

What the Path Gives and What Its Description Gives

Working through open problem O1 of the Vacuum.Net theory

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Abstract

Open problem O1 asks for three things to come out of coarse-graining the knot network: the lever $\gamma = 2\pi$, the sign of the coupling g_2 , and the reason for “four”, the 4π of a closing rotation.

This essay works the problem through in five steps. Each step comes with its rule and its numbers, and every number was computed for this essay.

The work produces three results.

- **γ depends on the observer.** When the observer is a knot on the path, γ is exact: $\gamma = -(1+P)/P$. Here P is the probability that the free path returns to a point it has already visited. In three dimensions $P = 0.3405$, so $\gamma = -3.94$. γ is therefore a state function of the path’s freedom. It is not one universal number.
 - **Four belongs to the description.** A rotation is two reflections. The moving object closes after 2π , the rotor that describes it after 4π . Built from the path’s own turns, 98.4% of single loops through a knot close for the object only. 78.8% of double loops close for the rotor as well. The trit closure rule is blind to this distinction.
 - **The sign of g_2 follows from the choice of density.** It does not follow from the rules alone. $\gamma = +2\pi$ with $g_2 < 0$ needs one extra datum: what the knot counts. The axioms do not yet supply it.
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1. The problem

The coupling law of the Foundational Paper (v7) writes the closing energy of a winding as a function of a density n :

$$e(n) = (g_2/2) \cdot n^2 + (g_3/6) \cdot n^3$$

Around the mean density $\bar{n}$ this gives an effective coupling:

$$g_{\text{eff}} = \bar{g} \cdot (1 + \gamma \cdot \delta), \text{ with } \bar{g} = g_2 + g_3 \cdot \bar{n}, \gamma = g_3 \cdot \bar{n} / \bar{g}, \delta = (n - \bar{n}) / \bar{n}$$

It follows that $g_2 = \bar{g} \cdot (1 - \gamma)$. If $\gamma = 2\pi$, then g_2 is negative and the relative change of the coupling is $\Delta g/g = 2\pi \cdot \delta$.

O1 asks where $\gamma = 2\pi$ comes from. The law itself leaves n undefined, so O1 also asks what n is. Third, O1 asks why closing takes 4π and not 2π .

The axioms available for the answer are these five:

- **N1 Strand.** There is one strand. Every distinction is the strand touching itself.
- **N2 Closure.** What closes in phase stays.
- **N3 Memory.** What closed is carried forward.
- **N4 Context.** Constants are state functions.
- **N5 Scale.** The same winding occurs at every depth.

The register is the trit, $+1$, 0 or -1 . A window is three trits. It closes when their sum is $0 \bmod 3$. Of the 27 possible windows, 9 close.

2. How γ is read from fluctuations

γ can be measured without knowing g_2 and g_3 beforehand. The method uses the fluctuations of the density.

Suppose the density n fluctuates around $\bar{n}$ with weight $\exp(-e(n))$. Write $x = n - \bar{n}$. Then:

$$-\ln P(x) = (a/2) \cdot x^2 + (b/6) \cdot x^3 + \dots$$

The coefficients are the second and third derivatives of e at $\bar{n}$:

$$a = e''(\bar{n}) = g_2 + g_3 \cdot \bar{n} = \bar{g} \quad b = e'''(\bar{n}) = g_3$$

For a weakly skewed distribution the cumulants are $\kappa_2 = 1/a$ and $\kappa_3 = -b/a^3$. Hence:

$$-\kappa_3/\kappa_2^2 = b/a = g_3/\bar{g}$$

Multiplying by $\bar{n}$ gives γ :

$$\gamma = -\bar{n} \cdot \kappa_3/\kappa_2^2$$

This is the estimator used throughout. It needs only the mean, the variance and the third cumulant of the density that the observer counts.

