at depth  $n$  has length  $3^n \cdot S_0$  and time  $3^n \cdot T_0$ . At depth 2 the length is  $9S_0 = 410.29 \text{ nm}$ , the vacuum wavelength of the Balmer  $H\delta$  line of hydrogen with its reduced mass. That identity holds by construction of  $S_0$ .

## Part VIII — Correspondences

Correspondences identify model elements with observed structures. They are neither proved nor predicted; they are declared, and they can fail.

C1 (Mark). Spencer-Brown's mark of distinction = the self-touching of the strand. Absorbed into N1.

C2 (Rotor). The local state is a quaternion rotor; spin- $\frac{1}{2}$  is the rotation label of T2.

C3 (Nilpotence). Rowlands' nilpotent condition  $Q^2\psi = 0$  with  $Q^2 = E^2 - p^2 - m^2$  corresponds to closure: a configuration closes exactly when its rotor squares to zero in the Cl(1,2) fragment.

C4 (Knot = matter). Stable particles are closed windings. The electron corresponds to the Williamson-van der Mark toroidal configuration: a photon closed on itself in a double loop.

C5 (Crossover) — revised in this edition. Two roles of  $2\pi$  are separated. (i) The coupling exponent  $\gamma$  of T4 is a state function (N4), not a fixed number. For a free path T18 gives the lever  $\gamma = -(1 + P_d)/P_d$ ; that this lever is the  $\gamma$  of T4 is open (O1). The earlier value  $\gamma = 2\pi$  is withdrawn as a correspondence. (ii) The crossover keeps one geometric turn per mesh: one mesh around is one full turn,  $2\pi$ . The crossover rule  $\chi_a = 1/2\pi$  gives  $a_0 = cH_0/2\pi$  (I3).

C5a (Bridge B1, observable-state).  $\delta = k \chi_a$  near the reference point, with  $k \equiv d\delta/d\chi_a$ .  $k = 1$  is an additional choice, not a result. The measured threshold tests I3; it does not fix  $\gamma$  or  $k$  separately. Validity: near the reference point only.

C6 (Trit = strand state). The trit values  $\{1, 0, T\}$  correspond to  $\{\text{winding, rest, counterwinding}\}$  at a site. This identification carries T7 from arithmetic onto the net. It is a correspondence: the arithmetic is proved, the mapping is declared.

## Part IX — Identifications

---

I1. Electron = Williamson-van der Mark double-loop photon. I2. State variable  $\chi_a = a/(cH_0)$ : the only independent dimensionless ratio of local acceleration to background. I3.  $a_0 = cH_0/2\pi$ , with  $2\pi$  the geometric turn of C5 (ii). With  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} = 2.27 \times 10^{-18} \text{ s}^{-1}$ :  $a_0 = (3.00 \times 10^8 \times 2.27 \times 10^{-18})/2\pi = 1.08 \times 10^{-10} \text{ m s}^{-2}$ . Measured threshold (rotation-curve fits):  $1.2 \times 10^{-10} \text{ m s}^{-2}$ . Agreement within 10%. I4.  $g = g(\alpha(\chi))$ : context sets  $\alpha$ ,  $\alpha$  sets interaction.  $\Delta\alpha/\alpha = \beta \cdot \Delta\chi$  is the inner function in the weak-field tail.

## Part X — Predictions

---

Each prediction carries its pre-commitments. A prediction without a pre-registered map is not a prediction.

