

What Three Adds

Phase-Encoded Ternary Light in Symmetric Couplers and Counter-Coupled Resonator Chains

J. Konstapel, Constable Research, Leiden

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Abstract

Silicon nitride photonics is moving from laboratory to foundry, with a dedicated plant under construction in Enschede. It has been proposed as the carrier of an oscillator-based computing layer. This paper asks which properties of phase-encoded light in such a carrier belong to the number three, and which hold at every base.

Two systems are treated exactly. The first is a symmetric m -port coupler fed with m beams whose phases encode a window of m digits mod m . The second is a pair of rings of phase-quantised elements, each element reading its neighbours and its partner in counter-phase.

Five results follow, each proven in full.

1. A window sends all its light to one port if and only if it is an arithmetic progression. This holds at every m .
 2. Only at $m = 3$ do the windows that send all light to one port coincide with the windows whose digit sum vanishes.
 3. Only at $m = 3$ does every other window spread its light exactly evenly. The reason is $\cos(2\pi/3) = -1/2$.
 4. Two counter-coupled rings of length N over $\mathbb{Z}_3$ lock for exactly $3^{(3^v)}$ of the 3^N initial differences, with v the 3-adic valuation of N . The same law holds over every prime p . Continuous phases lock with probability zero.
 5. One million resonators at 1–10 GHz need 750–15,000 cm² of silicon nitride, more than a 300 mm wafer. A single time-multiplexed fibre loop has no such limit.
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1. Introduction

1.1 Silicon nitride as a platform

Stoichiometric silicon nitride is transparent from about 405 to 2350 nm. Its waveguides reach a propagation loss of 0.034 dB/m. The corresponding 1/e intensity length is $10 / (0.034 \cdot \ln 10) = 127.7$ m. Ring resonators reach an intrinsic quality factor of 7.2×10^8 at 1615 nm. The photon lifetime is then $Q \cdot \lambda / (2\pi c) = 0.62$ μ s.

Its thermo-optic coefficient is 2.45×10^{-5} K⁻¹, about seven times below that of silicon. At 1550 nm and group index 2.0, a ring resonance moves by $\lambda \cdot (dn/dT) / n_g = 19$ pm/K. Silicon, with $n_g = 4.2$, moves by 66 pm/K.

Silicon nitride does not emit light. Indium phosphide does. The two platforms are complementary. In the Netherlands they are being industrialised side by side, in Enschede and Eindhoven.

1.2 Occasion

On 19 October 2026 the Financieele Dagblad reported that the New Origin foundry in Enschede had resolved its grid and water constraints. The plant is a pure-play foundry for silicon nitride. Imec is transferring its LPCVD process to it. The investment is about €200 million. Opening is planned for 2027.

An oscillator-based computing architecture, Right-Brain AI, named this platform as its substrate in 2025. Its first layer encodes information in the phases of coupled optical modes. Its second phase specifies 10^6 to 10^9 evanescently coupled silicon nitride resonators per chip, each at 1 to 10 GHz. Its admission rule accepts a configuration only when a balanced sum vanishes.

That rule is ternary. It raises a precise question. When digits are written into optical phases in thirds of a turn, what does light do that it would not do with halves, quarters or fifths?

1.3 The ternary frame

The question comes from the Vacuum.Net theory. Its image is a fishnet stretched in water, with strands always under tension. Where strands wind around one another, knots form. Between them lie meshes.

At a crossing the strand passes over, under, or not at all: +1, -1 or 0. This is a trit. Three neighbouring crossings form a window (x, y, z). The window closes when $x + y + z \equiv 0 \pmod{3}$. Of the 27 windows, 9 close.

A window closes exactly when its three trits are all equal or all different. If $x = y \neq z$, the sum is $2x + z$, which vanishes only for $z \equiv x$, a contradiction. All equal gives $3x \equiv 0$. All different gives $0 + 1 + 2 \equiv 0$.

This paper does not assume the theory. It asks which of its ternary statements are properties of three, and which are properties of every base.

2. Symmetric couplers with phase-encoded input

2.1 Setting

Let $m \geq 2$. A window $t = (t_0, \dots, t_{m-1})$ has digits in $\mathbb{Z}_m$. It **closes** when $\sum t_j \equiv 0 \pmod{m}$.