A reference case. Let the counted density be a fraction of independent sites, each counted with probability p . Over a stretch of ℓ sites, $\kappa_2 = p(1-p)/\ell$ and $\kappa_3 = p(1-p)(1-2p)/\ell^2$. Then:

$$\gamma = (2p - 1)/(1 - p)$$

The length ℓ drops out. Fully random trits counted as “non-zero” have $p = 2/3$, which gives $\gamma = 1$. For $\gamma = 2\pi$, p must equal $(1+2\pi)/(2+2\pi) = 0.879$.

3. First step: the trit lattice

The first model puts the closure rule on a cubic lattice:

- each site carries a trit;
- along each axis, three consecutive sites form a window;
- the weight of a configuration is $\exp(-H/T)$, with $H = -(\text{number of closed windows})$;
- the lattice size is $L = 9$ or 12 , with periodic boundaries.

The updates are Metropolis moves on 27 sublattices. Sites in one sublattice share no window.

The lattice has a sharp transition with memory.

- **Disordered side:** about 50% of the windows are closed. At $T = \infty$ the share is exactly $1/3$, which is 9 of 27.
- **Ordered side:** about 98% are closed.
- **Coexistence:** a lattice started half ordered and half random ends ordered at every T up to 1.52. The ordered side wins up to that point.

γ was measured on the disordered side for two densities.

T	non-zero trits, L=9	non-zero trits, L=12	closed windows, L=9	closed windows, L=12
1.60	8.4 ± 0.4	6.1 ± 1.8	-2.3 ± 1.4	-5.0 ± 0.8
1.55	7.4 ± 0.5	8.4 ± 2.2	-3.6 ± 0.9	-2.6 ± 1.5
1.50	7.5 ± 0.7	5.9 ± 0.8	-3.9 ± 1.1	-2.4 ± 1.7
1.475	6.7 ± 1.2	12.1 ± 1.1	-6.1 ± 1.2	-1.8 ± 1.7
1.45	—	9.1 ± 1.7	—	-6.3 ± 1.3

What the lattice shows.

- γ for non-zero trits rises from exactly 1 at $T = \infty$ to the range 6–8 near the transition.
- γ for closed windows has a similar size but the opposite sign.

The size lies in the neighbourhood of 2π . The value does not settle, and the sign depends on what is counted.

What the lattice puts in from outside. Three elements of this model are not part of the net:

- the temperature T , a bath that shakes the lattice;
- the ensemble of configurations, from which κ_2 and κ_3 are taken;
- the weight H , which was chosen, not given.

The next step removes all three.

4. Second step: who observes

N1 leaves no room for anything outside the strand. The observer must be part of it. Two older sources say the same in their own terms.

Grassmann (1844). He defines an extended form as the totality of all elements through which a generating element passes under continuous change (Einleitung §9). The form is the path of a motion. A law of development is required for the product to be definite (§10); in the net, that law is the closure rule. Grassmann removed spatial intuition from the foundations. A segment is a difference of two points, $b - a$, with no origin. A fixed frame is not needed.

Larson (1959 onwards). He separates two reference systems.

- **The natural reference system.** In it the primary motions cause no change of position. It progresses at unit speed, the speed of light. Its datum is unity, not zero.
- **The stationary spatial reference system.** This is the observer's frame. In it the progression appears as expansion.

Scalar motion has no direction of its own. A direction appears only when the motion is coupled to a point treated as fixed. Such a point is a displacement from unit speed: an inward unit set against the outward progression. In the language of the net it is a knot.

The reading used from here on.

- The net is the path that the motion follows: Larson's natural reference system and Grassmann's generated form.
 - The observer is a knot on that path.
 - At a knot the observer sees every passage that meets there.
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5. Third step: the path with nothing outside

The rules.

- The path takes N unit steps on the cubic lattice. Each step is one of six directions, with equal weight, because nothing outside the path prefers one.
- A **knot** is a site the path passes $k \geq 2$ times.
- For each pair of passages it sees, the observer compares the outgoing steps. Same direction gives $+1$, opposite gives -1 , perpendicular gives 0 . The trit thus compares the path with itself, not with an axis.
- Each pair gives an event at both of its passage times. Events are ordered along the path.
- A **window** is three consecutive events. It closes when its trit sum is $0 \bmod 3$.