P1 (Clock-spectroscopy benchmark, conditional). In potential form:  $\Delta\alpha/\alpha \approx +3 \times 10^{-8}$  at the white dwarf G191-B2B, conditional on the pair  $(\chi_\Phi, \beta)$ . The inputs, as in Edition 5:  $\Delta\chi_\Phi = GM/(Rc^2) = 5.3 \times 10^{-5}$  at the surface of G191-B2B (with  $M = 1.0 \times 10^{30} \text{ kg}$  and  $R = 1.4 \times 10^7 \text{ m}$ :  $6.67 \times 10^{-11} \times 1.0 \times 10^{30} / (1.4 \times 10^7 \times 9.0 \times 10^{16}) = 5.3 \times 10^{-5}$ ), and  $\beta = \alpha/4\pi = 0.0072974/12.566 = 5.81 \times 10^{-4}$ . By I4,  $\Delta\alpha/\alpha = \beta \cdot \Delta\chi_\Phi = 5.81 \times 10^{-4} \times 5.3 \times 10^{-5} = 3.08 \times 10^{-8}$ . A null result refutes that pair, not the theory. Pre-commitment: the  $\chi$ -map of the line of sight, fixed before analysis.

P2 (Overdetermination).  $\gamma k = \beta \cdot p_\alpha$ , with  $p_\alpha \equiv d \ln g / d \ln \alpha$ . Two independent routes — rotation curves (SPARC) and  $\alpha$ -spectroscopy — must return the same product  $\gamma k$  at

the same reference state (which requires one  $\chi$ , O4). Sign conditions stated; preregistration required.

P3 (Solar winding edge). The edge lies where internal acceleration meets the background:  $GM/r^2 = a_0$ , hence  $r_{\text{edge}} = \sqrt{GM/a_0}$ . Every input is in this text:  $GM_{\odot} = 1.327 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$ ,  $a_0 = 1.08 \times 10^{-10} \text{ m s}^{-2}$  (I3). Then  $r_{\text{edge}} = \sqrt{(1.23 \times 10^{30})} = 1.11 \times 10^{15} \text{ m} = 7,400 \text{ AU}$ . (Edition 5 printed 5,350 AU; the formula was right, the arithmetic was not.) Scaling:  $r_{\text{edge}} \propto \sqrt{M}$ . Honest label: the scaling follows from the threshold identification; the theory's own contribution is the value of the threshold (I3).

P4 (No dark-matter particle). Operational form: no context-independent particle will ever be detected. Pre-registered function:  $\Delta m_{\text{eff}}/m_{\text{eff}} = s_m \cdot \gamma k \cdot \Delta \chi_a$ , with susceptibility  $s_m \equiv d \ln m_{\text{eff}} / d \ln g_{\text{eff}}$ , to be computed in O1. A collective excitation that tracks  $\chi$  does not count as a particle. Only refutable in this operational form.

P5 (Forbidden outcomes). (i) A quantized spin-2 graviton. (ii) Any measured  $\alpha$ -variation must survive the  $\chi$ -regression; a residual dipole after regression refutes the model. Four pre-commitments fixed in Edition 5 apply unchanged.

## Part XI — Open calculations

---

O1. Coarse-graining of the knot network. Part of it is closed in this edition:  $\gamma$  is a state function, and for a free path T18 gives its value. Open: that the lever of the path is the coupling of the medium; the sign of  $g_2 = \bar{g}(1 - \gamma)$ , which for  $\bar{g} > 0$  is negative for  $\gamma > 1$  and positive for  $\gamma < 1$  (as for the free-path lever), which waits on the value of  $\gamma$  at the reference state; and P4, which waits on  $s_m$ . "Why four" is answered by T2: a free frame returns after  $2\pi$ ; the rotor, and an attached strand, return only after  $4\pi$ . O2. Closure dynamics (Rosen): not "which action does the net minimize" but "which rewrite dynamics is its own fixed point". The action principle is an inherited external mover; the fixed-point form replaces it. On the register this is solved: the fixed points of the rewrite are the rings that close everywhere (T13, T14), the eigenforms of Part II.2. Open for the whole net. O3. The mass spectrum from the winding alphabet, including  $m_p/m_e = 1836$ , and the electric charge. O4. One  $\chi$ : unify  $\chi_a = a/(cH_0)$  and  $\chi_\Phi = |\Phi|/c^2$ , now including  $k$  (C5a). O5. Maxwell's equations from N1–N5, beyond the translation of Part VI. O6. Dirac's equation from N1–N5 without input (Part V). O7. The fine-structure constant  $\alpha$ , now an input (P1).