Each digit sets the phase of one input beam of unit power: $a_j = \omega^{(t_j)}$, with $\omega = e^{(2\pi i/m)}$. A lossless symmetric m-port coupler acts as the discrete Fourier transform:

$$A_k = m^{(-1/2)} \cdot \sum_j a_j \cdot \omega^{(jk)}, \quad P_k = |A_k|^2.$$

At $m = 3$ this device is a tritter. Integrated tritters have been fabricated and characterised with single photons.

A window is **concentrated** when one output carries power m . It is **uniform** when every output carries power 1.

Write $u_j(k) = t_j + jk \pmod{m}$. Then $\sqrt{m} \cdot A_k = \sum_j \omega^{(u_j(k))}$.

2.2 Three lemmas

Lemma 1 (power conservation). $\sum_k P_k = m$.

Proof. $\sum_k |A_k|^2 = m^{-1} \sum_{(j,l)} a_j \bar{a}_l \sum_k \omega^{((j-l)k)}$. The inner sum is m for $j = l$ and 0 otherwise. So the total is $\sum_j |a_j|^2 = m$. ■

Lemma 2 (global phase). Adding a constant c to every digit multiplies every A_k by ω^c . Powers and closure are unchanged, since the sum changes by $m\omega^c$. ■

Lemma 3. $\sum_{j=0}^{m-1} j = m(m-1)/2$. Modulo m this is 0 for odd m and $m/2$ for even m .

Proof. For odd m , $(m-1)/2$ is an integer. For even m , $(m/2)(m-1) \equiv -m/2 \equiv m/2$. ■

2.3 Theorem 1: single-port output

Theorem 1. For every $m \geq 2$, a window delivers power m to one output if and only if it is an arithmetic progression $t_j \equiv c + jd$. The output is $k \equiv -d$. All other outputs are dark. There are exactly m^2 such windows.

Proof.

1. $\sqrt{m} \cdot A_k$ is a sum of m unit phasors. Its modulus is at most m .
2. Equality holds only when all phasors are equal, that is when $u_j(k) \equiv c$ for all j .
3. This means $t_j \equiv c - jk$: a progression with $d \equiv -k$.
4. Conversely, a progression with step d gives $u_j(-d) \equiv c$ for all j , so $P_-(-d) = m$.
5. Lemma 1 then forces all other outputs to zero.
6. The map $(c, d) \mapsto (c + jd)_j$ is injective, since $t_0 = c$ and $t_1 - t_0 = d$. So there are m^2 progressions. ■

This is the equality case of the triangle inequality for the discrete Fourier transform. It holds at every base.

2.4 Theorem 2: concentration equals closure only at $m = 3$

Theorem 2. For odd m every progression closes. For even m exactly $m^2/2$ progressions close. The set of progressions equals the set of closing windows only for $m = 3$.

Proof.

1. A progression sums to $mc + d \cdot m(m-1)/2$.
2. By Lemma 3 this vanishes mod m for odd m . For even m it equals $dm/2$, which vanishes exactly for even d . Half the steps are even, so $m^2/2$ progressions close.
3. There are m^{m-1} closing windows: the first $m - 1$ digits are free and the last is fixed.
4. For even m some progression does not close, so the sets differ.
5. For odd m the sets are equal only if $m^2 = m^{m-1}$, that is $m = 3$. ■

m	Progressions	Closing windows	Closing progressions
2	4	2	2
3	9	9	9
4	16	64	8
5	25	625	25
6	36	7,776	18
7	49	117,649	49

Physical reading. A tritter is the only symmetric coupler whose bright single-port outputs mark exactly the closing windows. At every other base, closure and single-port output come apart. The equality at three rests on the count $m^2 = m^{m-1}$, not on the coupler.

2.5 Theorem 3: even spreading only at $m = 3$

Theorem 3.

1. At $m = 3$, every window that is not concentrated delivers power 1 to each output.
2. At every $m \geq 4$, the window $w = (0, \dots, 0, 1)$ is neither concentrated nor uniform.

Proof of 1. At $m = 3$, $3P_k = |\omega^{u_0} + \omega^{u_1} + \omega^{u_2}|^2$. Three cases arise.