- **What closes stays:** an event stays when it lies in at least one closed window.

There is no temperature, no weight and no ensemble.

The numbers follow from the rules. Each one below was checked against 16 paths of 4 million steps.

- **Trit shares.** Given one outgoing step, the other is the same with probability $1/6$, opposite with $1/6$, and perpendicular with $4/6$. Measured: 0.667, 0.167, 0.167.
- **Knots per step.** A free walk in three dimensions returns to a visited site with probability $P_3 = 0.3405$ (Pólya). New sites appear at rate $1 - P_3 = 0.660$ per step, and a share P_3 of them is visited again. That gives $0.660 \times 0.3405 = 0.225$ knots per step. Measured: 0.224.
- **Window closure.** For independent trits with these shares, the probability that a window closes is $(1 + 2 \cdot (1/2)^3)/3 = 5/12 = 0.417$. Here $1/2$ is the value of $2/3 + \omega/6 + \omega^2/6$ at $\omega = e^{(2\pi i/3)}$. Measured along the path: 0.424. The small excess comes from correlations along the path.
- **What stays.** 72% of the events lie in a closed window.

γ along the path. The observer is now carried along the path, and stretches of length ℓ play the role of the ensemble.

ℓ	staying events per step	fraction of events that stay
1	-1.282 ± 0.001	$+1.718 \pm 0.002$
16	-2.011 ± 0.006	$+3.072 \pm 0.004$
256	-3.61 ± 0.04	$+3.43 \pm 0.07$
1024	-4.91 ± 0.11	$+3.86 \pm 0.26$
4096	-6.71 ± 0.37	$+3.3 \pm 1.2$
16384	-9.8 ± 0.9	—

What the path shows.

- For independent counts, γ would not change with ℓ (section 2). Here it does. That is the signature of the path's long memory: returns can come after any delay.
- Staying events per step have no fixed point. They pass through -2π near $\ell \approx 3000$ and keep growing.
- The staying fraction levels off at about 3.4 to 3.9.

Seen from a moving observer, γ is therefore a function of the observer's span ℓ .

6. Fourth step: Larson's displacement as n

v_7 leaves n open. Larson gives a candidate: the speed displacement, counted in units. For an observer on a knot the natural count is the number of passages it sees, minus one:

$$n = k - 1$$

The derivation. A free path in d dimensions returns to a visited site with probability P_d . Each return is independent of the ones before. The number of visits K to a site is therefore geometric: $P(K = k) = P_d^{k-1} \cdot (1 - P_d)$. For knots, $n = K - 1 \geq 1$ has the same geometric law with ratio P_d .

Write $q = 1 - P_d$. For a geometric variable starting at 1:

- the mean is $1/q$;
- the variance is $(1 - q)/q^2$;
- the third cumulant is $(1 - q)(2 - q)/q^3$.

Inserting these in $\gamma = -\bar{n} \cdot \kappa_3 / \kappa_2^2$ gives:

$$\gamma = -(1 + P_d)/P_d$$

Check. For $d = 3$, $P_3 = 0.340537$, so $\gamma = -1.3405/0.3405 = -3.937$. Measured over 7.2 million knots: -3.946 ± 0.006 . Restricted to knots where every event stays, the value is -7.57 ± 0.02 . Selection by closure doubles the size.

d	P _d	$\gamma = -(1+P_d)/P_d$
3	0.3405	-3.94
4	0.1932	-6.18
5	0.1352	-8.40

What follows.

- γ is fixed by the freedom of the path: how often it returns, and so how many directions it has. This is N^4 in exact form. The constant is a state function of the path's depth.
- With $\gamma < 0$, $g_2 = \bar{g} \cdot (1 - \gamma) = \bar{g} \cdot (1 + 2P_d)/P_d$. That is positive, so there is no cancellation.
- In four dimensions $|\gamma| = 6.18$, 1.7% below 2π . The numbers lie close together. They are not equal.

7. Fifth step: why four

The algebra. The algebra built on Grassmann's extension theory writes a rotation as two reflections.