## Part XII — Position

---

Wolfram/Gorard. Rule selection by the criterion that known physics emerges; the ruliad is an external standard. Program length now gives a neutral comparison (Part III.1): richness produced per axiom. Five axioms here; the challenge to any rival is to produce the same richness from fewer. Verlinde. Moves the foundation one floor (holography, de Sitter) but keeps it external. Loop quantum gravity, CDT, causal sets. Keep causal ordering or a background condition as primitive. N4 treats universality as an empirical limiting case; T3 supplies one topological conservation law independently of Noether. Chaitin. Supplies the metric on which all of the above can be compared without an

external judge: comprehensibility = compressibility. The comparison is internal to the programs themselves.

## Part XIII — Epistemic statuses

---

Every statement in this paper carries exactly one of five statuses:

Assumed — the axioms N1-N5.

Conditionally proved — T1-T23, valid under their stated hypotheses. Where a hypothesis is an external result (Dirac's equation in T19), it is named in the hypothesis. T20-T22 are identities of quaternion algebra; T23 is a linear map whose reading as Larson's dimensions is stated in Part VII.1.

Corresponded — C1-C6, declared identifications that can fail. I1-I4 are identifications of the same kind.

Predicted — P1-P5, with pre-commitments.

Open — O1-O7, prioritized; the reduction claim remains a working conjecture.

By T7.2 the status system itself gains a small foundation: readout at finite depth is well-defined without external convention. The paper's honesty rule — say at which depth you are reading — is now backed by arithmetic.

---

## Appendix A — The Origin of the Chain

---

The steps of Part 0 appeared in this order in the author's published texts on constable.blog.

| Step                                                                 | First appearance | Text                                                                                                                                           |
|----------------------------------------------------------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| Void $\rightarrow$ distinction $\rightarrow -1, 0, +1$ with sum zero | 2008-2009        | "About the Void" (1-12-2008); "About the Logic of Creation" (12-10-2009); "About Dark Matter" (27-1-2009)                                      |
| Closure: "Closed in Itself"                                          | 2008-2009        | "Why Numbers are Waves" (3-9-2008); "About the Logic of Creation"                                                                              |
| The same structure at every level                                    | 2008             | "About the Trinity" (29-12-2008); $\times 3$ per depth as $\Delta \rightarrow \Delta \times \Delta$ in "About Number and Magnitude" (9-1-2012) |
| Two ladders, $\times 2$ and $\times 3$                               | 2011             | "About the Sum of Things" (22-9-2011); "About the Vedic Square" (19-12-2011)                                                                   |

| Step                                              | First appearance | Text                                                                                                                                                 |
|---------------------------------------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| Larson's reciprocity of space and time            | 2012             | "About Number and Magnitude"; units in "De BewegingsRuimte van Dewey Larson" (29-12-2024)                                                            |
| Rowlands' nilpotence; rewriting from nothing      | 2020             | "The Simple Geometry of the Big Transformation" (7-12-2020); "Behind the Wall of the Aether lies Another Universe" (3-10-2021)                       |
| Maxwell's quaternions, later replaced by vectors  | 2021             | "Behind the Wall of the Aether lies Another Universe"; "The Universe Lives For Ever" (30-8-2023)                                                     |
| Balanced ternary beside Rowlands                  | 2022             | "About Antifragile Design" (19-11-2022); radix economy in "De Verborgen Boodschap van de Sri-Yantra" (3-5-2025)                                      |
| The toroidal photon, $4\pi$ , harmonics $f_0/3^k$ | 2025             | "Het Universum Organiseert Zichzelf" (26-10-2025); "De Vele Mogelijkheden van het Licht" (4-11-2025)                                                 |
| Selection by locking; constants as selected modes | 2025             | "The Resonant Universe" (15-11-2025)                                                                                                                 |
| $\Psi^2 = 0 \Rightarrow E^2 = p^2 + m^2$          | January 2026     | "The Dual Space Foundation of Consciousness" (11-1-2026); "Strategic Insights into Unified Field Theory" (12-1-2026)                                 |
| Mark, re-entry, eigenform $O(A) = A$              | January 2026     | "Louis H. Kauffman: A Complete Vision of Reality" (13-1-2026); $O(A) = A$ in "Exploring Knot Theory: Kauffman's Vision and Applications" (13-1-2026) |
| The self-rewriting net                            | February 2026    | "Het universum als zichzelf herschrijvend topologisch netwerk" (17-2-2026)                                                                           |
| Why three and not two                             | May 2026         | "Licht voordat licht gemaakt werd" (10-5-2026)                                                                                                       |
| One strand, N1-N5                                 | July-August 2026 |                                                                                                                                                      |