- All exponents equal: 9, so $P_k = 3$.
- Exactly two equal: $|2 + \omega^{(\pm 1)}|^2 = 4 + 1 + 4\cos(2\pi/3) = 5 - 2 = 3$, so $P_k = 1$.
- All different: $1 + \omega + \omega^2 = 0$, so $P_k = 0$.

Every output power is therefore 0, 1 or 3. A window without a power-3 output has every

$P_k \leq 1$. By Lemma 1 they sum to 3. So each equals 1. ■

Proof of 2. The window w has $t_0 = t_1 = 0$, so a progression through it would have $d = 0$ and be all zeros. It is not, so by Theorem 1 it is not concentrated. At output 0:

$$m \cdot P_0 = |(m-1) + \omega|^2 = (m-1)^2 + 2(m-1)\cos(2\pi/m) + 1.$$

For $m \geq 4$, $\cos(2\pi/m) \geq 0$. Then $m \cdot P_0 \geq (m-1)^2 + 1 > m$, since $(m-1)(m-2) > 0$. So $P_0 > 1$. ■

At $m = 3$ the same expression gives $4 - 2 + 1 = 3$, so $P_0 = 1$. One number separates three from every larger base: $\cos(2\pi/3) = -1/2$. Two equal phasors and one shifted by a third of a turn always produce exactly one third of the total power.

A second reading. Every function from Z_3 to Z_3 is a polynomial of degree at most 2. A window at $m = 3$ that is not a progression is therefore quadratic. Quadratic windows spread evenly at every odd base:

Lemma 4. For odd m and $\gcd(a, m) = 1$, the window $t_j = aj^2 + bj + c$ is uniform.

Proof. $\sqrt{m} \cdot A_k = \omega^c \sum_x \omega^{(ax^2 + (b+k)x)}$. Since 2 and a are units mod m , complete the square. A shift of x permutes Z_m . So $|\sqrt{m} \cdot A_k|$ equals the modulus of the quadratic Gauss sum $G(a, m) = \sum_x \omega^{(ax^2)}$. For odd m with $\gcd(a, m) = 1$, $|G(a, m)|^2 = m$. So $P_k = 1$. ■

2.6 Theorem 4: uniform windows at odd primes

Theorem 4. For an odd prime p , a window is uniform if and only if it is quadratic with $a \neq 0$. The number of uniform windows is $U(p) = p^2(p-1)$.

Proof.

1. The output powers are the Fourier transform of the periodic autocorrelation $C(s) = \sum_x \omega^{(t(x+s) - t(x))}$. Uniform output means $C(s) = 0$ for all $s \neq 0$.
2. For prime p , a sum of p roots of unity of order p vanishes only if each appears exactly once. So $C(s) = 0$ exactly when $x \mapsto t(x+s) - t(x)$ is a bijection of Z_p .
3. A function whose every difference map is a bijection is planar. So uniform means planar.
4. On a prime field the planar functions are exactly the quadratic ones (Gluck; Hiramene; Rónyai and Szőnyi).
5. Lemma 4 gives the converse. Counting $a \in \{1, \dots, p-1\}$, b and c gives $p^2(p-1)$. ■

For $p \leq 13$ step 4 was also verified by exhaustive enumeration in integer arithmetic. The counts are 18, 100, 294, 1,210 and 2,028. Uniform windows are generalized bent functions, a known class.

2.7 Theorem 5: mirror symmetry

Theorem 5. Negating every digit, or reversing the window, maps the power at output k to output $-k$. At $m = 3$ output 0 is fixed and outputs 1 and 2 are exchanged.

Proof. Negation gives $A'_k = \text{conj}(A(-k))$. Reversal gives $A'_k = \omega^{((m-1)k)} A(-k)$. Both have modulus $|A(-k)|$. ■

2.8 The tritter as a sorter

At $m = 3$ the 27 windows fall into four classes:

Class	Windows	Output powers
Homogeneous closure	(0,0,0), (1,1,1), (2,2,2)	(3, 0, 0)
Mixed closure, one hand	(0,1,2), (1,2,0), (2,0,1)	(0, 0, 3)
Mixed closure, other hand	(0,2,1), (1,0,2), (2,1,0)	(0, 3, 0)
Open	the other 18	(1, 1, 1)

A passive tritter with three phase shifters and three detectors therefore sorts every trit

window in one pass. Closure is read as concentration. Its kind is read as the output index. The two cyclic orders, mirror images of one another, leave by different outputs. Open windows give a flat signal.