- A reflection of v in the plane normal to a is $v \rightarrow -a \cdot v \cdot a$.
- A second reflection, in b , gives $v \rightarrow b \cdot a \cdot v \cdot a \cdot b = R \cdot v \cdot \tilde{R}$, with $R = b \cdot a$.
- If a and b enclose the angle $\theta/2$, then v turns by θ .
- The rotor is $R(\theta) = \cos(\theta/2) - B \cdot \sin(\theta/2)$, where B is the plane of the turn.

Hence:

- **the object v** returns after 2π ;
- **the rotor R** equals -1 after 2π and returns to $+1$ only after 4π ;
- R and $-R$ move v identically. Seen from the object, they cannot be told apart.

Four is $2 \times 2\pi$: two reflections, each counted once. In the trit cube the same fact appears as two closing cells per rotation. $(1, -1, 0)$ and $(-1, 1, 0)$ both give $-e_{12}$.

The test on the path. Each turn of the path, from a step a to a perpendicular step b , is a rotation by $\pi/2$ about $a \times b$. Its rotor is $\cos(\pi/4) + \sin(\pi/4) \cdot (a \times b)$. This is built from the path's own successive directions only.

Around a loop from a knot back to the same knot, the rotors multiply to Q , including the closing turn. When $Q = +1$, the loop closes for the rotor. When $Q = -1$, it closes for the object but not for the rotor. Loops containing a reversal are left out, because a 180-degree turn has no defined axis. The sample is 4 paths of 1 million steps.

loops through the knot	loops with $Q = \pm 1$	share with $Q = +1$
one	104,590	1.6%
two	1,238	78.8%
three	29	31%
events that stay	63,249	2.5%
events that do not stay	42,612	2.6%

What the path shows.

- A single loop through a knot almost always closes for the object only: $Q = -1$ in 98.4% of cases. The simplest such loop is a square. Its tangent turns once, through 2π , and the rotor flips sign.
- Two loops through the same knot restore the rotor in most cases. The sign then alternates with the number of loops.
- The closure rule does not distinguish the two classes: 2.5% against 2.6%.

“Four” is therefore a property of the description of the path. It follows from the path's own turns, and the trit closure does not touch it. The knot needs two passages of a 2π loop before its rotor is back.

8. What O1 now contains

Part of O1	Status	Basis
Estimator $\gamma = -\tilde{n} \cdot K_3 / K_2^2$	derived	section 2
Observer as a knot on the path	adopted reading	N1, Grassmann, Larson; section 4

Part of O1	Status	Basis
$\gamma = -(1 + P_d)/P_d$ for Larson's n	derived and checked	section 6
γ as a state function of the path's freedom	follows from the above	N4; sections 5-6
$4\pi = \text{two reflections} \times 2\pi$	derived	section 7
Rotor class independent of trit closure	measured	section 7
$\gamma = +2\pi$ and $g_2 < 0$	not derived from N1-N5	sections 3, 5, 6
4π closure from the axioms alone	hypothesis	section 7

The coupling law. v_7 needs $\gamma = +2\pi$. With the five axioms and the observer on the path, γ comes out as $-(1 + P_d)/P_d$, a negative number fixed by the path's freedom. $\gamma = +2\pi$ requires one further datum: what the knot counts as n .

Two possible routes, both untested:

- **A density that grows when the path closes.** In the lattice this was the density of non-zero trits, and it gave positive γ near 6-8.
- **A selection that ties n to the rotor class.** The rotor class is the one place where 2π enters the path itself.

The four. "Four" needs no extra datum. It follows from the path describing its own turning. The knot is where that description is read, which is why the observer had to be placed there.

The image. In the fishnet image of the vacuum, the net is the trace of one moving thread. A knot is where the thread meets itself. The observer sits in the knot and sees the strands that pass through it. What it counts sets γ . How the thread turned on its way back sets the four.

9. How the numbers were produced

All runs used Python with NumPy.