| Step                              | First appearance       | Text                                                                                                                                                                                                              |
|-----------------------------------|------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                   |                        | one strand in "The Strand That Knots Itself" (9-7-2026); N1-N5 in Foundational Paper, Edition 5 ("De Fundamenten van de Vacuum.Net Theorie", 6-8-2026)                                                            |
| Window rule mod 3; closure depths | September-October 2026 | "Closure Is Zero Force" (PDF, 23-9-2026); near-closures 3, 8, 19, 65, 84 in "The Comma in the Net" (2-9-2026); the denominators 1, 2, 5, 12, 41, 53, 306 in "The Climate on the Ladder of Three" (PDF, 3-10-2026) |
| y as a state function             | October 2026           | "The Vacuum.Net Theory: A Formal Statement" (16-10-2026), Theorem 11; "What the Path Gives" (PDF, 16-10-2026)                                                                                                     |

All texts are at <https://constable.blog> under their dates.

## Appendix B — Record of the Checks

Every theorem and every number of this edition was checked by independent judges, each with its own code, on ranges fixed before running, in three rounds on 17 October 2026. Exact arithmetic (integers, fractions, sympy) was used wherever the values are rational.

| Item        | What was computed                                                       | Range or method                                                      | Result                                                                                 |
|-------------|-------------------------------------------------------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| III.1-III.2 | $r/\ln r$ ; balanced-ternary facts (a)-(d); digit sets of radix $\pm 3$ | all 3-digit sets in $[-4, 4]$ ; strings to length 11                 | agrees; only $\{T, 0, 1\}$ has both properties                                         |
| T7          | existence and uniqueness of balanced ternary                            | all 177,146 strings of length $\leq 11$ ; Euler's product to $K = 8$ | agrees                                                                                 |
| T1, T8      | Fox three-colourings; Kauffman bracket; Jones polynomial                | all Gauss words to 3 crossings; state sum over 8 states              | unknot 3, trefoil 9; $\langle K \rangle = A^{-7} - A^{-3} - A^5$ ; $V = t + t^3 - t^4$ |
| T2          | quaternion double cover                                                 | $10^4$ random unit quaternions                                       | deviation $\leq 2 \times 10^{-15}$                                                     |
| T3          |                                                                         |                                                                      | equals pq                                                                              |