Worked example, window (0, 0, 1):

- Output 0: $1 + 1 + \omega = 1.5 + 0.866i$, squared modulus 3, $P_0 = 1$.
- Output 1: $1 + \omega + \omega^3 = 2 + \omega$, $P_1 = 1$.
- Output 2: $1 + \omega^2 + \omega^5 = 1 + 2\omega^2 = -1.732i$, squared modulus 3, $P_2 = 1$.

Theorems 1 and 5 hold at every base. Theorems 2 and 3 are what three adds.

3. Counter-coupled rings

3.1 Model

Two rings a and b of N phase-quantised elements carry digits in $\mathbb{Z}_3$. Indices run mod N . Both update at once:

$$a'_i = -(a_{i-1} + a_{i+1}) - k \cdot b_i, \quad b'_i = -(b_{i-1} + b_{i+1}) - k \cdot a_i \pmod{3}.$$

For $k = 2 \equiv -1$, each ring reads its partner in counter-phase. The rings **lock** when $a = b$.

This is an idealised discrete-time model. It describes oscillators whose phase is restricted to thirds of a turn and whose update is exact. Section 3.7 discusses what it implies for physical resonators.

3.2 The difference evolves alone

Let $d = a - b$. Subtracting the two rules gives $d'_i = -(d_{i-1} + d_{i+1}) + k \cdot d_i$. Only d appears.

Represent d as $\sum d_i x^i$ in $R = \mathbb{F}_3[x]/(x^N - 1)$. A shift is multiplication by x . Then $d' = P \cdot d$ with

$$P = k - x - x^{-1} = -x^{-1}(x^2 - kx + 1).$$

For $k = 2$ this is $P = -x^{-1}(x - 1)^2$: the cycle Laplacian $2 - x - x^{-1}$, read mod 3.

3.3 Theorem 6: the lock law

Theorem 6. Let $N = 3^v N'$ with $3 \nmid N'$, and $k = 2$. Exactly $3^{(3^v)}$ of the 3^N initial differences reach zero. All pairs lock if and only if N is a power of 3. The longest lock time is $\lceil 3^{v/2} \rceil$ steps.

Proof.

1. In characteristic three the Frobenius map is additive. So $(x^{N'} - 1)^{(3^v)} = x^N - 1$.
2. The derivative of $x^{N'} - 1$ is $N'x^{N'-1}$, nonzero since $3 \nmid N'$. So $x^{N'} - 1$ is square-free.
3. Since 1 is a root, $x^{N'} - 1 = (x - 1)Q$ with $Q(1) = N' \neq 0$. Hence $x^N - 1 = (x - 1)^{(3^v)} \cdot Q^{(3^v)}$, with coprime factors.
4. By the Chinese remainder theorem $R \cong A \times B$, with $A = \mathbb{F}_3[x]/((x - 1)^{(3^v)})$ and $B = \mathbb{F}_3[x]/(Q^{(3^v)})$.
5. On B , both x and $x - 1$ are units, so P is a unit. No nonzero difference in B reaches zero.
6. On A , $x - 1$ is nilpotent of index exactly 3^v . P is a unit times $(x - 1)^2$, so P^t vanishes on A once $2t \geq 3^v$.
7. The differences that reach zero are exactly A , of dimension 3^v . Their number is $3^{(3^v)}$.
8. $A = R$ exactly when $3^v = N$. ■

Worked examples:

- $N = 6$: $x^6 - 1 = (x - 1)^3(x + 1)^3$. 27 of 729 differences lock (3.7 %).
- $N = 9$: $x^9 - 1 = (x - 1)^9$. All 19,683 differences lock.
- $N = 10$: only the 3 constant differences lock, of 59,049.

The method is that of Martin, Odlyzko and Wolfram for additive cellular automata.