- **Lattice:** L^3 sites, trits in $\{0, 1, 2\}$. Metropolis on 27 sublattices. Coexistence runs started with half the lattice set to 0 and half random, for 1500 sweeps. γ runs used 8,000 to 10,000 measurement sweeps after 500 sweeps of thermalisation, with errors from 5 blocks.
- **Path:** independent uniform steps in six directions. Knots are found by sorting positions. Trits come from the outgoing steps at the two passages. Events are sorted by time, and windows are taken cyclically.
- **γ along the path:** 16 paths of 4×10^6 steps, cut into stretches of length ℓ , with errors from 16 blocks.

- **Larson's n:** 8 paths of 4×10^6 steps.
 - **Rotors:** prefix products of unit quaternions along the path. A loop's Q is $P(j) \cdot P(i+1)^{-1}$, multiplied by the closing turn. Loops with reversals are excluded.
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Annotated references

Grassmann, H. (1844). *Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik*. Leipzig: Wigand. The source of the extended form as the trace of a generating element (Einleitung §§7–10). It also frees the theory from spatial intuition (Vorrede). Read the Einleitung; it is short and states the whole programme. Available at archive.org.

Clifford, W. K. (1878). Applications of Grassmann's extensive algebra. *American Journal of Mathematics* 1, 350–358. Joins Grassmann's exterior product with Hamilton's quaternions. This is where the product of two vectors becomes a rotor. Read it for the step from extension to rotation.

Hestenes, D. (1966). *Space-Time Algebra*. New York: Gordon and Breach. The modern statement of rotation as $R \cdot v \cdot \tilde{R}$ with half-angle rotors, and of the sign change of R after 2π . Read the chapters on rotations for section 7.

Larson, D. B. *Outline of the Deductive Development of the Theory of the Universe of Motion*. In: The Reciprocal System of Theory collection, archive.org. Larson's postulates in numbered form. Items 9–13 give the natural reference system. Items 25–27 explain how scalar motion acquires direction from its coupling to a reference point. Items 59–62 define speed displacement and its limit of two units per dimension. Read it whole; it is compact.

Nehru, K. V. K. *On the Nature of Rotation and Birotation*. In: The Reciprocal System of Theory collection, archive.org. Argues that the conventional reference system cannot tell θ from $\theta + 360^\circ$, and so imputes a phase. Read it for the observer's side of the $2\pi/4\pi$ question.

Nehru, K. V. K. *Birotation and the Doubts of Thomas*. Same collection. Shows that a balanced pair of opposite rotations carries no net angular momentum. Read it with the previous paper.

Pólya, G. (1921). Über eine Aufgabe der Wahrscheinlichkeitsrechnung betreffend die Irrfahrt im Straßennetz. *Mathematische Annalen* 84, 149–160. Proves that a free lattice walk returns with certainty in one and two dimensions, and with probability below one in three or more. The basis of section 6.

Watson, G. N. (1939). Three triple integrals. *Quarterly Journal of Mathematics* 10, 266–276. Evaluates the integral that gives $P_3 = 0.3405$. Read it for the exact constant.

Montroll, E. W. (1956). Random walks in multidimensional spaces, especially on periodic lattices. *Journal of the SIAM* 4, 241–260. Gives the return

probabilities in four and more dimensions, including $P_4 = 0.1932$ and $P_5 = 0.1352$. Read it for the table in section 6.

Konstapel, J. (2026). *Larson, Grassmann en de Vacuum.Net-theorie*. constable.blog, 27 September 2026. Maps strand, winding, knot and mesh onto Larson's S/T, T/S and T^3/S^3 and onto Grassmann's grades. Read it for the dictionary used in section 4.

Konstapel, J. (2026). *De Kubus Waarin de volledige Natuurkunde Past*. constable.blog, 2 October 2026. The trit cube as the combinatorial skeleton of the Clifford algebra of space, with 9 closing cells and the double cover. Read it for the cube form of section 7.

Konstapel, J. (2026). *Foundational Paper, version 7*. States the coupling law, theorem T5 and open problem O1 in their original form. Read it for section 1.