| Item       | What was computed                                                                 | Range or method                                                             | Result                                                                     |
|------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------|----------------------------------------------------------------------------|
|            | Hopf invariant as a linking number                                                | maps $[z_1^p : z_2^q]$ , coprime $p, q \leq 3$                              |                                                                            |
| T4-T6      | constitutive identity; sign theorem; entropy production                           | symbolic; 735 exact-rational cases; $10^4$ random positive semidefinite $L$ | agrees                                                                     |
| T9-T14     | window counts; trit cube; closure probability; closed rings; rewrite fixed points | exhaustive; rings $n = 3-13$ ; dynamics $n = 3-10$                          | agrees                                                                     |
| T15-T17    | records of $\ k \cdot \log_2 3\ $ ; comma; residues                               | $k \leq 10^6$ , exact integer arithmetic                                    | 1, 2, 5, 12, 41, 53, 306, 665, 15601, ...; 1.013643                        |
| T18        | Watson integrals; cumulants; random walks                                         | 30 digits; symbolic; $10^6$ walks of $10^4$ steps in $d = 3$                | $P_3, P_4, P_5$ as printed; $-3.939 \pm 0.013$ against exact $-3.961$      |
| T19        | $Q^2$ and its three cross terms                                                   | $4 \times 4$ matrices; 1000 random points on and off the shell              | agrees to $10^{-14}$                                                       |
| T20-T22    | $4c$ identity; pure-quaternion identities; ring measure                           | symbolic; $10^3$ random triples                                             | agrees; $R_Q = 0.827$ in the example                                       |
| T23, VII.2 | bridge map; conversion units; $9S_0$                                              | Smith normal form; 40-digit decimals                                        | agrees; 410.29 nm                                                          |
| I3, P1, P3 | $a_0$ ; $\Delta\alpha/\alpha$ ; solar edge                                        | stated inputs                                                               | $1.08 \times 10^{-10} \text{ m s}^{-2}$ ; $3.08 \times 10^{-8}$ ; 7,410 AU |
| Appendix A | dates and content of the cited posts                                              | WordPress search on constable.blog                                          | 13 rows confirmed                                                          |

The judges' reports and code are kept with this edition.

## Annotated references

Leibniz, G.W. (1686), Discours de métaphysique, §5-6. The design principle: maximal richness from minimal means; comprehensibility as consequence. The starting point of Part III.1.

Chaitin, G. (2005), Meta Math! The Quest for Omega; (2006), "The Limits of Reason", Scientific American 294(3). Reads Leibniz as proto-algorithmic information theory: a

theory is a program shorter than its data. Supplies the comparison metric of Parts III.1 and XII.

Stifel, M. (1544), *Arithmetica Integra*, p. 38. An early appearance of balanced ternary. The notation predates the theory by five centuries.

Knuth, D. (1997), *The Art of Computer Programming*, Vol. 2, pp. 195–213. Balanced ternary arithmetic; truncation-rounding equivalence; "perhaps the symmetric properties ... will prove to be quite important some day."

Andrews, G.E. (2007), "Euler's *De Partitio numerorum*", *Bull. AMS* 44(4), 561–573. Euler's use of the unique-representation theorem behind T7(ii).

Hayes, B. (2001), "Third Base", *American Scientist* 89(6). Accessible survey of ternary systems, radix economy, and the balance-scale reading.

Krinitzky, N.A., Mironov, G.A. & Frolov, G.D. (1963), "Programmno-upravlyaemaya mashina Setun", Chapter 10 of *Programmirovaniye*, ed. M.R. Shura-Bura, Fizmatgiz, Moscow. The balanced-ternary computer of Brusentsov and Sobolev. Hardware precedent for the MAZE kernel.

Ma, S. et al. (2024), "The era of 1-bit LLMs: all large language models are in 1.58 bits", [arXiv:2402.17764](https://arxiv.org/abs/2402.17764). Modern neural networks on the alphabet  $\{-1, 0, 1\}$ . The trinity alphabet is current engineering practice.

Lelli, F., McGaugh, S.S. & Schombert, J.M. (2016), *SPARC*, *AJ* 152, 157; McGaugh, Lelli & Schombert (2016), *PRL* 117, 201101; Milgrom, M. (1983), *ApJ* 270, 365; Chae, K.-H. et al. (2020), *ApJ* 904, 51. The measured acceleration threshold, the radial-acceleration relation, and the external-field effect: the data side of C5, C5a, I3, and P2.

Konstapel, J. (2026), working papers of 9 July 2026 and *The Assumptions Nobody Declares* (26 July 2026). The strand programme and the assumptions inventory that Part III.2 draws on.

New in Edition 8:

Spencer-Brown, G. (1969), *Laws of Form*, Allen & Unwin. The mark of distinction, the laws of calling and crossing, and re-entry. The ground of Part II. Reading advice: chapters 1–2 and 11.