3.4 Theorem 7: lock time on the ladder

Theorem 7. Let $N = 3^k$ and let $\text{val}(d)$ be the largest j with $(x - 1)^j \mid d$. The lock time is exactly

$$T(d) = \max(0, \lceil (N - \text{val}(d))/2 \rceil).$$

The number of differences locking within t steps is $3^{\min(2t, N)}$. The bound $\lceil N/2 \rceil$ is reached by exactly the differences with nonzero digit sum: two thirds of all.

Proof. For $N = 3^k$, $R = F_3[x]/((x - 1)^N)$ has a single chain of ideals. Every nonzero d is a unit times $(x - 1)^{\text{val}(d)}$. Then $P^t d$ is a unit times $(x - 1)^{(2t + \text{val}(d))}$, which vanishes when $2t + \text{val}(d) \geq N$. Evaluation at $x = 1$ gives $\sum d_i$, so $\text{val}(d) = 0$ exactly when the digit sum is nonzero. ■

N	$\lceil N/2 \rceil$	Longest lock found
3	2	2
9	5	5
27	14	14
81	41	41
243	122	122

At $N = 9$ the cumulative counts for $t = 0, \dots, 5$ are 1, 9, 81, 729, 6,561 and 19,683.

3.5 Theorem 8: dependence on the coupling

Theorem 8. Over Z_3 :

- $k = 2$ gives $3^{(3^v)}$ locking differences for every N .
- $k = 1$ gives $x^2 - x + 1 = (x + 1)^2$. The count is $3^{(3^v)}$ for even N and 1 for odd N .
- $k = 0$ gives $x^2 + 1$, irreducible over F_3 . The count is $3^{(2 \cdot 3^v)}$ when $4 \mid N$ and 1 otherwise.

Proof. The argument of Theorem 6 applies to each irreducible factor of $x^2 - kx + 1$. A factor contributes only if it divides $x^{(N')} - 1$. For $k = 1$, $x + 1$ divides exactly when N is even. For $k = 0$, the roots of $x^2 + 1$ are the primitive fourth roots of unity in F_9 , present exactly when $4 \mid N$. ■

Reversing only the sign of the rewrite at $k = 2$ is equivalent to $k = 1$. On $N = 3^j$ only the zero difference then locks.

3.6 Theorem 9: every prime has its ladder

Theorem 9. Replace Z_3 by Z/p^e for any prime p and any $e \geq 1$, with $k = 2$. Exactly $p^{(e \cdot p^{(v_p(N))})}$ differences reach zero. All pairs lock if and only if N is a power of p .

Proof. Over F_p , $x^N - 1 = (x^{(N')} - 1)^{(p^v)}$ with $p \nmid N'$, as in Theorem 6. Hensel's lemma lifts this coprime factorisation to Z/p^e . P is nilpotent on the part belonging to $x - 1$ and a unit elsewhere. That part is free of rank p^v over Z/p^e . ■

Consequence. The ladder of three is the $p = 3$ case of a ladder for every prime. Phases in ninths ($Z/9$) lock on $N = 3^k$. Binary phases ($p = 2$) lock on $N = 2^k$. Two-phase parametric oscillators, as used in coherent Ising machines, would lock under this map on lengths 2^k .

3.7 Theorem 10: continuous phases

Apply the same rule to real phases mod 2π , with integer coupling 2. The difference now evolves under the real cycle Laplacian L on the torus T^N . Its eigenvalues are $2 - 2\cos(2\pi j/N)$. Only $j = 0$ gives zero.

Theorem 10. For every $N \geq 2$ the set E of initial differences that ever reach zero has measure zero. E is dense in T^N if and only if N is a prime power.

Proof of measure zero. The real kernel of L^m is the diagonal for every m . The differences reaching zero within m steps therefore lift to a countable union of lines in R^N . Lines are null for $N \geq 2$. A countable union of null sets is null. ■

Proof of density for $N = p^j$. By Theorem 9 every point of $(2\pi/p^e)Z^N$ reaches zero, for every e . These points are dense. ■

Proof of non-density otherwise. If N is not a prime power, the cyclotomic polynomial satisfies $\Phi_N(1) = 1$. So $x - 1$ is a unit modulo Φ_N . Let $y = (x^N - 1)/\Phi_N$. Then y lies in the image of L^m for every m . The character $d \mapsto \langle y, d \rangle \bmod 2\pi$ is nontrivial and vanishes on E . So E lies in a proper closed subgroup. ■

The set reaching zero within m steps has $N^{(2m-1)}$ connected components. This follows from Kirchhoff's theorem: the cycle has N spanning trees, so $\prod_{j \neq 0} \lambda_j = N^2$.

Physical reading. Exact lock under this map needs a phase alphabet of finite order. With continuous phases it occurs with probability zero. Physical resonators also relax toward a steady state rather than iterating a map. A lock law for real devices therefore needs both a discrete phase alphabet and a dynamics close to this map. Three stable phases arise in period-tripling oscillators. They have been measured in superconducting microwave resonators. In optics they have been proposed through three-photon down-conversion, but not yet shown in a time-multiplexed loop.

4. Carrier scaling

4.1 One chip

A ring of circumference L and group index n_g has free spectral range $c/(n_g \cdot L)$. With $n_g = 2.0$:

Target	Circumference
10 GHz	$2.998 \times 10^8 / (2.0 \times 10^{10}) = 1.5 \text{ cm}$
1 GHz	15 cm

A ring of radius $50 \mu\text{m}$ has $L = 314 \mu\text{m}$ and a free spectral range of about 477 GHz. On a $120 \mu\text{m}$ pitch such rings fill about 6,900 per cm^2 . One million need about 144 cm^2 .

Rings at 1–10 GHz must be spirals of 1.5–15 cm. At a waveguide pitch of 5–10 μm each occupies 7.5×10^{-4} to $1.5 \times 10^{-2} \text{ cm}^2$. One million need 750 to 15,000 cm^2 . A 300 mm wafer has 707 cm^2 ; a lithographic reticle about 8.6 cm^2 .

A specification of 10^6 – 10^9 resonators per chip at 1–10 GHz is therefore not met on one chip. The estimate assumes one oscillator per resonator and one photonic layer.

4.2 Four carriers

Carrier	Measured	Strength
Silicon nitride chip	Loss 0.034 dB/m; $Q \ 7.2 \times 10^8$; two rings synchronised through 20 m of fibre	Stable phase on small area
Fibre loop, time-multiplexed	100,512 parametric oscillators in one 5 km ring, phases 0 or π	Number set by length and rate
Free space	40,000 binary phases on a spatial light modulator, read through one lens	Parallel global transform

Ferrite cores	Setun, Moscow, 1958–1965: balanced ternary, 50 machines including prototypes	Ternary register without light
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A lens performs one global two-dimensional transform. Reading many trit windows in parallel would need a lenslet per window, readout at three diffraction orders, correction of the pixel envelope, fan-out of shared trits, and three phase levels.

5. Discussion

5.1 What three adds

Specific to three	Holds at every base or prime
Single-port outputs coincide with closing windows (Thm 2)	Single-port output = progression (Thm 1)
Every non-closing window spreads evenly (Thm 3)	Quadratic windows spread evenly at odd m (Lemma 4)
Dependence on $k = 1$ and $k = 0$ (Thm 8)	All pairs lock $\Leftrightarrow N$ is a power of p (Thms 6, 9)
Effect of reversing the rewrite (Sec. 3.5)	Measure zero and density for continuous phases (Thm 10)

The sharpest ternary result is Theorem 3. Only at three does every window that fails to close spread its light perfectly evenly. Every function on three points is quadratic, and quadratic phases are flat in Fourier space. Closure at three thus has an exact counterpart: what does not close prefers no output.

5.2 Relation to earlier statements

The register result R6 of *Beyond Energy* states that two counter-coupled rings lock only on lengths 3^k . Theorem 6 confirms it over $\mathbb{Z}_3$. Theorem 9 shows it is the $p = 3$ case of a law for every prime.

An earlier reading presented the tritter as a passive closure reader that demonstrates the ternary rule. Theorem 2 confirms that it reads closure exactly at $m = 3$. Theorem 1 shows that it does so through Fourier algebra valid at every base.

5.3 Device tests

One multi-project silicon nitride die can carry four structures.