Kauffman, L.H. (1987), "State models and the Jones polynomial", *Topology* 26(3), 395–407. The bracket and its state sum, behind T8.

Kauffman, L.H. (2005), "EigenForm", *Kybernetes* 34(1/2), 129–150. The eigenform  $O(A) = A$  as the fixed point of observation; the formal side of N2.

Przytycki, J.H. (1998), "3-coloring and other elementary invariants of knots", *Banach Center Publications* 42, 275–295. The link between three-colourings and the Jones polynomial in Part II.3.

Lickorish, W.B.R. and Millett, K.C. (1986), "Some evaluations of link polynomials", *Commentarii Mathematici Helvetici* 61, 349–359. Evaluations of the Jones polynomial at roots of unity.

Jones, V.F.R. (1985), "A polynomial invariant for knots via von Neumann algebras", *Bulletin of the AMS* 12(1), 103–111. The polynomial that T8 computes.

Reidemeister, K. (1927), "Elementare Begründung der Knotentheorie", *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* 5, 24–32. The three moves: the grammar of knot equivalence.

Dirac, P.A.M. (1928), "The Quantum Theory of the Electron", *Proceedings of the Royal Society A* 117, 610–624. The input of Part V.

Rowlands, P. (2007), *Zero to Infinity: The Foundations of Physics*, World Scientific. The nilpotent Dirac operator and the rewrite system. Reading advice: the chapters on the nilpotent Dirac equation.

Hamilton, W.R. (1853), *Lectures on Quaternions*, Dublin. The operator  $\nabla$  and the one product with two parts.

Maxwell, J.C. (1873), *A Treatise on Electricity and Magnetism*, Clarendon Press, Vol. II, Arts. 591–619. The grouped equations in quaternion form, the source of Part VI.

Larson, D.B. (1959), *The Structure of the Physical Universe*, North Pacific Publishers, Portland. Space and time as the two aspects of motion; the dimensions of Part VII.

Hopf, H. (1931), "Über die Abbildungen der dreidimensionalen Sphäre auf die Kugelfläche", *Mathematische Annalen* 104, 637–665. The invariant behind T3.

Fox, R.H. and Crowell, R.H. (1963), *Introduction to Knot Theory*, Ginn; Springer GTM 57 (1977). Three-colouring, the rule of T1 and T10. Reading advice: Chapter VI, Exercise 6 (p. 92).

Khinchin, A.Ya. (1964), *Continued Fractions*, University of Chicago Press. Best approximations and Lagrange's theorem, behind T16. Reading advice: Chapter II, §6.

Hardy, G.H. and Wright, E.M. (2008), *An Introduction to the Theory of Numbers*, 6th ed., Oxford University Press. Kronecker's theorem (Theorem 439), behind T15.

Pólya, G. (1921), "Über eine Aufgabe der Wahrscheinlichkeitsrechnung betreffend die Irrfahrt im Straßennetz", *Mathematische Annalen* 84, 149–160. Return of the random walk; the basis of T18.

Watson, G.N. (1939), "Three triple integrals", *Quarterly Journal of Mathematics (Oxford Series)* 10, 266–276.  $P_3 = 0.340537$  in T18.

OEIS A086232 and A086233. The return probabilities  $P_4 = 0.1932017$  and  $P_5 = 0.1351786$  in T18.

de Groot, S.R. and Mazur, P. (1962), *Non-Equilibrium Thermodynamics*, North-Holland. Entropy production as  $\Sigma J \cdot X$  and the Onsager relations used in T6.

Tiesinga, E. et al. (2021), "CODATA recommended values of the fundamental physical constants: 2018", *Reviews of Modern Physics* 93, 025010. The inputs of Part VII.2.

Williamson, J.G. and van der Mark, M.B. (1997), "Is the electron a photon with toroidal topology?", *Annales de la Fondation Louis de Broglie* 22(2), 133–160. The electron of C4 and I1.

Edition 8 status: the derivations are written out; O1–O7 are open.