Structure	Measures	Expected
Tritter with three thermo-optic phase shifters	Output powers for all 27 windows	Four classes of Section 2.8
Same, three-level versus four-level encoding	Spread of non-concentrated outputs	Flat only at three levels (Thm 3)
Chains of 9 and 10 coupled rings	Mutual phase after injection	Distinguishes discrete lock from relaxation
Ring pumped at 1231 nm	Third harmonic at 410 nm	Depth-3 to depth-2 conversion through $\chi^{(3)}$

5.4 People

A foundry that fabricates outside designs opens photonics to small teams. A student group, a small firm or a hospital department can have a chip made without owning a cleanroom. In Enschede, design, wafer and packaging lie within a few kilometres. The results above give such designers exact facts to build on, and a clear statement of where one chip ends.

6. Computational verification

All closed forms were checked numerically before publication. Predictions for each check were fixed and hash-sealed before the computation. Each count was reproduced by at least two independent programs, one of them in exact integer or cyclotomic arithmetic.

Result	Proof scope	Numerical check
Theorem 1	all $m \geq 2$	all windows $m = 2-9$; exact arithmetic in $\mathbb{Z}[\omega]$ $m = 2-9$; integer pair differences $m = 2-7$
Theorem 2	all $m \geq 2$	as above; $m = 10$ exact reduction; $m = 11, 12$ sampled; set counts $m = 2-50$
Theorem 3	all m	all windows $m = 3-9$; $m = 10$ exact reduction
Theorem 4	all odd primes (step 4 cited)	exhaustive planar enumeration $p = 3, 5, 7, 11, 13$
Theorem 5	all m	2,620,833 window-operation pairs, $m = 2-7$
Theorem 6	all $N \geq 2$	all 3^N differences $N = 2-16$ (43,046,721 at $N = 16$); raw pairs $N = 2-8$; rank over $\text{GF}(3)$ $N = 2-60$
Theorem 7	$N = 3^k$	exhaustive $N = 3, 9$; samples and nilpotency index $N = 27, 81, 243$
Theorem 8	all N	exhaustive $k = 0, 1, 2$, $N = 3-16$; rank $N = 2-40$
Theorem 9	all p, e	moduli 4, 8, 9, 12, 16, 25, 27, 49, 81, 125; F_2 and F_5 ranks $N = 2-19$
Theorem 10	all $N \geq 2$	Smith normal form $N = 3-16$; component counts $N = 3-12$, $m = 1-3$
Section 4.1	arithmetic	recomputed independently

The largest floating-point deviation in the output powers was 3.9×10^{-14} . Code and data are available on request.

Appendix: the tritter sorting theorem

A.1 Statement

At $m = 3$ the 27 phase-encoded trit windows fall into exactly the four classes of Section 2.8. At every $m \geq 4$ the closing windows differ from the single-port windows, and some non-closing window is not uniform.

A.2 Proof

1. By Theorem 1 a window is concentrated exactly when it is a progression $c + jd$, and its output is $-d$.
2. At $m = 3$ there are three kinds of progression. Step $d = 0$ gives the constant windows, to output 0. Step $d = 1$ gives the cyclic shifts of $(0, 1, 2)$, to output 2. Step $d = 2$ gives the cyclic shifts of $(0, 2, 1)$, to output 1.
3. These are the windows with all trits equal or all different, which by Section 1.3 are exactly the closing windows.
4. By Theorem 3(1) every other window has output powers $(1, 1, 1)$.
5. Counting: $3 + 3 + 3 + 18 = 27$.
6. For $m \geq 4$, Theorem 2 gives $m^{(m-1)} > m^2$ closing windows, and Theorem 3(2) gives the non-uniform window $(0, \dots, 0, 1)$. ■

A.3 Literature

The single-port law is the equality case of the triangle inequality for the discrete Fourier transform. Flat spreading of quadratic phases follows from Gauss sums. The classification of planar functions on prime fields is due to Gluck, Hiramane, and Rónyai and Szőnyi. No source was found that states the four-class sorting of trit windows by a tritter, or the coincidence of closing and single-port sets at $m = 3$. The search covered the tritter literature, multiport beam splitters, optical ternary logic, optical Potts machines and additive cellular automata.

Annotated references

Platform and occasion

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